INVERSE DYNAMICS OF THE AGILE WRIST SPHERICAL PARALLEL ROBOT

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Some iterative matrix relations for the kinematics and dynamics of a spherical Agile Wrist parallel robot are established. The manipulator prototype is a three-degree-of-freedom mechanical system with three parallel legs. Controlled by concurrent torques, which are generated by some electric motors, three active elements of the robot have three independent rotations. Supposing that the position and the rotational motion of the platform are known, an inverse dynamic problem is developed using a new matrix approach to compute the actuating torques. Finally, some recursive matrix relations and some graphs for the time-history of the torques are determined.

LIST OF SYMBOLS

$a_{k,k-1}$ – orthogonal relative transformation matrix

$\varphi_{k,k-1}$ – relative rotation angle of T_k rigid body

$\vec{\omega}_{k,k-1}$ – relative angular velocity of T_k

$\vec{\omega}_{k0}$ – absolute angular velocity of T_k

$\tilde{\omega}_{k,k-1}$ – skew symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$

$\vec{\varepsilon}_{k,k-1}$ – relative angular acceleration of T_k

$\tilde{\varepsilon}_{k0}$ – absolute angular acceleration of T_k

$\tilde{\varepsilon}_{k,k-1}$ – skew symmetric matrix associated to the angular acceleration $\vec{\varepsilon}_{k,k-1}$

$\vec{r}^{\,A}_{k,k-1}$ – relative position vector of the centre A_k of joint

$\vec{v}^{\,A}_{k,k-1}$ – relative velocity of the centre A_k

$\vec{\gamma}^{\,A}_{k,k-1}$ – relative acceleration of the centre A_k

m_k – mass of T_k rigid body

$\hat{J}_k$ – symmetric matrix of tensor of inertia of T_k about the link-frame $A_k x_k y_k z_k$

$m_{q,q-1}$ – torque of the actuator T_{q-1}

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Rev. Roum. Sci. Tech. – Méc. Appl., Tome 52, No. 1, P. 27–38, Bucarest, 2007

1. INTRODUCTION

Compared with serial mechanisms, parallel robot is a spatial mechanical structure, behaving special characteristics such as: greater structural rigidity, larger load carrying capacity, better positioning accuracy, stabile functioning, control on the limits of velocities or accelerations and a lower workspace volume also. Parallel manipulators have a robust construction and they can move bodies of considerable masses and dimensions with high speeds and accelerations. These mechanisms consist of two main bodies coupled via numerous legs. One body is arbitrarily designated as fixed, while the other is regarded as mobile, and hence they are respectively called the *base* and the *mobile platform*.

Recently considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. Therefore, they have received increased interest from both researchers and industries. Among these, the class of manipulators known as Stewart-Gough-like platform focused great attention (Stewart, 1965; Merlet, 2000). They are used in flight simulators, pointing devices and more recently for Parallel Kinematic Machines. The Star manipulator (Hervé and Sparacino, 1992; Tremblay and Baron, 1999) and the Delta parallel robot (Clavel, 1988; Tsai, 1999; Staicu and Carp-Ciocârdia, 2003) are equipped with three engines and train on the mobile platform in a three degree of freedom general translation motion. Angeles (2002), Gosselin, Gagné and Wang (1989, 1995, 2001) developed the direct kinematics and dynamics of an Agile Wrist prototype of spherical manipulator that presents three concurrent rotations.

Spherical parallel manipulators provide high stiffness and accuracy due to their mechanical arrangement in which all actuators are fixed to the base and the end-effector is supported by three kinematical chains.

Parallel spherical wrists are used in technical applications where it is desired to orient a rigid body at high speed. Accuracy and precision in the execution of the task of the wrist are essential since the robot is intended to operate on fragile objects; where positioning errors of the tool could end in costly damage. An example of such applications includes the orientation of a camera at high velocity for the tracking of fast objects.

The isotropic spherical parallel manipulator prototype was described in [3] and a closed-form solution of the direct kinematical problem and a vector dynamic model are given in [4].

