ON THE LOADS LIMIT ACCORDING TO CLASSICAL FAILURE THEORIES OF MATERIALS

Sergiu T. CHIRIACESCU

In this paper, a general method for determining a domain $\mathcal{D}$ for the principal stresses which assure the strength of the elastic structure for any position of the principal stresses directions is presented. Such this domain permits to calculate the limit loads acting on the elastic structure or to determine the geometric characteristics of the critical structure section for any classical failure theory. In the case of the plane stress state this domain is a circle, and in the space stress state this is a sphere. In both situations, the radius of this domain is calculated for each failure theory.

1. INTRODUCTION

For each failure theory of materials an admissible domain $\mathcal{D}$ for the principal stresses $\sigma_i \left(i = \overline{1,3} \right)$ is proposed [1, 5]. In this domain it is assumed that a reliable strength prediction be available for various combinations of multiaxial stresses. As a rule, these theories are applied to verify a mechanical elastic structure (machine element, element of a civil building a.s.o.) subjected to a system of loads (forces, bending moments or torque moments). Due to a lot of difficulties for establishing the mechanism which transfers the current stress point $P(\sigma_1, \sigma_2, \sigma_3)$ to the limit point $P_l(\sigma_{1l}, \sigma_{2l}, \sigma_{3l})$ located on the boundary surface Γ of the domain $\mathcal{D}$, the failure theories are rarely used to calculate the load limits, the geometric characteristics of the critical structure section or for the safety factor.

Assuming that failure theories are available at least for some materials and loads, we present in what follows, some procedures to calculate the principal stresses limit. The proposed methods permit to evaluate the load limits acting on the elastic structure and/or to estimate the geometric characteristics of the critical section of the elastic structure.

2. ADMISIBLE STRESS. EQUIVALENT STRESS

The strength calculus of elastic structures is based on the concepts of *stress limit or load limit***.**

These performance indicators take into consideration a tensile specimen of the investigated material and represent the maximum value of the normal or shear stress, or the maximum value of the axial load or shear load , respectively.

"Transilvania" University of Braşov, 29 Eroilor Bld., 2200 Brasov, c. sergiu@univtbv.ro

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The failure of the structure becomes visible in the presence of plastic deformation (in the case of ductile materials), or in the presence of fracture (in the case of brittle materials).

For isotropic materials, the stress limit can be determined by using the stress – strain characteristic curve. For example, in the case of ductile materials the characteristic curve $\sigma = \sigma(\varepsilon)$ can be as in Fig. 1, where ε is the strain, σ the normal stress, S_y the yield stress, S_u the ultimate stress. For brittle materials $S_y = \sigma_{0.2}$, i.e. the stress which corresponds to $\varepsilon = 0.2\%$.

The stress limit S_l can be S_y or S_u.

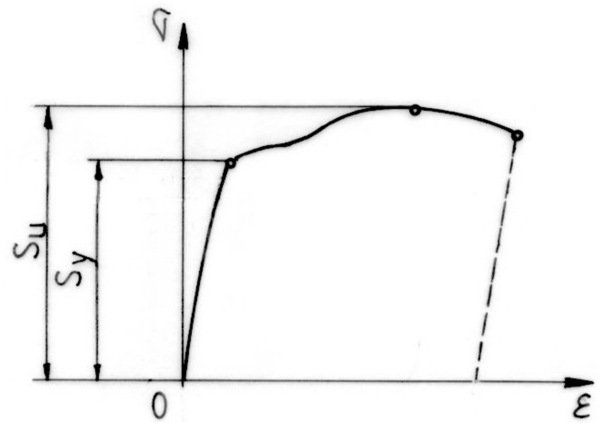


Fig. 1

Taking into consideration the deviation from hypothesis of isotropic materials, the applied thermal treatment, the predictive life duration of the structure a.s.o., for strength of materials calculus the stresses S_y or S_u must be replaced with *admissible stress* σ_a or τ_a , respectively, where σ_a is the *admissible tensile stress and* τ_a *is the admissible shear stress.*

The admissible stress can be defined as follows:

$$\sigma_a \left(\text{or } \tau_a\right) = \frac{S_l}{c}, \tag{1}$$

where $c > 1$ is the safety coefficient.

