GIMBALS FOR VECTOR CONTROL OF NERVA LIQUID MOTOR TRANSPORTERS

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The team for space technology of UPB presents the project for achieving a proprietary orbital access launching system under the NERVA readiness opportunity. The NERVA vehicle is a readily available and low cost means to demonstrate the feasibility to achieve secured high altitude sub-orbital flights with a final delivered velocity of one half the orbital velocity at 100 miles altitude, a typical low Earth orbit (LEO), by the conversion of dismissed soil-air SA-2 Guideline systems. These weapons became obsolete recently in Romania. The main technological feature regards the gimbal-type support of the rocket motor, presently within a fastened mounting. The problems arouse by the spherical bearing and its controls are revised in the paper. Application focuses on the mechanics of the motor.

1. SMALL LAUNCHERS

A true space research program can not be structured beyond the development of proprietary orbital injection launchers. Unfortunately, Romanian involvement in building space launchers is still very weak, despite the great potential of the national aerospace industry and the well-known inheritance in astronautics. During the last years the team of space engineering from the University “Politehnica” of Bucharest (UPB) had repeatedly proposed a series of solutions for building small and cheap orbital launchers for applications satellites, under the *NERVA* project [1]. The most accessible solution starts from the available SA-2 military anti-aircraft rocket vehicles, planed for withdrawal, and involves a series of precise and limited modifications that allow achieving half of the LEO injection velocity with ease. It is the purpose of the authors to present here the analysis of these modifications regarding the thrust vector control, through a gimbal mounting of the existing liquid propellant rocket engine (LRE) and the corresponding driver control. The resulting sub-orbital launcher must be further extended with an upper, third stage, to finally achieve the full orbital injection capability. Construction of that vehicle will introduce Romania among the few countries in the world with orbital transportation capacity at very low development costs.

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Besides the interest in building a national readiness launcher, the international interest in low cost satellites and orbital transporters for responsive space access is now growing strong. The number of offered small to medium sized orbital transporters is increasing as a real and practical business, due to a large number of profitable applications. The field of *small satellites* became an important challenge for any country with aerospace industry potential. A yearly international conference on this subject became a tradition at the US Iowa State University, where the subject devised a serious attention, never predicted in the past. Not only large national space agencies but also private companies with the capability of developing cheep space transporters are already developing such projects. It is to mention only the international “Sea Launch” and US “SpaceX” [2] companies, or “Orbital” [3], “t/Space” [4] and the Russian “Air Launch Corporation”. Claimed for long as the optimal solution for LEO injection, the return to expandable launchers was bolstered by the final retirement of the previously intended low cost, secure Space Shuttle system in US. In the meantime the high expectations in a low cost system had completely vanished. Not only US are in the phase of retiring the shuttle fleet, still Russia had ceased for long to continue work on its Buran space shuttle version, otherwise very similar to the American one. Other similar reusable orbital systems from Japan, China and Europe had also been cancelled due to uncertainties exhibited during their own national developments [6] and due to the half failure of the American program with reusable transporters. An early demonstration of Ruppe that single-shot launchers are more logical finally proved correct [7].

2. NERVA LAUNCHER DYNAMICS

Small vehicles were considered in Spain by “INTA”, within the AQUARIUS air-launch project. Very light structure solid rocket engines of Spanish technology are desired and the Eurofighter as an air launch platform [9]. The foreground of the NERVA vehicle is the common soil-air weapon under the NATO designation of SA-2 or Guideline, now obsolete in Romania. This missile will be subjected to a simple and precisely controlled re-conversion from military into a research and possibly commercial vehicle. SA-2 is a derivation on its turn of the brilliant *Rheintochter* missile, successfully developed at Peenemunde in early 1945 by the rocket team of Wernher von Braun. This development adds to the previous firsts of the ADDA team of space propulsion and space technology division of UPB. The developments include the design, manufacturing and successful test firing of the first Romanian liquid propellant rocket engine in 1969 and of the first air-breathing rocket motor in 1987 in Fagaras. Another first of the UPB team is the project of a low mass, 3-axes stabilized (3-AS) *PUBSAT* nanosatellite [9]. The involvement of

