DISCRETE MATHEMATICAL FILTERS OF VIDEO INFORMATION PROCESSING FOR AUTOMATIC IMAGE COMPONENTS DETECTION AND MARKING

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Abstract. In this paper we shall discretize two classes of mathematical filters intended for the video image analysis with the purpose to automatically detect and mark the components that these images contain. Unlike the classical mathematical filters, their discrete correspondent defined by us in this paper has the advantage of being more adaptable to the specific character of a video image, which is discrete by its nature. From this point of view the research that we have undertaken will present not only theoretical interest but also a strong practical character, simplifying considerably the work of software developers.

1. INTRODUCTION

In the field of computerized video image sequence analysis, one of the most intensive studied problems for the time being, is, due to its numerous practical applications, the problem of detecting and automatic marking of the forms (shapes) within a given image. During the time the problem has been approached in various ways [1, 3, 4, 5]. In most cases the approaches have been of mathematical nature. Within our researches in this field we have chosen to extend the method proposed by [1]. More exactly, the path we have chosen to study this problem refers to the use of some mathematically natured filters wherein the analyzed image undertakes a transformation meant to highlight its contrast related differences by which the various details that populate it distinguish from the general context. In this respect in [3] we have proposed a new type of filter meant to cover the shortages of the initial filter proposed by M. Demi in the above mentioned paper.

If, from a theoretical point of view the problem of detecting and marking the forms of a video image is solved by any of the instruments developed in papers [1, 3, 6], from a practical point of view things are not the same. This fact is caused by a mismatch in the general compatibility between a highly idealized mathematical nature of the algorithm used and the limitations of current

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technologies in developing devices that reproduce video images. More precisely it is about the mismatch between the “continuous” nature of the filters applied by the algorithm (in the absence of prior adjustments) and the “discrete” nature of displays which are made of a finite number of pixels.

In order to help programmers, in this paper we shall indicate a way in which detecting and automatic marking algorithms of the forms of a given image, presented in [1] or [3] can be expressed in computer language, by thus obtaining discretized variants of filters defined in the aforementioned papers. The filters obtained in this paper by discretizing the filters presented in [1] and [3] also allow a direct definition. This thing is extremely important in the development of practical applications, since it simplifies and reduces the effort of applying the algorithm.

2. DETECTION AND AUTOMATIC MARKING OF FORMS IN A VIDEO IMAGE

Within this section we shall present a general algorithm that detects and automatically marks details (forms) in a video image that relies on the use of special mathematical objects meant to separate the points on the outline of various details or figures of the analyzed image from the rest of the points of that image named, by this reason, image filters.

The mathematical processing of any video image $\mathcal{I}$ starts by defining the linking objects between the hardware components meant to visualize the video images and the notions specific for the mathematical apparatus that we are going to use. Denoting by $I(x, y)$ the luminous intensity (the gray level) of the pixel of coordinates (x, y), we obtain a function I defined on the surface on which image $\mathcal{I}$ is projected (i.e. the screen of a monitor). The components that form the image (objects or beings) are demarcated by the contrast differences between them and the background. The boundaries that mark these contrasts are mathematically represented by extreme values or by the singularities of function I. In this way, the process of automatic detection of the components of a certain image $\mathcal{I}$ is based on the use of some special filters aimed to separate or to amplify the extremals or the discontinuities of function I from the rest of the points of the image $\mathcal{I}$.

For the time being let us dispose of such a filter and let us, based on it, define a function $d : I \to \mathbb{R}_+$ that expresses the distance from a certain point $P(x, y) \in \mathcal{I}$ to the frontier (outline) of image $\mathcal{I}$'s nearest detail (Fig. 1). By using the relation

$$\bar{s}_P = \frac{d(x,y)}{\sqrt{d_x'^2(x,y) + d_y'^2(x,y)}} \left(d_x'(x,y)\bar{i} + d_y'(x,y)\bar{j} \right), \tag{1}$$

where $\bar{i}, \bar{j}$ are the unit vectors directed along the coordinate axes $0x$, and respectvely $0y$, we obtain a simple possibility to determine the shifting direction from a certain point $P(x,y)$ on image I 's surface to the frontier of the nearest detail D that populates image I (Fig. 1).

