HYSTERESIS IN THE SHAPE MEMORY ALLOYS

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Abstract. This paper discusses the hysteretic behaviour of a shape memory strip under uniaxial tension. The generalized play hysteresis operator is coupled with the exponential evolution law of 1D constitutive model of the shape memory material. The model considers two phase transformations: conversion of austenite into single-variant martensite and conversion of single-variant martensite into austenite. This paper also discusses the modeling and feedforward control for the strip hysteresis for some unstable cases. For these cases, the generalized play operator is analyzed in connection with the feedforward control. Results show that hysteresis can be reduced to less than 20% when applying the feedforward control.

Key words: hysteresis operator, shape memory alloys, exponential evolution law, feedforward control.

1. **INTRODUCTION**

In the 1970s, Krasnoselskii and Pokrovskii studied the concept of *hysteresis operator*, acting in spaces of time dependent functions [1]. Further researches were developed in a series of monographies dedicated to the hysteresis in connection with PDEs and applicative problems [2–4]. A useful survey can be found in [5,6]. Several models of mechanical and magnetic hysteresis may be represented via analogical models, namely the rheological models in mechanics, circuital models in electromagnetism, by arranging elementary components in series and/or in parallel [7–9]. In this paper, the generalized play operator is analyzed in connection with the variational inequalities arising in the shape memory alloys hysteretic behavior.

The properties of nickel (Ni) titanium (Ti) alloys (NiTi) were discovered in the 1960s, at the Naval Ordnance Laboratory (Buehler and Wiley). NiTi has excellent corrosion resistance, provided by a naturally formed thin adherent layer known as passive film.

Researchers are using NiTi and others shape memory alloys (SMA) to build biomimetic systems that mimic the behaviour of biological organisms such as fish or insects [10]. The ability of SMAs components to change shape in response to thermal or electrical stimuli considerably simplifies construction of biomimetic systems. SMAs even have the ability to recover from large deformations once heated. Shape memory alloys (SMAs) are a special class of adaptive materials which can convert thermal energy directly into mechanical work. In the 1960s were

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developed some nickel-titanium alloys with a composition of 53–57% nickel by weight that exhibited an unusual effect: the deformed specimens with residual strains of 6–10% regained their original shape after a thermal cycle. This effect became as the shape-memory effect. Shape memory alloys are materials capable of very large recoverable inelastic strain (of the order of 10%). The source of this mechanical behaviour is a crystalline phase transformation between the austenite and martensite. There are two main phases associated with the shape memory effect, austenite and martensite. Austenite is the high temperature phase where the alloy has a cubic crystal structure. Martensite, the low temperature phase, has a monoclinic crystal structure [11]. SMAs are able to return to either phase by either gaining or losing heat. The specific transformation temperature varies depending upon the exact chemical composition. For example, slight variances in Nitinol's composition, which is made from approximately equal amounts of Nickel and Titanium, can cause the transformation temperatures to vary from below 0 °C to above 100 °C. A useful survey on the SMA's can be found in [12]. A mathematical formulation for the hysteretic behaviour of a two-phase thermoelastic material undergoing stress-induced coherent martensitic phase transformations is presented by Kuczma, Mielke and Stein [13]. Their results show the influence of the phase transformation strain and boundary conditions on the propagation of the transformation front and the deformation mode of the structure.

In this paper, the simulated hysteretic behaviour of a shape memory strip under uniaxial tension is investigated. The generalized play hysteresis operator is coupled with the exponential evolution law of 1D constitutive model for the shape memory alloy. Singularities which may appear for certain values of the single-variant martensite fraction can make the strip to lose its stability if not controlled. In these cases, a possible solution can be the coupling of the generalized play operator to a control law, for example the feedforward control with no sensor requirement. This work is in the framework of the Visintin researches on models of hysteresis phenomena and on related PDEs [4–6], [15, 16].