The failure theories are based on the concept of *equivalent stress* σ_e or τ_e. The equivalent stress is uniaxial stress which has the same failure tendency on the tensile specimen as the actual stresses acting on the elastic structure.

3. FAILURE THEORIES

Relating the elastic structure to a tri-rectangular reference system 0*xyz*, in the point 0 of the stresses plane, the relation between the vector-stress **p** and the unit vector **v** perpendicular to this plane can be written as:

$$\mathbf{p} = \mathbf{T}_p \nu , \tag{2}$$

where:

$$\mathbf{p} = \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ \mathbf{p}_z \end{bmatrix}, \quad \mathbf{T}_p = \begin{bmatrix} \sigma_x & \tau_{yx} = \tau_{xy} & \tau_{zx} = \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} = \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} . \tag{3}$$

The eigenvalues of the matrix $\mathbf{T_p}$ **,** i.e. the solutions $\lambda_1=\sigma_1$, $\lambda_2=\sigma_2$, $\lambda_3=\sigma_3$ of the equation:

$$\left| \mathbf{T}_p - \lambda \mathbf{E} \right| = 0 \tag{4}$$

represent the principal stresses. In eq. (4) **E** is the unit matrix.

The calculus of the principal stresses can be found in many books [5], [10] a.s.o. of strength of materials. In particular, for plane stress state for which $\mathbf{T_p}$ is:

$$\mathbf{T}_p = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \tag{5}$$

the principal stresses are:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} . \tag{6}$$

The directions of the principal stresses are given by:

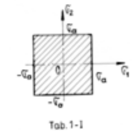
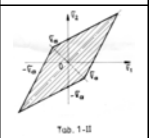
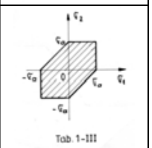
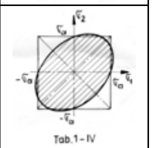
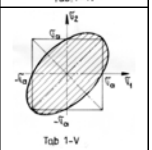
$$\alpha_1 = \arctan \frac{2\tau_{xy}}{\sigma_x - \sigma_y} , \quad \alpha_2 = \alpha_1 + \frac{\pi}{2} . \tag{7}$$

According to failure theories, the equivalent stress is determined as a function of principal stresses. The elastic structure is accepted in terms of strength of materials if $\sigma_l \le \sigma_a$.

For ductile materials, in Table 1 the possible domain $\mathcal{D}$ of the principal stresses is presented for each theory of failure. The graphic representation of these domains for the plane stress state is also presented. The contour line Γ around each domain $\mathcal{D}$ can be named the *limit contour line.*

In the following we present some modalities for passing from the current point P to the limit point P_l located (Table 1) on the limit surface (for triaxial stresses) or limit contour line (for biaxial Table 1 stresses).

Table 1

No.	FAILURE THEORY	EQUATION OF THE DOMEIN $\mathcal{D}$	GRAPHIC REPRESENTATION OF THE DOMEIN $\mathcal{D}$ FOR THE PLANE STRESS STATE
I	MAX. NORMAL STRESS	$\sigma_e = \max\left\{\lvert\sigma_1\rvert, \lvert\sigma_2\rvert, \lvert\sigma_3\rvert\right\} \le \sigma_a$	
II	MAX. NORMAL STRAIN	$\sigma_e = \max\left\{\begin{matrix}\lvert\sigma_1 - \nu(\sigma_2+\sigma_3)\rvert, \lvert\sigma_2 - \nu(\sigma_3+\sigma_1)\rvert, \\ \lvert\sigma_3 - \nu(\sigma_1+\sigma_2)\rvert\end{matrix}\right\} \le \sigma_a$	
III	MAX. SHEAR STRESS	$\tau_e = \max\left\{\lvert\sigma_1 - \sigma_2\rvert, \lvert\sigma_1 - \sigma_3\rvert, \lvert\sigma_3 - \sigma_1\rvert\right\} \le \sigma_a$	
IV	MAX. SPECIFIC ENERGY	$\sigma_e = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\nu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)} \le \sigma_a$	
V	MAX. DISTORSION ENERGY	$\sigma_e = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)} \le \sigma_a$	

4. PASSING FROM CURRENT STRESS STATE TO LIMIT STRESS STATE

In the first stage we consider the biaxial stress state of the elastic structure. In this case, we calculate the principal stresses (Fig.2) and their directions with relations (5) and (6).