2. INVERSE GEOMETRIC MODEL

In the present paper some recursive matrix relations for the kinematics and dynamics of a spherical parallel three-degree-of-freedom robot are established.

This mechanism has a structure with the axes of all nine revolute joints concurring in a common *center of rotation* (Fig. 1, after J. Angeles), the fixed point O in Fig. 2.

Let $Ox_0y_0z_0(T_0)$ be a fixed Cartesian frame, about which the manipulator moves. It has three legs of known size and mass. The wrist architecture consists of two main elements, the base and the moving platform, which is free to undergo arbitrary rotations with respect to the center O of three identical serial legs, each of these composed of two links coupled each to other by means of a revolute joint.

The first element of leg A, one of the three driving parts of the robot, is called *proximal link*. It is a homogenous rod, rotating about the axis Oz_1^A with the angular velocity $\omega_{10}^A = \dot{\varphi}_{10}^A$ and the angular acceleration $\varepsilon_{10}^A = \ddot{\varphi}_{10}^A$. It has the radius r, masse m_1 and tensor of inertia $\hat{J}_1$. The *upper link* T_2 of radius r is connected to the $A_2x_2^Ay_2^Az_2^A$ frame and has a relative rotation about the axis Oz_2^A with the angle φ_{21}^A and angular velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$; it has the masse m_2 and the tensor of inertia $\hat{J}_2$.

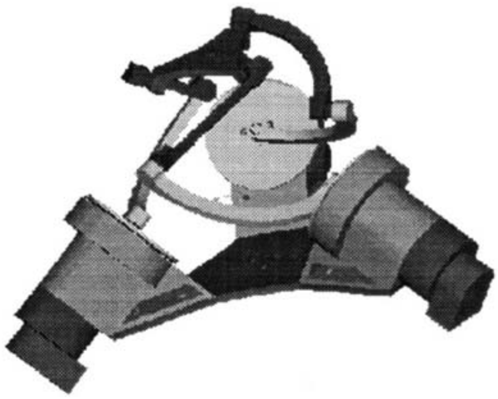

Fig. 1 – Spherical parallel robot.

The platform of the robot is an equilateral triangle of side dimension $l = r\sqrt{2+\sin^2\delta}$, mass m_3 and tensor of inertia $\hat{J}_3$, which rotates with the angular velocity $\omega_{32}^A = \dot{\varphi}_{32}^A$ and angular acceleration $\varepsilon_{32}^A = \ddot{\varphi}_{32}^A$ with respect to the neighboring body. The link angles of the manipulator are assumed to be the same on each of the legs connecting the end-effector to the base and are noted β, for the proximal link and γ for the upper link. Moreover, the angles $\alpha_A = 0$, $\alpha_B = 2\pi/3$,

$\alpha_C = -2\pi/3$, $\delta = \pi/6$ and $\sin\theta = 1/\sqrt{3}$, $\cos\theta = \sqrt{2/3}$ give the geometry of the base and the initial position of the robot arm (Fig. 2).

Each leg of the Agile Wrist is a serial $3R$ spherical wrist: the first joint couples the base with the proximal link, the second joint couples the proximal link with the upper link and the last one couples the upper link with the platform. The serial wrist is orthogonal: its neighboring revolute axes lay out at $\beta = \pi/2$, $\gamma = \pi/2$.

Let us consider the rotation angles $\varphi_{10}^{A}, \varphi_{10}^{B}, \varphi_{10}^{C}$ of the actuators A_1, B_1, C_1 of three legs A, B, C as variables that give the input vector $\vec{\varphi}_{10} = [\varphi_{10}^{A} \quad \varphi_{10}^{B} \quad \varphi_{10}^{C}]^{\mathrm{T}}$ of the instantaneous position of the mechanism. In the inverse geometric problem however, it can be considered that the three Euler angles $\alpha_1, \alpha_2, \alpha_3$ give the absolute orientation of the mobile platform.