2. GENERALIZED PLAY HYSTERESIS OPERATOR

Let us consider a system whose state is characterized by two scalar variables, the input function $u(t)$ and the output function $w(t)$, confined to a set $L \subset R^2$, $\forall t \in [0,T]$. The function $w(t)$ depends on the previous evolution of $u(t)$ (memory effect) and on the initial state w_0, such as

$$w(t) = A(u, w_0)(t)\,,\quad \forall t \in [0,T]\,,\quad (u(0), w_0) \in L\,,\quad A(u, w_0)(0) = w_0\,, \tag{1}$$

where $A(u, w_0)$ is a memory operator defined in a Banach space of time-dependent functions for any fixed w_0. The memory operator is causal: for $\forall (u_1, w_0), (u_2, w_0)$ with $u_1 = u_2$ in $[0,T]$, then $A(u_1, w_0)(t) = A(u_2, w_0)(t)$. Most typical hysteresis phenomena exhibit not purely rate-independent memory and as consequence, the rate-dependent effects are superposed to hysteresis. In the memory rate-dependent case, the *hysteresis operator* is not invariant with reference to any increasing diffeomorphism $\varphi:[0,T]\to[0,T]$, i.e. $A(u\circ\varphi,w_0)\neq A(u,w_0)\circ\varphi$, $\forall t\in[0,T]$. The generalized play operator $w:=A(u,w_0):R^+\to R$ is defined in the sense of Visintin [14-16]. Let $u(t)$ be any continuous, piecewise linear function on R^+, linear on $[t_{i-1},t_i]$, $i=1,2,...$ We define $w(t)=A(u,w_0)(t)$ by

$$w(t)=\min\left\{\gamma_l(u(0)),\max\left\{\gamma_r(u(0)),w_0\right\}\right\}\ \text{ for }\ t=0\ \text{ and }\ w_0\in R,$$
$$w(t)=\min\left\{\gamma_l(u(t_i)),\max\left\{\gamma_r(u(t_i)),w(t_{i-1})\right\}\right\}\ \text{ for }\ t\in(t_{i-1},t_i),\ i=1,2,...,\tag{2}$$

where $\gamma_l,\gamma_r:R\to R$ are maximal monotone, possible multivalued functions with

$$\inf\gamma_r(u)\leq\sup\gamma_l(u),\ \forall u\in R.\tag{3}$$

Note that $w(0)=w_0$ only if $\gamma_r(u(0))\leq w_0\leq\gamma_l(u(0))$. The classical play operator can be obtained from the general play operator by choosing

$$\gamma_l(u)=u+r,\ \ \gamma_r(u)=u-r\ ,\tag{4}$$

with $r\geq 0$ a parameter, $u(t)$ a continuous input function on $[0,T]$ and $w_{r0}\in[-r,r]$ an initial state. The hysteresis relationship with the PDEs can be written as [17]

$$w(x,t)=\left[A(u(x,...),w_0(x))\right](t)\ \text{ in }\ Q=\Omega\times[0,T],\tag{5}$$

where Ω is a bounded subset of R^n. The generalized play operator discussed in this paper is dissipative, in the sense that $\|(\lambda\mathrm{I}-A)x\|\geq\lambda\|x\|$ for $\forall\lambda>0$, where I is the identity mapping. The PDEs with hysteresis can be transformed into systems of differential inclusions. The generalized play operator can be defined as a solution in the Sobolev space $W^{1,1}(0,T)$, $w\in W^{1,1}(0,T)$ of a variational inclusion of the type [16]

$$w_{,t}\in\phi(u,w)\ \text{ in }\ (0,T),\quad w(0)=w_0.\tag{6}$$

The norm in $W^{1,1}(0,T)$ is defined as $\|f\|_{k,p}=\left(\sum\limits_{i=0}^{k}\|f^{(i)}\|_p^p\right)^{1/p}=$ $=\left(\sum\limits_{i=0}^{k}\int f^{(i)}|^p\,\mathrm{d}t\right)^{1/p}$. The rate-independent differential inclusion is

$$w_{,t} \in \phi(u,w) = \begin{cases} \{\infty\} & \text{if } w < \inf \gamma_r(u), \\ [0,+\infty] & \text{if } w \in \gamma_r(u) \setminus \gamma_l(u), \\ \{0\} & \text{if } \sup \gamma_r(u) < w < \inf \gamma_l(u), \\ [-\infty, 0] & \text{if } w \in \gamma_l(u) \setminus \gamma_r(u), \\ \{-\infty\} & \text{if } w > \sup \gamma_r(u), \\ [-\infty,+\infty] & \text{if } w \in \gamma_l(u) \cap \gamma_r(u). \end{cases} \tag{7}$$