University “Politehnica” of Bucharest in FP7 European Space priority projects and in other project proposals with France and Italy adds extra motivation in the development of the NERVA launcher [10] for PUBSAT. It consists of a readily available, low cost demonstrator for achieving half of the local orbital velocity at 100 miles altitude for the envisaged 3-AS satellite. This requires however a drastic improvement of the structural efficiency of SA-2 Guideline system, to accommodate higher thrust enhancement of the solid rocket motor (SRM) booster, thrust vectorization of the LRE and lightweight sustainer structure for the second stage. This refers to extended propellant tanks and a new, lightweight guidance avionics. The improved rocket transporter remains within the accessible Romanian technology, addressed through a high level industrial consortium (Electromecanica, AEROFINA, Factory of Powders in Fagaras and ELAROM), with UPB as project coordinator. The expertise of the ADDA SME (Association Dedicated to Development in Astronautics) and the existing capabilities of the industrial partners motivate a strong confidence in this project. The dynamics of thrust vectorization is here approached including the mobility of liquid propellant in the tanks of the vehicle. The scheme of the basic rocket is drawn in Fig. 2 from below.

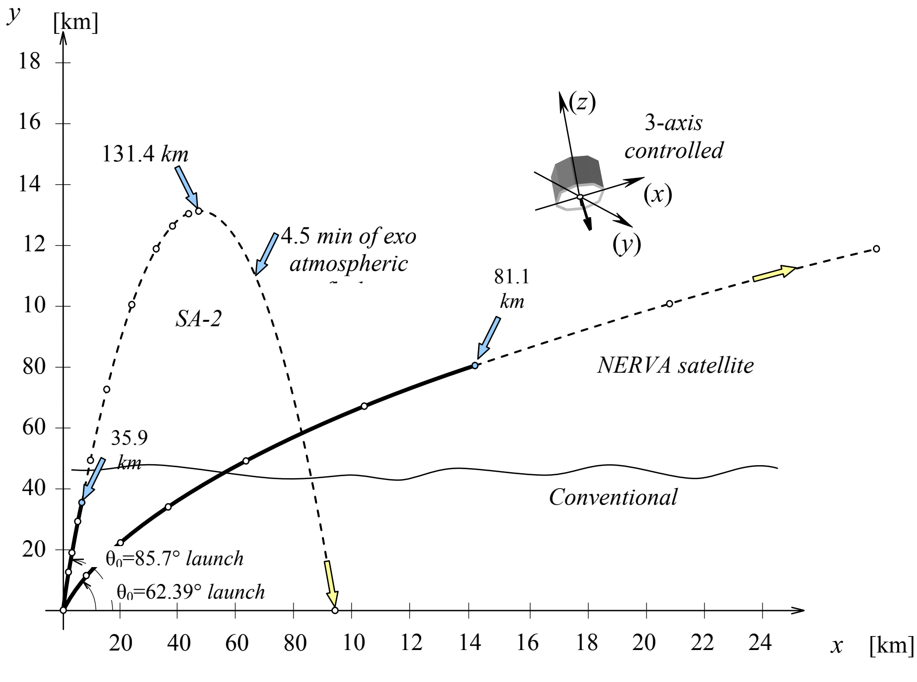

Fig. 1 – Trajectories and guidance requirements for SA-2 missile versus NERVA satellite launcher.

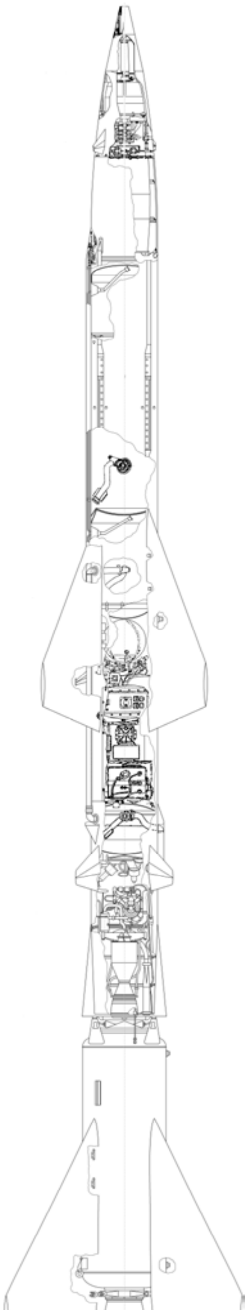

Fig. 2

3. VARIATIONAL SOLUTION FOR SLOSHING IN ALMOST EMPTY TANKS

Computation of eigen-oscillations of the inviscid liquid in rigid cavities are conveniently solved through a variational approach, followed by numerical calculations of the eigen-shapes and eigen-frequencies through a Rayleigh-Ritz procedure [12]. Asymptotic developments along the gravity were first used with incomplete, quadratic polynomial for deep tanks [12]. For shallow or almost empty rocket tanks the solution was first extended to full polynomial in [12] and is here applied to NERVA case with thrust vector control by gimbals bearing of the rocket engine.