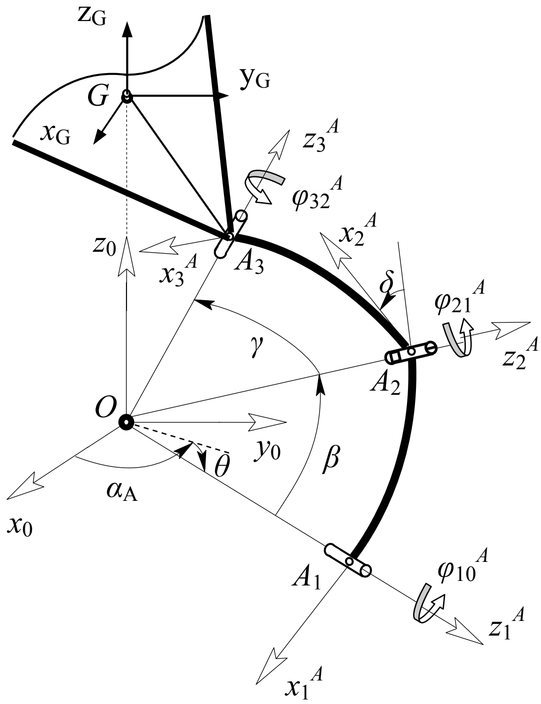

Fig. 2 – Kinematical scheme of first leg A of the mechanism.

Pursuing the first leg A along the way $OA_1A_2A_3$, we obtain the successive orthogonal transformation matrices

$$a_{10} = a_{10}^{\varphi}\theta_1 a_\theta a_\alpha^A,\ a_{21} = a_{21}^{\varphi} a_\delta \theta_1 \theta_2,\ a_{32} = a_{32}^{\varphi}\theta_1\ , \tag{1}$$

with

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_\theta = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$a_\alpha^A = \begin{bmatrix} \cos\alpha_A & \sin\alpha_A & 0 \\ -\sin\alpha_A & \cos\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix},\ a_\delta = \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^A & \sin\varphi_{k,k-1}^A & 0 \\ -\sin\varphi_{k,k-1}^A & \cos\varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_{k0} = \prod_{j=1}^{k} a_{k-j+1,k-j} \quad (k = 1,2,3)\,.$$

Some analogous relations can be written for the other two legs *B* and *C* of the mechanism. Let us suppose that the absolute motion of the platform is a rotation around the point O, expressed by the following Euler angles

$$\alpha_l = \alpha_l^*(1-\cos\frac{2\pi}{3}\ t),\ l = 1,2,3\,. \tag{3}$$

Representing the orientation of the platform in the fixed frame, successive rotations around the moving axes Ox, Oy, Oz lead to the commonly known rotation matrix a (Staicu, 1999):

$$a = a_3 a_2 a_1\,,$$

$$a_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix},\ a_2 = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix},$$

$$a_3 = \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Geometric conditions of rotation for the platform are given by the following identities

$$\overset{\circ}{a}{}_{30}^{\mathrm{T}} a_{30} = \overset{\circ}{b}{}_{30}^{\mathrm{T}} b_{30} = \overset{\circ}{c}{}_{30}^{\mathrm{T}} c_{30} = a\,, \tag{5}$$

with the notation

$$a_{30}^{\circ} = a_{\alpha}^{AT} a_{\theta}^{T} \theta_1^{T} \theta_2^{T} \theta_1^{T} a_{\delta}^{T} \theta_1^{T} . \tag{6}$$

From these matrix relations, we obtain the real time evolution of the characteristic angles φ_{10}^{A}, φ_{21}^{A}, φ_{32}^{A}, φ_{10}^{B}, φ_{21}^{B}, φ_{32}^{B}, φ_{10}^{C}, φ_{21}^{C}, φ_{32}^{C}.