3. ONE DIMENSIONAL CONSTITUTIVE MODEL

To illustrate the macroscale manifestation of these phenomena, typical uniaxial stress-strain diagrams for a polycrystalline SMA material are shown in Fig. 1 [24]. The temperatures are typically denoted as M_s, M_f, A_s and A_f ($M_f < M_s < A_s < A_f$). At a temperature $T > A_f$, a SMA material behaves pseudoelastically (Fig.1a). Applying stress induces transformation of austenite into martensite, it results an inelastic transformation strain. As the stress is reduced, after an initial elastic response the martensite formed during the loading process transforms back to austenite, the inelastic strain is therefore recovered, and the stress-strain diagram exhibits the characteristic hysteretic loop shown in Fig.1a. Fig.1b illustrates the shape memory effect for material starting as austenite at a temperature $T < A_s$. During the loading process ($A \to B$), the applied stress induces formation of martensite and inelastic strain. Upon unloading from *B* to *C*, the newly formed martensite remains stable. This one-dimensional theory models the superelastic effect for one-dimensional states of stress. The control variables are the uniaxial stress σ and the relative temperature T, which is related to the absolute temperature Θ and the reference temperature Θ_{ref} by $T = \Theta - \Theta_{ref}$.

As internal variables we may choose either the single-variant martensite fraction ξ_S or the austenite fraction ξ_A that satisfy the relation

$$\xi_S + \xi_A = 1. \tag{8}$$

We assume that the multiple-variant martensite fraction is zero $\xi_M = \dot{\xi}_M = 0$. From (8) we have

$$\dot{\xi}_S + \dot{\xi}_A = 0. \tag{9}$$

The model considers two phase transformations: conversion of austenite into single-variant martensite $A \to S$ and conversion of single-variant martensite into austenite $S \to A$.

We suppose that the regions in which phase transformations may occur are delimited by straight lines [25]. Assigning a fraction change to each process we set

$$\dot{\xi}_S = \dot{\xi}_S^{AS} + \dot{\xi}_S^{SA}, \quad \dot{\xi}_A = \dot{\xi}_A^{AS} + \dot{\xi}_A^{SA}. \tag{10}$$

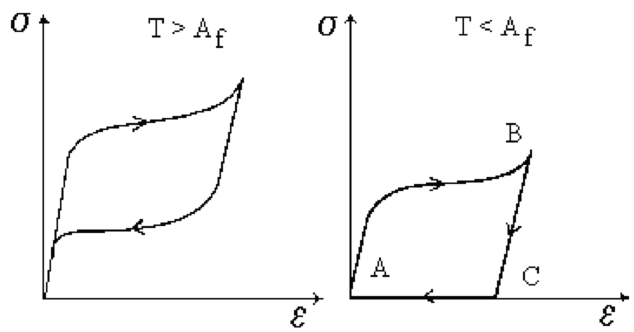

Fig. 1 – Pseudoelasticity (a) and shape memory effect (b) for a SMA material.

The superscript *AS* refers to the conversion of austenite into single-variant martensite, and *SA* to conversion of single-variant martensite into austenite. Let introduce the functions [22]

$$F^{AS} = \sigma - C^{AS}T, \quad F_s^{AS} = F^{AS} - R_s^{AS}, \quad F_f^{AS} = F^{AS} - R_f^{AS},$$
$$R_s^{AS} = C^{AS}T_s^{AS}, \quad R_f^{AS} = C^{AS}T_f^{AS},$$

and C^{AS} the material parameters, T_s^{AS} and T_f^{AS} the initial and final temperatures at which the transformations may occur. The region of transformations is described by

$$F_s^{AS} > 0, \quad F_f^{AS} < 0 \quad \Rightarrow \quad F_s^{AS} F_f^{AS} < 0. \tag{11}$$