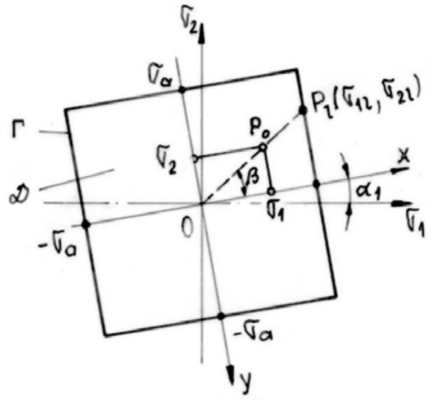


Fig. 2

If the current point P (σ_1, σ_2) is placed into the domain $\mathcal{D}$ then the elastic structure satisfies the strength condition for its functionality. However, if we want to use one or another failure theory to calculate the limit loads or to design the geometry of the critical section of the structure it is necessary to know how the trajectory of the point P to the limit point $P_l(\sigma_{1l}, \sigma_{2l})$ is.

a) There is a particular case in which the angles α_1 and β (Fig. 2) are constant while point P passes to point P_l is. This implies the following conditions:

$$\sigma_x = k_1\sigma_y\ ,\ k_1 = \text{const.}$$
$$\tau_{xy} = k_2\sigma_x = k_1k_2\sigma_y\ ,\ k_2 = \text{const.} \tag{8}$$

According to relations (8), the elements of the matrix T_p must be proportional with one of the stresses σ_x, σ_y, τ_{xy}. In this case, taking into consideration relations (5) and (6) we obtain:

$$k = \frac{1 + k_1 - \sqrt{(k_1 - 1)^2 + 4k_1^2k_2^2}}{1 + k_1 + \sqrt{(k_1 - 1)^2 + 4k_1^2k_2^2}}\ .$$

Referring to Fig. 2, for the first failure theory such a stress limit state corresponds to the point $P_l(\sigma_{1l}, \sigma_{2l})$.

Knowing the point P_l with the stresses σ_{1l} and σ_{2l} the limit loads can be calculated and/or the geometry of the critical structure section can be determined.

A gear transmission is shown as an illustrative example in Fig. 3. Neglecting the shear force, the loads of the rotating shaft are:

$$M_t = 9.54\frac{P}{n},\qquad F = \frac{M_t}{r\cos\alpha_0}\ , \tag{9}$$

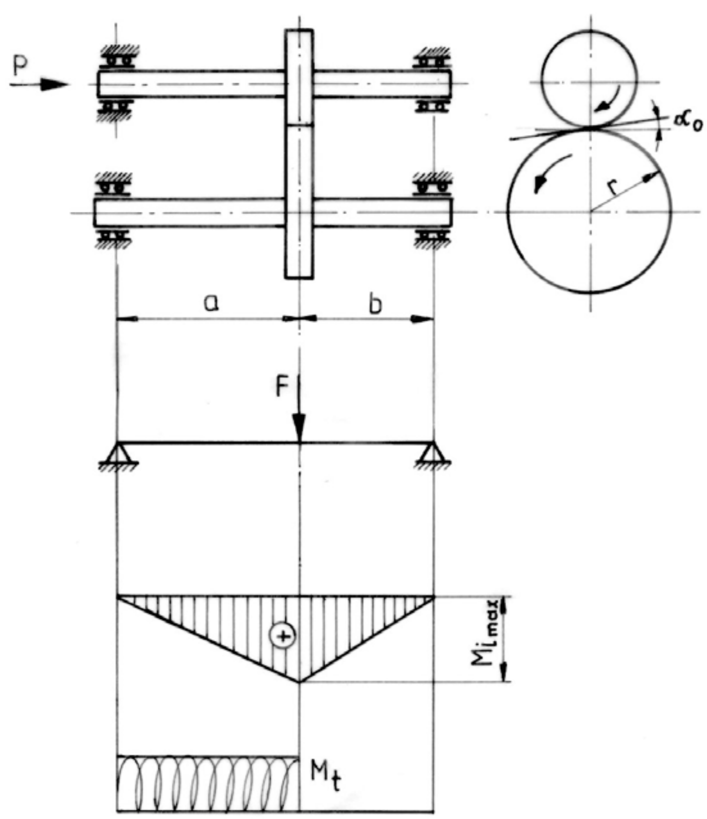


Fig. 3

where P [kW] is the power of transmission, r [mm] gear radius, and n[rev/min] the rotational frequency of the shaft.

According to relations (9) we have:

$$\sigma_x = \frac{M_{i\max}}{W_z} = \frac{ab}{r(a+b)} \cdot \frac{F}{W_z}, \quad \tau_{xy} = \frac{M_t}{W_p}, \tag{10}$$

where: $W_p = 2W_z = \pi d^{\;3}/16$.

The relations (10) show that:

$$\tau_{xy} = k_2 \sigma_x \ , \ k_2 = \frac{r(a+b)}{2b}.$$

In this case equations (6) (in which $\sigma_y = 0$) becomes:

$$\sigma_{1,2} = \frac{\sigma_x}{2}\left(1 \pm \sqrt{1+4k_2^2}\right). \tag{11}$$

From relations (8) and equations (11) we obtain:

$$\sigma_2 = k\sigma_1 \ , \ k = \frac{1-\sqrt{1+4k_2^2}}{1+\sqrt{1+4k_2^2}} \ .$$

Crossing of the contour line Γ with the straight-line $\sigma_2 = k\sigma_1$ give $\sigma_{2l} = k\sigma_{1l}$. In this case , one of the relations (11) permits the calculation of the limit value:

$$M_{tl} = \frac{r(a+b)}{b} \cdot \frac{\sigma_{1l}}{1+\sqrt{1+4k_2^2}} \ , \tag{12}$$

for a known W_z , or the calculation of a W_z when M_{tl} is given.

b) Another particular case is that in which the position of the axes 0 σ_1 and 0 σ_2 relative to the reference system 0*xy* remain the same during the transition of the point P (σ_1, σ_2) to the point $P_l(\sigma_{1l}, \sigma_{2l})$. Assuming that the position of the point P_l is predetermined and putting:

$$\tan(2\alpha) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\tau_{xyl}}{\sigma_{xl} - \sigma_{yl}} = \frac{1}{k_3} = \text{const.} \tag{13}$$

equations (6) permit to write:

$$\sigma_{1l,2l} = \frac{\sigma_{xl} + \sigma_{yl}}{2} \pm \frac{1}{2}\sqrt{(\sigma_{xl} - \sigma_{yl})^2 + 4\tau_{xyl}^2} \ . \tag{14}$$

Relations (13) and (14) give:

$$\sigma_{xl} = \frac{\sigma_{1l} + \sigma_{2l}}{2} \pm \frac{k_3}{2} \frac{\sigma_{1l} - \sigma_{2l}}{\sqrt{1+k_3^2}} \ ,$$

$$\sigma_{yl} = \frac{\sigma_{1l} + \sigma_{2l}}{2} \mp \frac{k_3}{2} \frac{\sigma_{1l} - \sigma_{2l}}{\sqrt{1+k_3^2}} \ , \tag{15}$$

$$\tau_{xyl} = \pm \frac{1}{2} \cdot \frac{\sigma_{1l} - \sigma_{2l}}{\sqrt{1+k_3^2}} \ .$$

The stresses σ_{xl}, σ_{yl} and τ_{xyl} can be determined from relations (15) as functions of principal stresses and k_3. Then, taking into consideration the dependences between the stresses σ_x ,σ_y ,τ_{xy} and the loads, the limit loads or the parameters of the cross-section of the structure can be obtained.