3. VELOCITIES AND ACCELERATIONS

The kinematics of the compounding elements of each leg (for example the leg A) are characterized by skew symmetric matrices given by the recurrence relations

$$\tilde{\omega}_{k0}^{A} = a_{k,k-1} \tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T} + \omega_{k,k-1}^{A} \tilde{u}_3 ,\ k = 1,2,3 . \tag{7}$$

These matrices are *associated* to the absolute angular velocities:

$$\vec{\omega}_{k0}^{A} = a_{k,k-1} \vec{\omega}_{k-1,0}^{A} + \omega_{k,k-1}^{A} \vec{u}_3 , \omega_{k,k-1}^{A} = \dot{\varphi}_{k,k-1}^{A} . \tag{8}$$

The following relations give the absolute linear velocities $\vec{v}_{k0}^{A}$ of the joints A_k :

$$\vec{v}_{k0}^{A} = a_{k,k-1} \vec{v}_{k-1,0}^{A} + a_{k,k-1} \tilde{\omega}_{k-1,0}^{A} \vec{r}_{k,k-1}^{A} ,\ \vec{v}_{k,k-1}^{A} = \vec{0}. \tag{9}$$

Knowing the rotation motion of the platform by the relations in (3), one develops the inverse kinematical problem and determines the velocities $\vec{v}_{k0}^{A}, \vec{\omega}_{k0}^{A}$ and accelerations $\vec{\gamma}_{k0}^{A}, \vec{\varepsilon}_{k0}^{A}$ of each of the moving links. The following *matrix relations of connectivity* form the inverse kinematics model

$$\omega_{10}^{A}\, \vec{u}_i^{T} a_{10}^{T}\, \vec{u}_3 + \omega_{21}^{A}\, \vec{u}_i^{T} a_{20}^{T}\, \vec{u}_3 + \omega_{32}^{A}\, \vec{u}_i^{T} a_{30}^{T} \vec{u}_3 =$$

$$= \vec{u}_i^{T} \{ \dot{\alpha}_1 a_1^{T} \vec{u}_1 + \dot{\alpha}_2 a_1^{T} a_2^{T} \vec{u}_2 + \dot{\alpha}_3 a_1^{T} a_2^{T} a_3^{T} \vec{u}_3 \} , \tag{10}$$

with

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

These relations give the relative angular velocities $\omega_{10}^{A}, \omega_{21}^{A}, \omega_{32}^{A}$ as a function of the angular velocities $\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3$ of the end-effector. If the other two chains of the manipulator are pursued, analogous relations can then be obtained.

Expression of a *complete* Jacobian matrix of the robot is easily written in invariant form from (10). This matrix is a fundamental element for the analysis of the robot workspace and determination of the particular configurations of singularities where the manipulator becomes uncontrollable. In [3, 4, 15] some families of isotropic spherical parallel manipulators were obtained using the expressions of the Jacobian matrices and the definition of the *dexterity*.

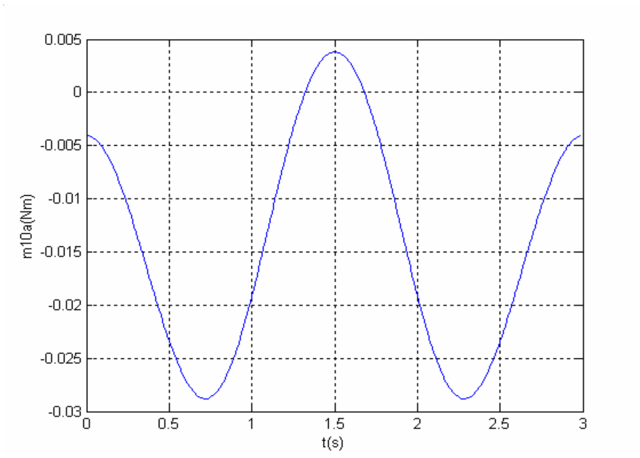

Fig. 3 – Torque m_{10}^{A} of first actuator.