Applying for instance the variational principle of Hamilton,

$$\delta I = \delta \int_{t_1}^{t_2} L \cdot \mathrm{d}t = \delta \int_{t_1}^{t_2} \left(T - V\right) \cdot \mathrm{d}t = 0\,, \tag{1}$$

the potential of velocities $\Phi(X, t)$ results by extremizing the functional:

$$I = \frac{1}{2} \cdot \int_{t_1}^{t_2} \mathrm{d}t \left[\int_V \left|\nabla\Phi\right|^2 \cdot \mathrm{d}V - \frac{1}{g} \cdot \int_S \Phi^2 \cdot \mathrm{d}S \right], \tag{2}$$

where V is the volume of the liquid and S is the free surface. The Fourier procedure of separation of variables is applied and the potential of velocities is considered as a harmonic time function

$$\Phi\left(X, t\right) = \Psi\left(X\right) \cdot \sin\left(\omega \cdot t + \varphi\right). \tag{3}$$

The integration in time is in this case immediately performed and the eigen-shape of the oscillation $\Psi(X)$ follows through the minimization of the functional:

$$2 \cdot I = \int_V \left|\nabla\Psi\right|^2 \cdot \mathrm{d}V - \frac{\omega^2}{g} \int_S \Psi^2 \cdot \mathrm{d}S\,. \tag{4}$$

For the sake to simplify the calculations, two limiting cases are profiling. First, whether the tank is very little filled with liquid, namely the liquid volume looks shallow, it may be supposed that the influence of the depth on the oscillations is low, negligible at the limit. In this case the potential of the motion does not depend on the ordinate and thus:

$$\Psi\left(r, \theta, z\right) = F\left(r, \theta\right), \tag{5}$$

the problem becoming bi-dimensional, fact that sensibly simplifies the calculations.

Secondly, whether the tank is filled with a high depth, it is likely to appreciate that the sloshing motion is exponentially diminishing through the interior of the liquid, put in the form

$$\Psi(r,\theta,z) = \mathrm{e}^{\lambda_H \cdot z} \cdot E(r,\theta), \tag{6}$$

where $\lambda_H = \dfrac{\omega^2}{g}$, once again the problem becoming bi-dimensional.

The axial symmetry of the tanks allows that the circular eigen-shape be separated and behave with its own harmonic components,

$$F(r,\theta) = G(r) \cdot \cos(s \cdot \theta), \tag{7}$$

where the even number $s \mid s = 0, 1, \ldots$ represents the rank of the circular harmonics of the eigen motion. This way the problem is reducing to one dimension only, namely to the determination of the radial eigen shape $G(r)$. The functional becomes:

$$I = \int_0^{r_0} r \left[h \left(\frac{\mathrm{d}G^2}{\mathrm{d}r} + \frac{s^2}{r^2} \cdot G^2 \right) - \frac{\omega^2}{g} \cdot G^2 \right] \cdot \mathrm{d}r, \tag{8}$$

where the shape of the generating line of the tank $h(r)$ follows to be introduced. It is convenient this generator geometry to be expressed in a polynomial manner:

$$h(r) = h_0 \sum_{i=0}^{K} k_i \cdot \left(\frac{r}{r_0} \right)^i, \tag{9}$$

fact mainly oriented to conical and parabolic tanks applicability, yet not only. For parabolic tanks, Lamb [12] had obtained an exact solution for the eigen shape $G(r)$ of quadratic polynomial type. Extending this supposition, we search a solution of the variational problem in the forma of:

$$G(r) = \sum_{j=0}^{N} a_j \left(\frac{r}{r_0} \right)^{s+2\cdot j} \tag{10}$$

compulsory for avoiding the singularity in the term $\dfrac{s^2}{r^2} \cdot G^2$ when $r = 0$ and $s \neq 0$. The problem reduces to finding the $(N+1)$ unknown coefficients a_j of the polynomial $G(r)$ by solving the simultaneous equations:

$$\frac{\partial I}{\partial a_j} = 0, \; j = 0, 1, \ldots, N.$$

This is the Rayleigh-Ritz procedure [20]. In order to develop it, the functional is put under the form:

$$I=\int_0^{r_0}\left[h\left(\frac{\mathrm{d}G^2}{\mathrm{d}r}\right)+h\cdot\frac{s^2}{r^2}\cdot G^2-\frac{\omega^2}{g}\cdot G^2\right]\cdot r\cdot\mathrm{d}r=I_1+I_2+I_3\,, \tag{11}$$

where the functions appear:

$$\frac{\mathrm{d}G}{\mathrm{d}r}=\frac{1}{r_0}\sum_{l=0}^{N}a_l\left(s+2\cdot l\right)\left(\frac{r}{r_0}\right)^{s+2\cdot l-1},$$
$$\left(\frac{\mathrm{d}G}{\mathrm{d}r}\right)^2=\frac{1}{r_0^{\,2}}\sum_{l=0}^{N}\left[a_l\cdot\left(s+2l\right)\left(\frac{r}{r_0}\right)^{s+2l-1}\cdot\sum_{m=0}^{N}a_m\left(s+2m\right)\left(\frac{r}{r_0}\right)^{s+2m-1}\right]. \tag{12}$$

The following integrals are immediately evaluated,

$$\int_0^1\rho^{2(s+l+m)+i-1}\cdot\mathrm{d}\rho=\frac{1}{2\left(s+l+m\right)+i},\quad \int_0^1\rho^{2(s+l+m)+1}\cdot\mathrm{d}\rho=\frac{1}{2\left(s+l+m+1\right)} \tag{13}$$

with the variable $\rho=r/r_0$. As far as

$$\frac{\partial I}{\partial a_j}=\frac{\partial I_1}{\partial a_j}+\frac{\partial I_2}{\partial a_j}+\frac{\partial I_3}{\partial a_j}=0 \tag{14}$$

and

$$\frac{\partial I_1}{\partial a_j}=h_0\left[\sum_{l=0}^{N}a_l\left(s+2l\right)\left(s+2j\right)\sum_{i=0}^{K}\frac{k_i}{2\left(s+l+j\right)+i}+\left(s+2j\right)\sum_{m=0}^{N}a_m\left(s+2m\right)\sum_{i=0}^{K}\frac{k_i}{2\left(s+m+j\right)+i}\right]$$
$$\frac{\partial I_2}{\partial a_j}=\frac{\partial}{\partial a_j}h_0s^2\sum_{l=0}^{N}\sum_{m=0}^{N}\sum_{i=0}^{K}a_la_m\frac{k_i}{2\left(s+l+m\right)+i}=2h_0s^2\sum_{m=0}^{N}a_m\sum_{i=0}^{K}\frac{k_i}{2\left(s+m+j\right)+i}$$
$$\frac{\partial I_3}{\partial a_j}=-\frac{\omega^2}{g}r_0^2\sum_{m=0}^{N}\frac{a_m}{s+m+j+1} \tag{15}$$

the following system of N+1 homogeneous equations result for the N+1 coefficients a_j:

$$\sum_{m=0}^{N}\left\{\left[\left(s+2j\right)\left(s+2m\right)+s^2\right]\cdot\sum_{i=0}^{K}\frac{k_i}{s+\frac{i}{2}+j+m}-\frac{\omega^2\cdot r_0^2}{g\cdot h_0}\cdot\frac{1}{s+j+m+1}\right\}\cdot a_m=0\,. \tag{16}$$

By generally dividing through 2 and denoting the quadratic nondimensional frequency as

$$b^2 = \frac{\omega^2}{g} \cdot \frac{r_0^2}{h_0}, \qquad \lambda = \frac{\omega^2}{g},$$

the system appears as

$$\sum_{m=0}^{N} A_{jm} \cdot a_m = 0, \qquad A_{jm} = \sum_{i=0}^{K} \frac{s^2 + (j+m) \cdot s + 2 \cdot j \cdot m}{s + \frac{i}{2} + j + m} \cdot k_i - \frac{b^2/2}{s+j+m+1}, \quad (17)$$

where j stands for the row and m for the column.