For example, if we consider the first failure theory, the point on the contour limit Γ can be chosen as in Fig. 4 and Table 2.

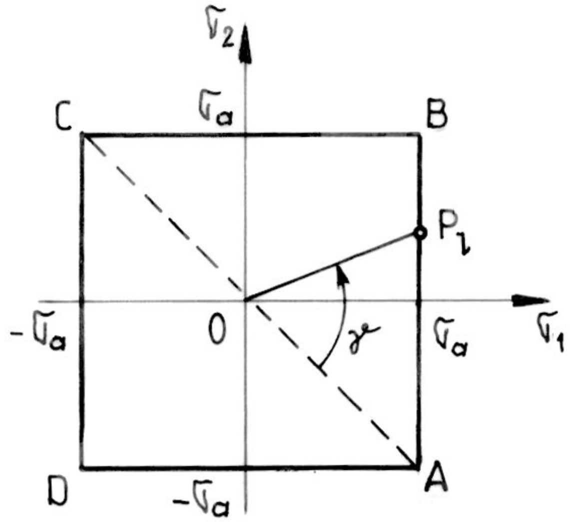


Fig. 4

Table 2

STREIGHT-LINE	γ	σ_{1l}	σ_{2l}
AB	$\left[0,\frac{\pi}{2}\right]$	σ_a	$\sigma_a \tan\left(\gamma-\frac{\pi}{4}\right)$
BC	$\left[\frac{\pi}{2},\pi\right]$	$\sigma_a \tan\left(\frac{3\pi}{4}-\gamma\right)$	σ_a
CD	$\left[\pi,\frac{3\pi}{2}\right]$	$-\sigma_a$	$\sigma_a \tan\left(\frac{5\pi}{4}-\gamma\right)$
DA	$\left[\frac{3\pi}{2},2\pi\right]$	$\sigma_a \tan\left(\frac{7\pi}{4}-\gamma\right)$	$-\sigma_a$

In the case of the fourth failure theory, according to Table 1 the equation of the contour line can be written as follows:

$$\frac{\sigma_{1l}^2}{\left(\frac{\sigma_a}{\sqrt{1+\nu}}\right)^2}+\frac{\sigma_{2l}^2}{\left(\frac{\sigma_a}{\sqrt{1-\nu}}\right)^2}-1=0\,, \tag{16}$$

where as Fig. 5 permits to calculate the coordinates of the point P :

$$\sigma_{1l} = \pm \frac{\sigma_a}{\sqrt{1+\nu+(1-\nu)\tan^2\gamma}},$$
$$\sigma_{2l} = \sigma_{1l}\tan\gamma, \tag{17}$$

where $\gamma \in [0,\ 2\pi]$.

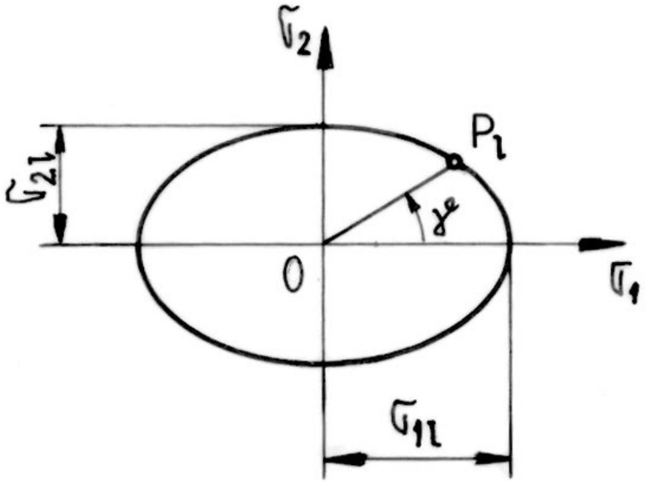


Fig. 5

In a similar mode the coordinates σ_{1l} and σ_{2l} can be obtained for any failure theory.