The production of single-variant martensite is activated by increasing the stress, or by decreasing the temperature or by a proper combination of them. We require

$$\dot{F}^{AS} > 0. \tag{12}$$

The fraction evolutionary equations relative to the $A \to S$ phase transformation are expressed by using (25) and (26)

$$\dot{\xi}_A^{AS} = K_1^{AS} < -F_s^{AS} F_f^{AS} >< \dot{F}^{AS} >, \quad \dot{\xi}_S^{AS} = -\dot{\xi}_A^{AS},$$

where K_1^{AS} is a scalar function of the control and internal variables, and $< >$ is the Macaulay bracket. For the austenite production, we introduce the functions

$$F^{SA} = \sigma - C^{SA}T, \quad F_s^{SA} = F^{SA} - R_s^{SA}, \quad F_f^{SA} = F^{SA} - R_f^{SA},$$

$$R_s^{SA} = C^{SA} T_s^{SA} , \quad R_f^{SA} = C^{SA} T_f^{SA} ,$$

where C^{SA} are the material parameters, and T_s^{SA} and T_f^{SA} the initial and final temperatures at which the transformations may occur at zero stress. The region of transformations is described by

$$F_s^{SA} < 0 , \; F_f^{SA} > 0 \; \Rightarrow \; F_s^{SA} F_f^{SA} < 0 . \tag{13}$$

The production of austenite is activated by decreasing the stress, or by increasing the temperature or by a proper combination of them. We require

$$\dot{F}^{SA} < 0 . \tag{14}$$

The fraction evolutionary equations relative to the $S \to A$ phase transformation are expressed by using (13) and (14)

$$\dot{\xi}_S^{SA} = K_1^{SA} < -F_s^{SA} F_f^{SA} >< -\dot{F}^{SA} > , \quad \dot{\xi}_A^{SA} = -\dot{\xi}_S^{SA} , \tag{15}$$

where K_1^{SA} is a scalar function of the control and internal variables. The functions K_1^{AS} and K_1^{SA} can be determined from certain time-continuous flow laws. The first one is the exponential flow rule

$$\dot{\xi}_A^{AS} = -\dot{\xi}_S^{AS} = -\beta^{AS} \xi_A \frac{< -F_s^{AS} F_f^{AS} >}{| F_s^{AS} F_f^{AS} |} \frac{< \dot{F}^{AS} >}{(F_f^{AS})^2} , \tag{16}$$

$$\dot{\xi}_S^{SA} = -\dot{\xi}_A^{SA} = -\beta^{SA} \xi_S \frac{< -F_s^{SA} F_f^{SA} >}{| F_s^{SA} F_f^{SA} |} \frac{< -\dot{F}^{SA} >}{(F_f^{SA})^2} , \tag{17}$$

where the scalars constants β^{AS} and β^{SA} measure the rates at which the transformations proceed. The second rule is the linear flow rule

$$\dot{\xi}_A^{AS} = -\dot{\xi}_S^{AS} = \pi^{AS} \xi_A \frac{< -F_s^{AS} F_f^{AS} >}{| F_s^{AS} F_f^{AS} |} \frac{< \dot{F}^{AS} >}{F_f^{AS}} ,$$

$$\dot{\xi}_S^{SA} = -\dot{\xi}_A^{SA} = -\pi^{SA} \xi_S \frac{< -F_s^{SA} F_f^{SA} >}{| F_s^{SA} F_f^{SA} |} \frac{< -\dot{F}^{SA} >}{F_f^{SA}} .$$

Since the model has only one independent internal variable ξ_S we need only one evolution equation. From (8)-(10) we obtain for the exponential law

$$\dot{\xi}_S = \dot{\xi}_S^{AS} + \dot{\xi}_S^{SA} = \beta^{AS} (1 - \xi_S) \frac{< -F_s^{AS} F_f^{AS} >}{| F_s^{AS} F_f^{AS} |} \frac{< \dot{F}^{AS} >}{(F_f^{AS})^2} - \beta^{SA} \xi_S \frac{< -F_s^{SA} F_f^{SA} >}{| F_s^{SA} F_f^{SA} |} \frac{< -\dot{F}^{SA} >}{(F_f^{SA})^2} , \tag{18}$$