Let us assume now that the robot has a virtual motion defined by the following angular velocities $\omega_{10a}^{Av} = 1, \omega_{10a}^{Bv} = 0, \omega_{10a}^{Cv} = 0$. Characteristic virtual velocities expressed as function of robot's position are given by the conditions of connectivity concerning the relative velocities of two independent loops $A_1A_2A_3OB_3B_2B_1$ and $B_1B_2B_3OC_3C_2C_1$:

$$\vec{u}_i^{\mathrm{T}} a_{30}^{\mathrm{T}}\ \vec{\omega}_{30a}^{Av} = \vec{u}_i^{\mathrm{T}} b_{30}^{\mathrm{T}}\ \vec{\omega}_{30a}^{Bv} = \vec{u}_i^{\mathrm{T}} c_{30}^{\mathrm{T}}\ \vec{\omega}_{30a}^{Cv}, \qquad (i=1, 2, 3). \tag{12}$$

Concerning the relative angular accelerations $\varepsilon_{10}^{A}, \varepsilon_{21}^{A}, \varepsilon_{32}^{A}$ of the elements of first leg A, for example, these are given by some other *conditions of connectivity* obtained by deriving the relations (10); it results:

$$\begin{aligned}
&\varepsilon_{10}^{A}\ \vec{u}_i^{\mathrm{T}} a_{10}^{\mathrm{T}} \vec{u}_3 + \varepsilon_{21}^{A}\ \vec{u}_i^{\mathrm{T}} a_{20}^{\mathrm{T}} \vec{u}_3 + \varepsilon_{54}^{A}\ \vec{u}_i^{\mathrm{T}} a_{30}^{\mathrm{T}} \vec{u}_3 = \\
&= \vec{u}_i^{\mathrm{T}} \{\ddot{\alpha}_1 a_1^{\mathrm{T}} \vec{u}_1 + \ddot{\alpha}_2 a_1^{\mathrm{T}} a_2^{\mathrm{T}} \vec{u}_2 + \ddot{\alpha}_3 a_1^{\mathrm{T}} a_2^{\mathrm{T}} a_3^{\mathrm{T}} \vec{u}_3 + \\
&\quad + \dot{\alpha}_1 \dot{\alpha}_2 a_1^{\mathrm{T}} \tilde{u}_1 a_2^{\mathrm{T}} \vec{u}_2 + \dot{\alpha}_2 \dot{\alpha}_3 a_1^{\mathrm{T}} a_2^{\mathrm{T}} \tilde{u}_2 a_3^{\mathrm{T}} \vec{u}_3 + \\
&\quad + \dot{\alpha}_3 \dot{\alpha}_1 a_1^{\mathrm{T}} \tilde{u}_1 a_2^{\mathrm{T}} a_3^{\mathrm{T}} \vec{u}_3 - \omega_{10}^{A} \omega_{21}^{A} a_{10}^{\mathrm{T}} \tilde{u}_3 a_{21}^{\mathrm{T}} \vec{u}_3 - \\
&- \omega_{21}^{A} \omega_{32}^{A} a_{20}^{\mathrm{T}} \tilde{u}_3 a_{32}^{\mathrm{T}} \vec{u}_3 - \omega_{32}^{A} \omega_{10}^{A} a_{10}^{\mathrm{T}} \tilde{u}_3 a_{21}^{\mathrm{T}} a_{32}^{\mathrm{T}} \vec{u}_3 \},
\end{aligned} \tag{13}$$

where $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3$ are three skew symmetric matrices associated with the unit vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$.

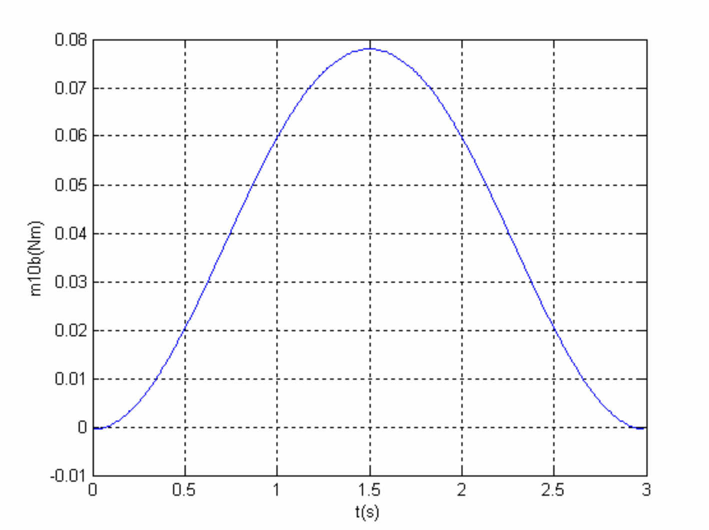

Fig. 4 – Torque m_{10}^{B} of second actuator.