As an illustrative example, we assume that M_i and M_t as in Fig. 3 are independent. In this case for $2\alpha = \pi/4$ relation (13) gives $k_3 = 1$. If we apply the fifth failure theory and if we choose the point P_l corresponding to $\gamma = \pi/4$, relations (13) and (14) give: $\sigma_{1l} = \sigma_{2l}$, $\tau_{xyl} = 0$, $\sigma_{xl} = \sigma_a/\sqrt{2}$, $\sigma_y = 0$. These values permit to calculate the limit loads F_l and M_{tl} .

c) Let us consider the general case in which the loads acting on the elastic structure have the independent variations. Now, the stresses σ_1 and σ_2 are independent, and the value of the angle α_1 depends on the value and the sense of loads acting on the structure. For example, for $\sigma_x = \sigma' = 60$ MPa, $\sigma_y = 0$, $\tau_{xy} = \tau' =$ = 8 MPa and $\sigma_x = \sigma'' = -60$ MPa, $\sigma_y = 0$, $\tau_{xy} = \tau'' = 80$ MPa we obtain the positions P' and P'' as in Fig 6. Therefore, in this case the point P_l on the contour limit Γ can not be determined and, in consequence, we can not calculate the limit loads.

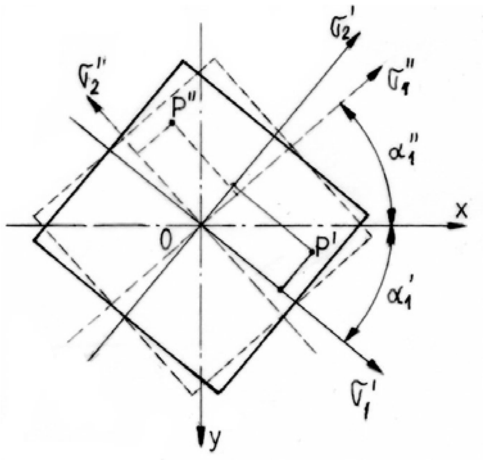


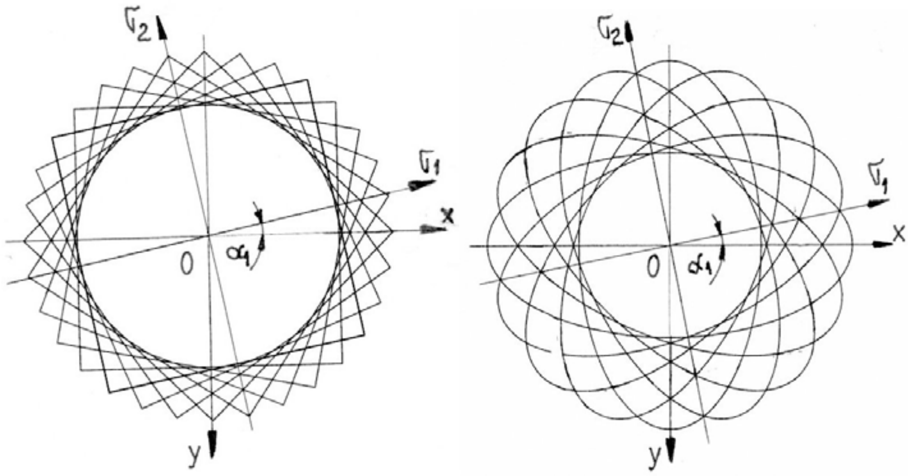
Fig. 6

Regarding the various cases of the position of the system 0 σ_1 σ_2 relative to the reference system *0xy*, *in the following we propose to determine the domain $\mathcal{D}_1 \subset \mathcal{D}$ so that the condition on the strength of the elastic structure be satisfied for all possible variations of the loads (as magnitude and sense) acting on the structure*.

For the failure theories I and IV in Fig. 7 we show the graphical procedure for determining the domain $\mathcal{D}_1$. It can be observe that his domain is :

$$\sigma_1^2 + \sigma_2^2 \le R^2, \tag{18}$$

where *R* is the radius of the new contour line.



a. b.

Fig. 7

Inequality (18) has the same form for all failure theories, but the radius R can be determined as in Table 3 for each theory.