The condition of compatibility $\left|A_{jm}\right| = 0$ constitutes the seculary equation of degree N+1 in

$$x = \frac{b_{sr}^2}{2}, \quad (18)$$

which supplies the N+1 eigen-frequencies of the radial modes r, corresponding to the circular eigen-mode s.

The seculary equation has the form:

$$\left|A_{jm}\right| = \left|a_{jm} + b_{jm} \cdot x\right| = \left|a_{jm}\right| + x \cdot \left|b_{jm}\right| = 0, \quad (19)$$

where:

$$a_{jm} = \sum_{i=0}^{K} \frac{s^2 + (j+m) \cdot s + 2 \cdot j \cdot m}{s + \frac{i}{2} + j + m} \cdot k_i, \quad b_{jm} = -\frac{1}{s+j+m+1}, \quad j, m = 0, 1, ..., N. \quad (20)$$

The matrix A_{jm} is simetrical, consequently $(N+1)\left(\frac{N}{2}+1\right)$ elements may only be computed. In this research the computation of determinants was preferred for calculating the coefficients of the secular polynomial. For example, in parabolic approximation N=2 of the radial eigen-shape, when A_{jm} is of rank 3, secular equation

$$C_1 + C_2 \cdot x + C_3 \cdot x^2 + C_4 \cdot x^3 = 0 \quad ($$

has the following coefficients (the indexes were omitted):

$$C_1 = \begin{vmatrix} a & a & a \\ a & a & a \\ a & a & a \end{vmatrix}, \qquad C_2 = \begin{vmatrix} a & a & a \\ a & a & a \\ b & b & b \end{vmatrix} + \begin{vmatrix} a & a & a \\ b & b & b \\ a & a & a \end{vmatrix} + \begin{vmatrix} b & b & b \\ a & a & a \\ a & a & a \end{vmatrix},$$

$$C_3 = \begin{vmatrix} a & a & a \\ b & b & b \\ b & b & b \end{vmatrix} + \begin{vmatrix} b & b & b \\ a & a & a \\ b & b & b \end{vmatrix} + \begin{vmatrix} b & b & b \\ b & b & b \\ a & a & a \end{vmatrix}, \qquad C_4 = \begin{vmatrix} b & b & b \\ b & b & b \\ b & b & b \end{vmatrix}, \quad (22)$$

where eight determinants follow.

The accuracy is further increased when a complete, not a quadratic, polynomial is used [13]

$$G(\rho) = \rho^s \cdot \sum_{n=0}^{N} a_n \cdot \rho^n \ . \tag{23}$$

Now the solving system (17) becomes

$$\sum_{m=0}^{N} \left\{ \left[s^2 + (s+n)(s+m) \right] \cdot \sum_{i=0}^{H} \frac{h_i}{2s+m+n+i} - \frac{\dfrac{\omega^2 \cdot r_0^2}{2gh_0}}{s+1+\dfrac{m+n}{2}} \right\} \cdot a_m = 0 \ . \tag{24}$$

The accuracy of the complete form for the polynomial regarding the contour of the axis-symmetrical tank was checked through a mass of numerical computations with different geometries of the tank bottom. The results are applicable to the NERVA dynamics computations when the final period of the ascent powered flight is considered. The fuel tanks are almost empty at that time.

The spherical or cross bearing is introduced for the vector control of the thrust (vectorization). The rocket vehicle becomes a controllable device and the stability of the control system essentially depends on the dynamic characteristic of the vehicle, where the mobility of the liquid, due to its high mass fraction in the vehicle total mass, plays an important role.

4. RESULTS AND CONCLUSIONS

The computation of the eigen-frequencies and eigen-shapes of oscillation in circular, conical tank bottom had been performed in double precision [13]. The results from running the "AZET–2" computer code are presented in Table 1.