The angular accelerations $\vec{\varepsilon}_{k0}^{A}$ and the linear accelerations $\vec{\gamma}_{k0}^{A}$ of the joints A_k are easily calculated with the recurrence relations

$$\begin{aligned}
&\vec{\varepsilon}_{k0}^{A} = a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{A} + \varepsilon_{k,k-1}^{A}\vec{u}_3 + \omega_{k,k-1}^{A} a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{\mathrm{T}}\vec{u}_3 \\
&\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A} = a_{k,k-1}\left(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A}\right)a_{k,k-1}^{\mathrm{T}} + \\
&+\omega_{k,k-1}^{A}\omega_{k,k-1}^{A}\tilde{u}_3\tilde{u}_3 + \varepsilon_{k,k-1}^{A}\tilde{u}_3 + 2\omega_{k,k-1}^{A} a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{\mathrm{T}}\tilde{u}_3 \\
&\vec{\gamma}_{k0}^{A} = a_{k,k-1}\left[\vec{\gamma}_{k-1,0}^{A} + \left(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A}\right)\ \vec{r}_{k,k-1}^{A}\right].
\end{aligned} \tag{14}$$

4. EQUATIONS OF MOTION

Since spherical parallel robots are mostly used in applications involving high-speed motion, the dynamics of the robot has a very important effect on the required actuator torques.

Three electric motors A_1, A_2, A_3, that generates the torques $\vec{m}_{10}^{A} = m_{10}^{A}\vec{u}_3$ $\vec{m}_{10}^{B} = m_{10}^{B}\vec{u}_3$, $\vec{m}_{10}^{C} = m_{10}^{C}\vec{u}_3$ having the directions of the axes $A_1 z_1^A$, $B_1 z_1^B$, $C_1 z_1^C$

control the motion of mechanism's legs. The motors are fixed to the base and are represented as cylinders in Fig. 1.

Suppose that the motion of the mobile platform is known. While the position, angular velocity, angular acceleration as well as the velocity and acceleration of the center of mass are known for each of the links, it follows that the forces and the moments acting on each body are also determined.

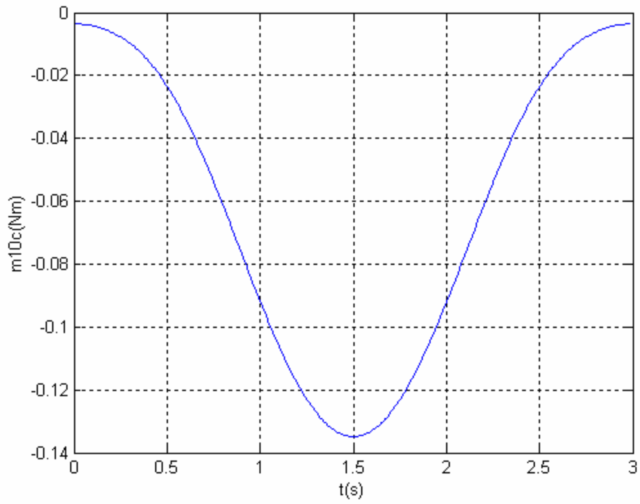

Fig. 5 – Torque m_{10}^{C} of third actuator.

The force of inertia and the resultant moment of forces of inertia, acting on the body T_k are evaluated with respect to the center of the A_k joint. On the other hand, the characteristic vectors $\vec{f}_k^{\,*}, \vec{m}_k^{\,*}$ designate the action of the weight $m_k \vec{g}$ or of any other external and internal applied forces at the same element of the robot. At this stage, only the torques applied on the proximal links by the three actuators is of interest.