In the analyzed image I, we denote by D that detail we want the "video-camera-computer" system to detect and to automatically mark. In order to signal out our intention to the system, we surround the image's area of interest by using a closed curve γ. Thus, curve γ becomes the first approximation of the searched contour. The problem that we now must solve consists in determining a transformation process of curve γ in the exact contour ∂D of the detail D. By using the previously obtained relation (1) we can solve this problem by describing the transformation $\gamma \ni (x,y) \to (x^*, y^*) \in \partial D$ by the following formulas

$$x^* = x + \frac{d(x,y)d_x^{'}(x,y)}{\sqrt{d_x^{'2}(x,y)+d_y^{'2}(x,y)}},\ y^* = y + \frac{d(x,y)d_y^{'}(x,y)}{\sqrt{d_x^{'2}(x,y)+d_y^{'2}(x,y)}} .(2)$$

If the expected results are not obtained from the first application, the procedure is repeated by considering curve γ's image γ^* obtained within the first step, as the first approximation of outline ∂D.

In order to be operational, the algorithm must meet other extra requirements. Namely, the way in which we move the points located in the neighborhood of a contour towards the points belonging to that contour must be conceived in such a way that the corresponding

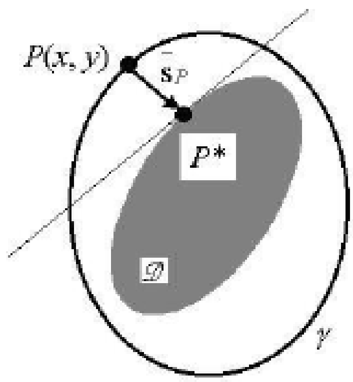

Fig. 1 – The intuitive presentation of the way the mathematical algorithm functions.

mathematical operations have as few references to function $I = I(x, y)$ as possible because function I is not previously known.

3. CLASSICAL MATHEMATICAL FILTERS

In this section we shall present two filters capable to fulfill the conditions implied by the applicability of the algorithm presented in the previous section. The first takes the form:

$$F_\sigma(x,\ y) = \left(\frac{x}{\sigma^2} \mathrm{e}^{-\frac{x^2+y^2}{2\sigma^2}} * I(x,y) \right)^2 + \left(\frac{y}{\sigma^2} \mathrm{e}^{-\frac{x^2+y^2}{2\sigma^2}} * I(x,y) \right)^2,\ \sigma > 0, \quad (3)$$

where "*" represents the convolution product, and was defined by M. Demi in [1]. In the particular case when the light intensity function $I = I(x,y)$ takes the form

$$I(x,y) = \begin{cases} 0, & y > mx, \\ \alpha, & y \le mx, \end{cases} \quad (4)$$

where $\alpha, m \in$, $\alpha \neq 0$, the filter (3) shows the essential property

$$F_\sigma(x,y) = 2\pi\alpha^2\sigma^2 \mathrm{e}^{-\frac{d^2(x,y)}{\sigma^2}}, \quad (5)$$

where $d(x,y)$ represent the Euclidian distance from point $P(x,y)$ to the straight line $d: y = mx$.

Relation (5) enables the determination of the distance function $d = d(x,y)$ in the following two ways

$$d(x,y) = \sigma\sqrt{\ln\left(2\pi\alpha^2\sigma^2\right) - \ln F_\sigma(x,y)}, \quad (6)$$

or

$$d(x,y) = \sigma^2 \frac{\sqrt{F'^{\,2}_{\sigma_x}(x,y) + F'^{\,2}_{\sigma_y}(x,y)}}{2F_\sigma(x,y)}. \quad (7)$$

The practical experiments realized by means of functions (6) or (7) have shown that these are adaptable to the requirements of the problem that we want to solve, the algorithm described in the previous section being quite functional.

The second filter that we want to present takes the form:

$$F_\sigma(x,\ y) = \left(\frac{x}{\sigma^2} \frac{1}{\left(1 + \frac{x^2+y^2}{2\sigma^2}\right)^2} * I(x,y) \right)^2 + \left(\frac{y}{\sigma^2} \frac{1}{\left(1 + \frac{x^2+y^2}{2\sigma^2}\right)^2} * I(x,y) \right)^2,\ \sigma > 0,$$

(8)

and was defined in [3] as an alternative to M. Demi's filter*. As in the case of the filter (3), when the light intensity function $I = I(x, y)$ takes the special form (4), we can show that filter (8) has the property

$$F_\sigma(x, y) = \frac{2\pi^2\alpha^2\sigma^2}{1 + \frac{d^2(x, y)}{2\sigma^2}},\ \sigma > 0, \tag{9}$$

where, as earlier, $d(x, y)$ represents the Euclidian distance from point $P(x, y)$ to the straight line $d : y = mx$. This last relation is also essential for our theory, since it provides another way to express the distance function d, namely

$$d(x, y) = \sigma\sqrt{\frac{4\pi^2\alpha^2\sigma^2 - 2F_\sigma(x, y)}{F_\sigma(x, y)}} \tag{10}$$

As in the case of the experiments realized by means of the distance functions given by relations (6) or (7), the use of function (10) within the algorithm that marks details of a video image, leads to acceptable results.