Table 3

FAILURE THEORY	R
I	σ_a
II	$\sigma_a \cos\left(\frac{\pi}{8}\right)$
III	$\sigma_a \cos\left(\frac{\pi}{4}\right)$
IV	$\frac{\sigma_a}{\sqrt{1+\nu}}$
V	$\frac{\sigma_a}{\sqrt{1.5}}$

Let us consider an example similar with those presented in Fig. 3, but where M_i (i.e. F) and M_t are the independent loads. According to Eq. (6) in which:

$$\sigma_x = \frac{ab}{a+b}\frac{F}{W_z}\,,\ \sigma_y = 0\,,\ \tau_{xy} = \frac{M_t}{W_p} = \frac{M_t}{2W_z}\,,$$

we obtain:

$$\sigma_1^2 + \sigma_2^2 = \frac{1}{W_z^2}\left[\frac{a^2b^2F^2}{(a+b)^2} + \frac{M_t^2}{2}\right].$$

For $R = \sigma_a$ the above relation permits to write:

$$\frac{F^2}{\left(\frac{a+b}{ab}W_z\right)^2} + \frac{M_t}{\left(\sqrt{2}W_z\right)^2} \le \sigma_a^2 \quad . \tag{19}$$

The loads limit domain (19) has an ellipse contour line presented in Fig. 8. The domain Γ shows that the limit-loads F and M_t must be in the intervals [– F_{max} , F_{max}] and [– $M_{t\,max}$, $M_{t\,max}$] respectively, where:

$$F_{\max} = \frac{a+b}{ab}W_z\sigma_a\ ,\ M_{t\max} = \sqrt{2}W_z\sigma_a\,.$$

in condition (18).

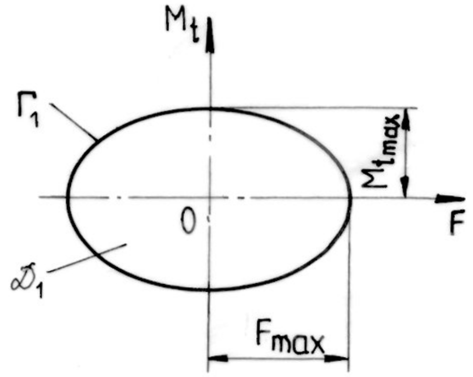


Fig. 8

5. THE SPACE STRESS STATE

All the facts presented so far can be extended for the space stress state. Thus, for the case c) of the Chapter 5, according to the first failure theory (Table 1), the domain $\mathcal{D}$ is a cube having $2\sigma_a$ as its edge. Therefore, the domain $\mathcal{D}_1 \subset \mathcal{D}$ is:

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 = R^2, \tag{20}$$

where $R = \sigma_a$.

According to the second failure theory, and assuming that $\sigma_1 > \sigma_2 > \sigma_3$ and $\sigma_l = \sigma_1 - \nu(\sigma_2 + \sigma_3)$. We obtain at the limit of the domain $\mathcal{D}$:

$$\sigma_{1l} - \nu(\sigma_{2l} + \sigma_{3l}) \le \sigma_a , \tag{21}$$

which represents a plane. The equation of the perpendicular straight line from the point 0 on this plane is:

$$\frac{1}{l} = -\frac{\nu}{m} = \frac{\nu}{n} = \frac{1}{k_4} = \text{const.} , \tag{22}$$

where: $l^2 + m^2 + n^2 = 1$.

Crossing this straight line with the plane (21)we get:

$$\sigma_{1l} = \frac{\sigma_a}{1+2\nu^2} , \ \sigma_{2l} = -\frac{\nu\sigma_a}{1+2\nu^2} , \ \sigma_{3l} = -\frac{\nu\sigma_a}{1+2\nu^2} , \tag{23}$$

and the distance d from the point 0 to the point P_l (σ_{1l}, σ_{2l}, σ_{3l}) is:

$$d = \sqrt{\sigma_{1l}^2 + \sigma_{2l}^2 + \sigma_{3l}^2} = \sigma_a \sqrt{1/\left(1+2\nu^2\right)} . \tag{24}$$

Taking into consideration relation (24) we observe that condition (20) is also valid for the failure theory II, but: $R = \sigma_a \sqrt{1/\left(1+2\nu^2\right)}$.