Regarding the robot control, the relevant objective of a dynamic model is to determine the torques that must be exerted by the actuators in order to produce a given trajectory of the robot. Such a dynamics model can be obtained using three main approaches, which provide the same results concerning these active moments. The first approach is the Newton-Euler classic procedure; the second one consists in the Lagrange equations with their multiplier's formalism and the third one is based on the principle of virtual works.

In the inverse dynamic problem, here presented, one applies the method of virtual powers in order to establish some recursive matrix relations for the torques of the three active couples. Some graphs of these moments are also obtained.

The fundamental principle of the virtual powers [1, 8, 11, 16] states that a mechanism is under dynamic equilibrium if and only if the virtual power developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Assuming that frictional forces at the joints are negligible, the virtual power produced by the forces of constraint at the joints is zero.

Applying *the fundamental equations of the parallel robots dynamics* established in compact form by Stefan Staicu [9, 10], the following matrix relation results

$$m_{10}^{A} = \vec{u}_3^{\mathrm{T}} \{\vec{M}_1^{A} + \omega_{21a}^{Av}\vec{M}_2^{A} + \omega_{32a}^{Av}\vec{M}_3^{A} + \omega_{21a}^{Bv}\vec{M}_{20}^{B} + \omega_{21a}^{Cv}\vec{M}_{20}^{C}\}, \quad (15)$$

where one denoted

$$\begin{aligned}
&\vec{F}_{k0}^{A} = m_k^{A}\left[\vec{\gamma}_{k0}^{A} + \left(\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A}\right)\ \vec{r}_k^{CA}\right] - \vec{f}_k^{*A},\\
&\vec{M}_{k0}^{A} = m_k^{A}\tilde{r}_k^{CA}\vec{\gamma}_{k0}^{A} + \hat{J}_k^{A}\vec{\varepsilon}_{k0}^{A} + \tilde{\omega}_{k0}^{A}\hat{J}_k^{A}\vec{\omega}_{k0}^{A} - \vec{m}_k^{*A},\\
&\vec{F}_k^{A} = \vec{F}_{k0}^{A} + a_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^{A} \quad (k = 1,\ 2,\ 3),\\
&\vec{M}_k^{A} = \vec{M}_{k0}^{A} + a_{k+1,k}^{\mathrm{T}}\vec{M}_{k+1}^{A} + \tilde{r}_{k+1,k}^{A}a_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^{A}.
\end{aligned} \quad (16)$$

The relations (15) and (16) represent the *inverse dynamic model* of the spherical parallel robot. Analogous expressions we can obtain for the torques m_{10}^{B}, m_{10}^{C} exerted by the other two actuators B_1, C_1.

The procedure above developed leads to very good estimates of the actuators torques for given displacement of end-effector, provided that the inertial properties of the links are known with sufficient accuracy and that friction is not significant. It is also remarked that, depending on the masses and inertias of the links, the present matrix dynamics model leads to interesting and useful results for purposes of control.

The new dynamic approach developed here is completely general and can be used for any spherical manipulator with revolute actuators.

Consider for example a robot that has the following characteristics:

$$m_1^{A} = m_1^{B} = m_1^{C} = m_2^{A} = m_2^{B} = m_2^{C} = 0.14\ \mathrm{kg}, m_3 = 0.7\ \mathrm{kg},$$

$$r = 0.074\mathrm{m},\ \Delta t = 3\ \mathrm{s},\ \alpha_1^{*} = \frac{\pi}{36}, \alpha_2^{*} = \frac{\pi}{18}, \alpha_3^{*} = \frac{\pi}{12}. \quad (17)$$

Finally, we obtain the time-history of the torques m_{10}^{A} (Fig. 3), m_{10}^{B} (Fig. 4), m_{10}^{C} (Fig. 5) of the three revolute actuators. According with the motion lows (3), the symmetry of the graphics of the active torques $m_{10}^{A}, m_{10}^{B}, m_{10}^{C}$ with respect to the maximum rotation of the platform is verified.