4. DISCRETE MATHEMATICAL FILTERS

The effective computation of expressions (3) and (8) that delivers one of the filters necessary for the automatic marking algorithm of the details in a video image, as well as the computation of the partial derivatives of these filters can be done only after having analyzed that image, namely while running the software program that puts in practice the algorithm described in the beginning of this paper because the light intensity function $I = I(x, y)$ is not a priori known. Since no computer can calculate defined integrals automatically otherwise but utilizing numerical approximation methods, we have to further on indicate a way to realize this.

The effective evaluation of filter (3) implie the computation of double, improper integrals of the form

$$\iint_{\mathbb{R}^2} (x - \xi) e^{-\frac{(x-\xi)^2 + (y-\eta)^2}{2\sigma^2}} I(\xi, \eta) d\xi d\eta,\ \iint_{\mathbb{R}^2} (y - \eta) e^{-\frac{(x-\xi)^2 + (y-\eta)^2}{2\sigma^2}} I(\xi, \eta) d\xi d\eta.$$

* The main disadvantages of Demi's filter (3) consist in the fact that, by greatly amplifying the discontinuities of function I, the background noises that accompany image I are inevitably amplified. As compared to filter (3), filter (8) has the advantage of amplifying this background noise to a lower extent.

Analogous, the effective evaluation of filter (8) implies the computation of double, improper integrals of the form

$$\iint_{\mathbb{R}^2} \frac{(x-\xi) I(\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta}{\left(1+\frac{(x-\xi)^2+(y-\eta)^2}{2\sigma^2}\right)^2}, \iint_{\mathbb{R}^2} \frac{(y-\eta) I(\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta}{\left(1+\frac{(x-\xi)^2+(y-\eta)^2}{2\sigma^2}\right)^2}.$$

Thus, the description of a procedure to numerically approximate the presented integrals becomes necessary. For this pourpose we shall generically denote by

$$\iint_{\mathbb{R}^2} f(x,y,\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta, \tag{11}$$

one of these integrals. Let (x,y) be an fixed point from $\mathbb{R}^2$. Because

$$\iint_{\mathbb{R}^2} f(x,y,\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta = \lim_{a\to\infty, b\to\infty} \int_{-a}^{a}\int_{-b}^{b} f(x,y,\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta,$$

for a fixed precision degree $\varepsilon > 0$, we can choose $a > 0,\ b > 0$, so that

$$\left| \iint_{\mathbb{R}^2} f(x,y,\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta - \int_{-a}^{a}\int_{-b}^{b} f(x,y,\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta \right| < \frac{\varepsilon}{2}.$$

By also using a cubature formula (in this pourpose see, for example, [2]), we can approximate the value of the proper integral

$$\int_{-a}^{a}\int_{-b}^{b} f(x,y,\xi,\eta) \mathrm{d}\xi \mathrm{d}\eta, \tag{12}$$

by an error of at most $\frac{\varepsilon}{2}$. This last aproximation shall express the value of the improper integral (11), with an error of at most ε.

In order to be more explicit, let us suppose that we use Simpson's cubature formula. In this case if we denote

$$h = \frac{a}{n},\ k = \frac{b}{m},\ \xi_i = -a + ih,\ \eta_j = -b + jk,\ i = 0, 1,.., 2n,\ j = 0, 1,.., 2m,$$

parameters m and n being determiend depending on the precision degree established for the approximation, then

$$S = \frac{hk}{9} \sum_{i=0}^{2n} \sum_{j=0}^{2m} \lambda_{ij} f(x,y,\xi_i,\eta_j), \tag{13}$$

where λ_{ij} are the corresponding elements of the matrix

$$\begin{pmatrix} 1 & 4 & 2 & 4 & 2 & \dots & 4 & 2 & 4 & 1 \\ 4 & 16 & 8 & 16 & 8 & \dots & 16 & 8 & 16 & 4 \\ 2 & 8 & 4 & 8 & 4 & \dots & 8 & 4 & 8 & 2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 2 & 8 & 4 & 8 & 4 & \dots & 8 & 4 & 8 & 2 \\ 4 & 16 & 8 & 16 & 8 & \dots & 16 & 8 & 16 & 4 \\ 1 & 4 & 2 & 4 & 2 & \dots & 4 & 2 & 4 & 1 \end{pmatrix},$$

(for more details see [2]) will approximate the value of integral (12), by an error of at most $\varepsilon/2$. Within the work hypothesis previously used, quantity (13) will approximate the value of integral (11) by an error of at most ε.