For the linear law we have

$$\dot{\xi}_S = \dot{\xi}_S^{AS} + \dot{\xi}_S^{SA} = -(1-\xi_S)\frac{<-F_s^{AS}F_f^{AS}>}{|F_s^{AS}F_f^{AS}|}\frac{<\dot{F}^{AS}>}{F_f^{AS}} - \xi_S\frac{<-F_s^{SA}F_f^{SA}>}{|F_s^{SA}F_f^{SA}|}\frac{<-\dot{F}^{SA}>}{F_f^{SA}}, \tag{19}$$

where for simplicity we considered $\pi^{AS} = \pi^{SA} = 1$. The flow laws (18) and (19) can be integrated in closed forms. For example a closed form for (18) is

$$\xi_S = 1 - \exp\left[\frac{\beta(\sigma-\sigma_s)}{(\sigma_f-\sigma_s)(\sigma-\sigma_f)}\right], \tag{20}$$

where σ_s and σ_f are the initial and final value of the stress at which the evolution process is active. The flow laws can be expressed in the equivalent forms

$$\dot{\xi}_S = H^{AS}\beta^{AS}(1-\xi_S)\frac{\dot{F}^{AS}}{(F_f^{AS})^2} + H^{SA}\beta^{SA}\xi_S\frac{\dot{F}^{SA}}{(F_f^{SA})^2}, \tag{21}$$

for the exponential law, and

$$\dot{\xi}_S = -H^{AS}(1-\xi_S)\frac{\dot{F}^{AS}}{F_f^{AS}} + H^{SA}\xi_S\frac{\dot{F}^{SA}}{F_f^{SA}},$$

for the linear law. The parameters H^{AS} and H^{SA} are defined as

$$\begin{aligned} &H^{AS}=1 \text{ if } \dot{F}_s^{AS}>0,\ \dot{F}_f^{AS}<0,\ \dot{F}^{AS}>0 \text{ and } H^{AS}=0 \text{ otherwise,}\\ &H^{SA}=1 \text{ if } \dot{F}_s^{SA}<0,\ \dot{F}_f^{SA}>0,\ \dot{F}^{SA}<0 \text{ and } H^{SA}=0 \text{ otherwise,}\end{aligned} \tag{22}$$

The well-defined regions where each transformation may occur are indicated by $S^{I,AM}$ and $S^{I,MA}$. Considering one phase transformation at a time, the elastic domains are

$$S^{E,AM} = S - S^{I,AM}, \quad S^{E,MA} = S - S^{I,MA},$$

$$S^I = S^{I,AM} \cup S^{I,MA}, \quad S^E = S^{E,AM} \cap S^{E,MA}.$$

The generalized play hysteresis operator $w := A(u,w_0): R^+ \to R$ defined by (2) is coupled with the exponential evolution law (21) and (22) under the form

$$\dot{\xi}_S + \dot{w} = H^{AS}\beta^{AS}(1-\xi_S-w)\frac{\dot{F}^{AS}}{(F_f^{AS})^2} + H^{SA}\beta^{SA}(\xi_S+w)\frac{\dot{F}^{SA}}{(F_f^{SA})^2}. \tag{23}$$

The accounting for the dissipation and the hysteresis effects, the phase transformation may take place only if its driving force X reaches some threshold values, which depend upon the single-variant martensite fraction ξ_S, and w, with

$$\dot{w}(x,t)=0 \quad \text{if} \quad X(x,t)\neq 0, \quad w(x,t_0)=w_0 \quad \text{if} \quad X(x,t_0)=0, \tag{24}$$

where t_0 is the time during the process at which the phase transformation starts state reaches.

4. UNIAXIAL TEST

Let us consider a strip of length a and width b, made from NiTi, subjected to a simulated uniaxial tension (Fig. 2). In the coordinate axis x and y, u and v are displacement components. The boundary conditions are

$$\begin{gathered} u(0,y)=0 \quad \text{for} \quad 0\le y\le b,\ v(0,b/2)=0, \\ u(a,y)=\varphi(t) \quad \text{for} \quad 0\le y\le b,\ v(a,b/2)=0. \end{gathered} \tag{25}$$

The function $\varphi(t)$ is the exponential decay function, $\alpha\exp(-\beta|t-t_0|)/2$ with α=0.2 and β=1.2. The material parameters of the NiTi strip are given in table 1 (subscripts s and f represent start and finish temperature with '0' standing for stress free state). We have calculated the strip for $a/b=5$, a=20 mm and the thickness of 0.4 mm [13]. Fig. 3 shows the major hysteresis loop between the force F at the side $x=a$, $0\le y\le b$ divided by the initial cross-sectional area A_0 of the strip versus the scaled elongation $\varphi(t)/a$, with $\xi_S(x,t_0)=0$, $w(x,t_0)=w_0$, $x\in[0,a]\times[0,b]$, where ξ_S is the current variant martensite fraction, and w is the play operator which plays the role of a discrete memory. The loop is independent of frequency. The line (A,B) corresponds to the same scaled elongation but with different histories.