5. CONCLUSIONS

Most of dynamical models based on the Lagrange formalism neglect the weight of intermediate bodies and take into consideration only the active forces or moments and the wrench of applied forces on the moving platform. The number of relations given by this approach is equal to the total number of the position variables and Lagrange multipliers inclusive. Also, the analytical calculi involved in these equations are very tedious, thus presenting an elevated risk of making errors.

The commonly known Newton-Euler method, which takes into account the free-body-diagrams of the mechanism, leads to a large number of equations with unknowns among which are also the connecting forces in the joints. Finally, the actuating torques could be obtained.

Within the inverse kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of the parallel robot have been established in present paper. The dynamics model takes into consideration the masses and forces of inertia introduced by all component elements of the parallel mechanism.

The new approach based on the principle of virtual powers can eliminate all forces of internal joints and establishes a direct determination of the time-history evolution of torques required by the actuators. The recursive matrix relations (15) and (16) represent the explicit equations of the dynamics simulation and can easily be transformed in a model for automatic command of the spherical parallel robot.

Received 20 June 2006

REFERENCES

1. J. ANGELES, *Fundamentals of Robotic Mechanical Systems.Theory, Methods and Algorithms*, Springer-Verlag, New York, 2002.
2. R. CLAVEL, *Delta: a fast robot with parallel geometry*, Proc. of 18th Int. Symposium on Industrial Robots, Lausanne, 1988.
3. C. GOSSELIN, J. ANGELES, *The optimum kinematics design of spherical three-degree-of-freedom parallel manipulator*, ASME Journal of Mech. Trans. and Automat. in Design, **111**, *2* (1989).
4. C. GOSSELIN, M. GAGNÉ, *Dynamic models for spherical parallel manipulators*, IEEE Int. Conference on Robotics and Automation, Milan, 1995.
5. J-M. HERVÉ, F. SPARACINO, *Star. A New Concept in Robotics*, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, 1992.
6. Y-W. LI, J. WANG, L-P. WANG, X-J. LIU, *Inverse dynamics and simulation of a 3-DOF spatial parallel manipulator*, Proceedings of the IEEE International Conference on Robotics & Automation, Taipei, Taiwan, 2003.
7. J-P. MERLET, *Parallel Robots*, Kluwer Academic Publishers, 2000.
8. ŞT. STAICU, D. ZHANG, R. RUGESCU, *Dynamic modelling of a 3-DOF parallel manipulator using recursive matrix relations*, Robotica, Cambridge University Press, **24**, *1*, (2006).
9. ŞT. STAICU, *Modèle dynamique en robotique*, UPB Scientific Bulletin, Series D: Mechanical Engineering, Bucharest**, 61**, *3–4* (1999).

10. ŞT. STAICU, *Relations matricielles de récurrence en dynamique des mécanismes*, Rev. Roum. Sci. Techn. – Méc. Appl., **50**, *1-3* (2005).
11. C-D. ZHANG, S-M. SONG, *An Efficient Method for Inverse Dynamics of Manipulators Based on Virtual Work Principle*, Journal of Robotic Systems, **19**, *5* (1999).
12. ŞT. STAICU, D.C. CARP-CIOCÂRDIA, *Dynamic analysis of Clavel's Delta parallel robot*, Proceedings of the IEEE International Conference on Robotics & Automation, Taipei, Taiwan, 2003.
13. D. STEWART, *A Platform with Six Degrees of Freedom*, Proc. Inst. Mech. Eng., **1**, *15* (1965).
14. A. TREMBLAY, L. BARON, *Geometrical Synthesis of Parallel Manipulators of Star–Like Topology with a Genetic Algorithm*, IEEE International Conference on Robotics and Automation, Detroit, Michigan, 1999.
15. J. WANG, C. GOSSELIN, *Representation of the Singularity Loci of a Special Class of Spherical 3-dof Parallel Manipulator with Revolute Actuators*, CCToM Symposium on Mechanisms, Machines and Mechatronics, Saint-Hubert (Montréal), 2001.
16. L-W. TSAI, *Robot analysis: the mechanics of serial and parallel manipulators*, John Wiley & Sons, Inc., 1999.