Table 1

Computed eigen-frequencies in the conical bottom tank [13]

S	N	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$R = 6$	$r = 7$
0	2	0	2,173171	7,8796	3,864514	–	–	–
	3	0	2,173030	7,0529	3,807779	5,521689	–	–
	4	0	2,173006	7,0329	3,807646	5,407531	7,184181	–
	5	0	2,173002	7,0159	3,807585	5,407530	6,995663	8,857246
1	2	1,145928	2,903234	11,9583	4,617685	–	–	–
	3	1,145868	2,902906	8,7962	4,538179	6,286465	–	–
	4	1,145849	2,902813	8,6487	4,538047	6,143658	7,960023	–
	5	1,145842	2,902787	8,5378	4,537930	6,143546	7,737451	9,643510

Table 1

(continued)

2	2	1,542404	3,418094	15,6648	5,184273	–	–	–
	3	1,542388	3,417967	10,1034	5,109796	6,887444	–	–
	4	1,542385	3,417933	9,9718	5,109776	6,750262	8,585337	–
	5	1,542389	3,417929	9,9714	5,109931	6,749491	8,369046	10,285653
3	2	1,845170	3,851150	11,6035	5,674274	–	–	–
	3	1,846512	3,851100	11,4431	5,607837	7,416262	–	–
	4	1,846511	3,851087	11,3548	5,607765	7,290673	9,140830	–
	5	1,846478	3,851008	11,3472	5,612761	7,248537	9,053733	10,767590

The values for the first radial modes are converging fast, especially for low rank circular modes. The first circular mode appears for $s = 0$, in the known order. The high rank eigen-frequencies are given for demonstration only, as far as in stability calculations the first oscillation modes are only important, due their significant amplitude. For less filled conical tanks, on one side and for deep, full tanks on the other, the normalized frequency of the first radial mode is confirming that the depletion of the liquid in the tank has the counterintuitive effect of reducing the frequency and amplitude of the eigen-oscillations [5]. The absolute frequencies depend on the absolute size of the tank [13].

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REFERENCES

1. * * *, *International Launch Services Press Release*, Tuesday, September 6, 2005.
2. * * *, SpaceX *Excellent Engineers Wanted*, Av. Week Space Techn., Sept. 25, 2006, p. 67.
3. SARIGUL-KLIJN, M., SARIGUL-KLIJN, N., MORGAN, B., TIGHE, J., LEON, A., HUDSON, G., MCKINNEY, B., GUMP, D., *Flight Testing of a New Air Launch Method for Safely Launching Personnel and Cargo into LEO*, AIAA-2006-1040.
4. * * *, http://www.nasa.gov/home/hqnews/2007/feb/HQ_0720_COTS_agreements.html, *NASA Signs Agreement with t / Space*. Last accessed on 02/01/2008.
5. MILES, J. W., *Note on the damping of free-surface oscillations due to drainage*, Journal of Fluid Mechanics, **12**, Part 3, pp. 438-440, March 1963.
6. KOELLE, H. H., *Handbook of Astronautical Engineering*, McGraw-Hill, N.Y., 1961.
7. LINDBERG, R. E., *Overview of the Pegasus Air-Launched Space Booster*, SAE, Aerospace Technology Conference and Exhibition, Anaheim, CA, Sept. 25-28, 1989.
8. MATSUO, H., KOHNO, W., MAKINO, T., NAGAO, Y., HIROSE, H., *Conceptual study of air-launch small satellite launcher system*, Proceedings of the 35th Space Sciences and Technology Conference, Nagaoka, Japan, Oct. 28-31, 1991 (A93-56251 24-12), pp. 121-122.

9. RUGESCU, R. D., *NERVA Vehicles, Romania's Access to Space*, U.P.B. Sci. Bull., Series D, **70**, *3*, Bucharest, Romania, 2008.
10. RUGESCU, R. D., *NERVA Romanian Non-Orbital Entry-Return Vehicle Assessor Project*, Texas A&M University, USA, Paper IAC-06-B5.5.11, Proceedings of the 57th IAC Congress, Valencia, Spain, Oct. 02-06, 2006.
11. RUGESCU, R. D., *Eigen Oscillations in Deep Rocket Tanks*, Sc. Bull. of I.P.B., Series in Mechanics, **43**, *4*, pp. 39-42, 1981.
12. LAWRENCE, H.R., WANG, C. J., REDDY, R.B., *Variational Solution of Fuel Sloshing Modes*, Jet Propulsion, **28**, *11*, pp. 729-736, November 1958.
13. RUGESCU, R. D., *Contributions in Optimization of Rocket Vehicles*, Ph.D. Thesis Draft, Inst. Politehnic, Bucharest, 1987, pp. 183-213.