By using this evaluation procedure, we can define the discrete form of filters (3) and (8) by

$$F_\sigma(x,y) = \lim_{m,n\to\infty}\left(\frac{hk}{9\sigma^2}\right)^2\left[\left(\sum_{i=0}^{2n}\sum_{j=0}^{2m}\lambda_{ij}(x-\xi_i)e^{-\frac{(x-\xi_i)^2+(y-\eta_j)^2}{2\sigma^2}}I(\xi_i,\eta_j)\right)^2 + \right.$$
$$\left. +\left(\sum_{i=0}^{2n}\sum_{j=0}^{2m}\lambda_{ij}(y-\eta_j)e^{-\frac{(x-\xi_i)^2+(y-\eta_j)^2}{2\sigma^2}}I(\xi_i,\eta_j)\right)^2\right], \tag{14}$$

and respectively, by

$$F_\sigma(x,y) = \lim_{m,n\to\infty}\left(\frac{hk}{9\sigma^2}\right)^2\left[\left(\sum_{i=0}^{2n}\sum_{j=0}^{2m}\lambda_{ij}\frac{(x-\xi_i)I(\xi_i,\eta_j)\mathrm{d}\xi\mathrm{d}\eta}{\left(1+\frac{(x-\xi_i)^2+(y-\eta_j)^2}{2\sigma^2}\right)^2}\right)^2 + \right.$$
$$\left. +\left(\sum_{i=0}^{2n}\sum_{j=0}^{2m}\lambda_{ij}\frac{(y-\eta_j)I(\xi_i,\eta_j)\mathrm{d}\xi\mathrm{d}\eta}{\left(1+\frac{(x-\xi_i)^2+(y-\eta_j)^2}{2\sigma^2}\right)^2}\right)^2\right]. \tag{15}$$

Observations. The effective calculus of formulas (2) requires the effective knowledge not only of one of the two filters (3) or (8), but also of the partial derivatives up to order two of the chosen filter. More precisely, when, in order to determine the transformation (2), we choose the function distance defined by the formula (6) or (10), we then only have to calculate the partial derivatives of first order, either of filter (3), or of filter (8), depending on the case, but, when we choose formula (7), we need to know the partial derivatives of second order of filter (3) as well. Determining these partial derivatives also implies the calculus of some integrals of the form (11). Indeed, if we choose as an example filter (3), then the calculus of its partial derivatives of first order implies the calculus of some integrals of the form

$$\iint_{\mathbb{R}^2} \left[(x-\xi)^2 - \sigma^2 \right] e^{-\frac{(x-\xi)^2+(y-\eta)^2}{2\sigma^2}} I(\xi,\eta)\,\mathrm{d}\xi\mathrm{d}\eta ,$$

$$\iint_{\mathbb{R}^2} (x-\xi)(y-\eta) e^{-\frac{(x-\xi)^2+(y-\eta)^2}{2\sigma^2}} I(\xi,\eta)\,\mathrm{d}\xi\mathrm{d}\eta ,$$

$$\iint_{\mathbb{R}^2} \left[(y-\eta)^2 - \sigma^2 \right] e^{-\frac{(x-\xi)^2+(y-\eta)^2}{2\sigma^2}} I(\xi,\eta)\,\mathrm{d}\xi\mathrm{d}\eta .$$

The theory exposed above clearly shows the way we can discretize the integrals of the form (11). Therefore, the same theory can be used as well, to discretize the calculus formulas of the derivatives we need in order to evaluate the transformation (2), and at last, this theory can be used to put into practice the detection and automatic marking algorithm of the target shapes within a given video image, describled in section 2.

5. COMMENTS

Within concrete applications, the programmer can not chose parameters n and m as large as he wont (which determine step h of the horizontal divisions and respectively, step k of the vertical divisions of the dimensions of the analyzed image), because he is limited by the resolution of the display on which the application is projected (product mn can not exceed the total number of pixels of the screen on which the images are being displayed in order to be analyzed). Due to the performance capacity of current hardware components, this restraint is not a major obstacle. In most cases the results of the tests that verified the algorithm presented in this paper have been satisfactory.

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