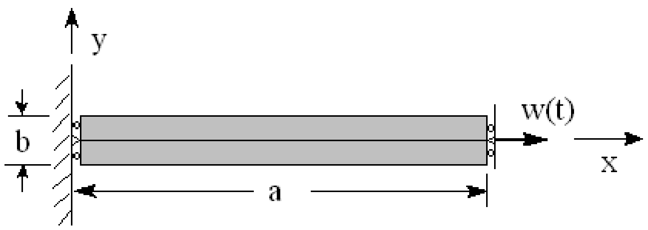

Fig. 2 – Strip subjected to uniaxial tension.

Distribution of the martensite fraction at the same corresponding states A or B can lead to instabilities in the hysteretic behaviour of the strip. For such unstable cases, direct hysteresis can be compensated by another hysteresis connected into cascade with it [27–32]. Such scheme is called feedforward control of the hysteresis and it is presented in Fig. 4, where w_r is the reference input to be tracked. For instance, direct hysteresis and its compensator can be symmetric relative to the linear curve (w_r, w), as shown in Fig. 4. To obtain a linear input-output (w_r, w) with a unit gain, the real system curve (u, w) and the compensator curve (w_r, u) should be symmetric [27]. This compensator is characterized by thresholds r'_k and the weightings p'_k. The calculation of these parameters follows the symmetry principle. The thresholds r'_k, $k = 1,2,...,n$, are computed as follows

$$r'_k = \sum_{j=1}^{k} p_j (r_k - r_j), \quad k = 1,2,...,n, \tag{26}$$

$$p'_1 = \frac{1}{p_1}, \quad p'_k = \frac{-p_k}{\left(p_1 + \sum_{j=2}^{k} p_j\right)\left(p_1 + \sum_{j=2}^{k-1} p_j\right)}, \quad k = 2,...,n. \tag{27}$$

Table 1

Material parameters of NiTi strip [26]

Parameter	Symbol	Value	Unit
length	L	7.5×10^{-2}	m
radius	r	1.5×10^{-4}	m
density	ρ_a	6.45×10^{3}	kg/m^3
Lamé moduli	λ^A, λ^M μ^A, μ^M	28.26, 6.3 18.85, 4.19	GPa GPa
coefficient of linear thermal expansions	α^A, α^M	12.5×10^{-6}	1/°C
thermal conductivity	k^A, k^M	18, 8.5	W/m°C
slope of stress versus temperature	C^A, C^M	13.5, 13.5	MPa/°C
transformation temperatures	A_{0s}, A_{of} M_{0s}, M_{0f}	57, 35 21, −12	°C
heat capacity	C_v	5.44×10^{6}	J m^{-3} °C
electrical resistivity	ρ_e	0.5×10^{6}	Ω m

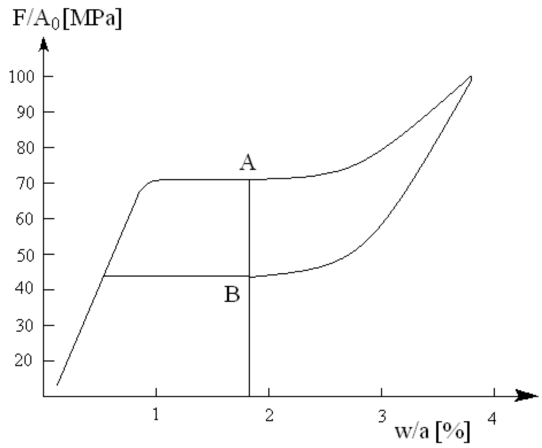

Fig. 3 – Major hysteresis loop in the scaled force-elongation space.

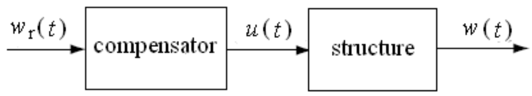

Fig. 4 – The scheme of the feedforward control.