In the same manner, if $\sigma_e = \sigma_1 - \sigma_2$ for the failure theory III (Table 1) we obtain the condition (19), in which $R = \left(\sqrt{2}/2\right)\sigma_a$.

The failure theories I and V (Table 1) shown that in the space $\sigma_1\sigma_2\sigma_3$ the domains $\mathcal{D}$ are two ellipsoids. Relative to the system $0\,\sigma_1\sigma_2\sigma_3$, the equation of the domain for the failure theory IV is:

$$\sigma_{1l}^2 + \sigma_{2l}^2 + \sigma_{3l}^2 - 2\nu\left(\sigma_{1l}\sigma_{2l} + \sigma_{2l}\sigma_{3l} + \sigma_{3l}\sigma_{1l}\right) \leq R^2 . \tag{25}$$

Relative to the proper axes of the ellipsoid this equation becomes:

$$\frac{\sigma_{1l}^2}{\left(\dfrac{\sigma_a}{\sqrt{1+\nu}}\right)^2} + \frac{\sigma_{2l}^2}{\left(\dfrac{\sigma_a}{\sqrt{1-\nu}}\right)^2} + \frac{\sigma_{3l}^2}{\left(\dfrac{\sigma_a}{\sqrt{1-\nu}}\right)^2} = 1 . \tag{26}$$

For $\nu = 0.5$, Eq.(26) corresponds to the failure theory V.

The small semi-axes of each ellipsoid represents the radius *R* of the sphere $\mathcal{D}$.

Conclusion. *All failure theories have a sphere with radius R as the domain $\mathcal{D}$. This assures the unconditioned strength of the elastic structure for different loads in a limited domain.*

6. CASE OF THE BRITTLE MATERIALS

In the some manner as in chapters 4 and 5, for brittle materials the domain $\mathcal{D}_I$ can be established. For these materials the admissible tensile stress σ_{at} differs from the admissible compressive stress σ_{ac}. For example, referring to the failure theory I, in Fig. 9 the domain for the Coulomb-Mohr theory (the straight 1) and for the modified Mohr theory (the straight 2) are presented [5, 6].

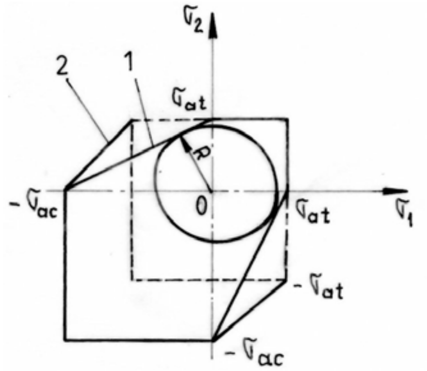


Fig. 9

It can be easily shown that for the Coulomb-Mohr theory we have:

$$R = \frac{\sigma_{at}\left|\sigma_{ac}\right|}{\sqrt{\sigma_{at}^2 + \sigma_{ac}^2}} ,$$

and that for the modified Mohr theory the value of the radius R is σ_a .

7. CONCLUSIONS

As a rule, at present classical failure theories are applied to verify the elastic structure subjected to a known system of loads.

If we want to apply these theories to determine the limit loads or to calculate the geometric characteristics of the critical section of the elastic structure it is necessary to anticipate the limit stresses from the contour surface (or line) Γ of the domain $\mathcal{D}$ of the possible principal stresses.

As we showed in this paper, only in special casses the position of the principal stresses from the contour surface (line) can be determined. In such situations, the position of the principal stress directions relative to the reference system *0xyz* remain immovable.

In this paper we proposed a restricted domain $\mathcal{D}_1 \subset \mathcal{D}$ which can assure the strength of the elastic structure for any position of the principal stresse directions. Thus, we can calculate the limit loads which are independent relative to their values and senses in the critical structure section.

The proposed inequation (20) has the same form for all failure theories.

The contour line (for the plane stress state) or the contour surface (for the space strain state) can be used for calculation of the limit loads or geometric characteristics of the critical structure section.

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