Figs. 5 and 6 show that an inhomogeneous field is induced with different evolution paths during the loading and unloading. The initially straight axis $0 \le x \le a$, $y = b/2$ of the strip does not remain straight in the xy-plane during the process. Singularities are depicted in these figures for some values of ξ_S, which can make the strip to lose its stability if not controlled.

For that, the generalized play operator is connected in this paper to the plate equations and the control is focused on feedforward or compensation technique with no sensor requirement.

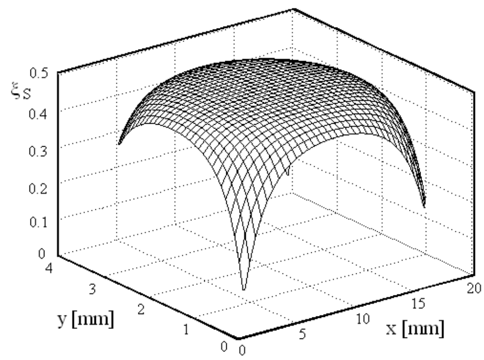

Fig. 5 – Distribution of martensite fraction at the same corresponding states A.

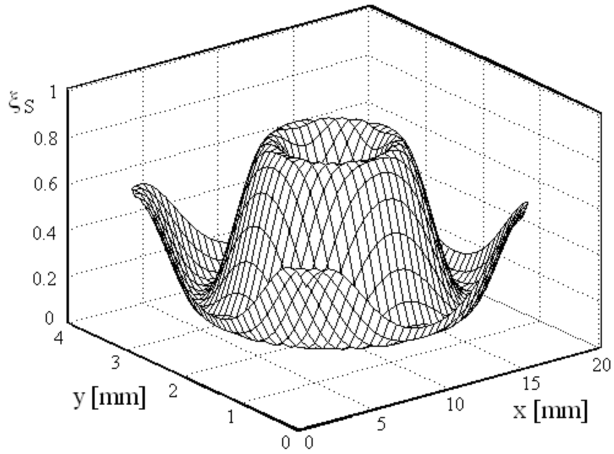

Fig. 6 – Distribution of martensite fraction at the same corresponding states B.

The control problem is applied next by using the hysteresis compensator (26) and (27). The diagram of the force F / A_0 *versus* the martensite fraction is illustrated in Fig. 7 for the state A, and both uncontrolled (right) and controlled (left) cases, respectively. The results show that hysteresis can be reduced to less than 20% when applying the feedforward control. The state B exhibits the same behavior for certain values of the martensite fraction.

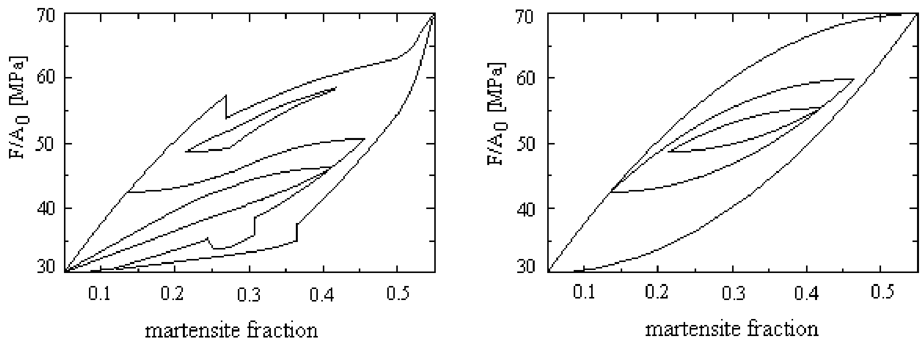

Fig. 7 – Loops force- martensite fraction for state A for both uncontrolled (right) and controlled (left) cases, respectively.

5. **CONCLUSIONS**

The hysteretic behaviour of a shape memory strip under uniaxial tension is simulated and analyzed by coupling the generalized play hysteresis operator with the exponential evolution law of the 1D constitutive model for the shape memory alloy. The results show that two states of the force-elongation law are connected with inhomogeneous states in the bulk of the material.

The behavior of the shape memory strip is accompanied by the hysteresis phenomenon which may lead to degradation of the motion by driving it to limit-cycle instability. The control problem is applied next by using the hysteresis compensator. Therefore, this paper is aimed to outline some of the basic elements of the hysteresis operators in connection with the evolution and the control laws, respectively.

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