APPLICATION OF CANONICAL REPRESENTATIONS TO RANDOM VIBRATIONS OF MDOF LINEAR SYSTEMS

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Abstract. In this paper is presented the canonical representations method for analyzing the random vibrations of a multiple degrees of freedom linear system. The general method is exemplified in the case of a two degrees of freedom quarter-car model for a given covariance function of road induced excitation.

Key words: canonical representation, random process, linear system, random vibrations.

1. INTRODUCTION

The application of the canonical representations method in the study of dynamical systems with random excitations is analogous with the function series expansion method used for deterministic dynamical systems [1–3].

Consider a mechanical linear system with the m random processes as inputs $\{x_0(t)\}_1,...,\{x_0(t)\}_m$. It is known that the state of the system is described by the r outputs random processes $\{x(t)\}_1,...,\{x(t)\}_r$. If the m random processes applied at the inputs of the system form a random stationary vector with normal repartition, than due to the linearity properties, the movement of the system will be described by a random stationary vector with normal repartition, too.

Denoting by $\mathbf{H}(\omega)$ the transfer matrix and by $\mathbf{S}_{\mathbf{x}_0}(\omega)$ the bilateral spectral density matrix of the random process $\{\mathbf{x}_0(t)\}$ with *m* inputs and *r* outputs, one can obtain the relations between the first and second order characteristics of the excitation and response of a mechanical system with *m* inputs and *r* outputs as [4]:

$$\boldsymbol{m}_{\mathbf{x}} = \mathbf{H}(0)\boldsymbol{m}_{\mathbf{x}_0}$$
$$\mathbf{R}_{\mathbf{x}}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\mathbf{H}(\omega)}\, \mathbf{S}_{\mathbf{x}_0}(\omega)\mathbf{H}^{\mathrm{T}}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega, \qquad (1)$$

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and

$$\mathbf{S}_{\mathbf{x}}(\omega) = \overline{\mathbf{H}(\omega)}\, \mathbf{S}_{\mathbf{x}_0}(\omega) \mathbf{H}^{\mathrm{T}}(\omega) . \tag{2}$$

The equation of motion for the linear systems used in vehicle dynamics can be written as[4]:

$$\begin{aligned} &\mathbf{M}\{\ddot{\mathbf{q}}(t)\} + \mathbf{A}^{\mathrm{T}}\mathbf{C}\mathbf{A}\{\dot{\mathbf{q}}(t)\} + \mathbf{A}^{\mathrm{T}}\mathbf{K}\mathbf{A}\{\mathbf{q}(t)\} = \\ &\qquad = -[\mathbf{A}^{\mathrm{T}}\mathbf{C}_0\{\dot{\mathbf{x}}_0^{\mathrm{v}}(t)\} + \mathbf{A}^{\mathrm{T}}\mathbf{K}_0\{\mathbf{x}_0^{\mathrm{v}}(t)\}], \end{aligned} \tag{3}$$

where matrix **M** covers the inertial properties of mechanical systems used in modeling the vehicles vibrations and **q** the vector of generalized coordinates is expressed as:

$$\mathbf{x} = \mathbf{A}\mathbf{q} , \tag{4}$$

using the matrix **A** of the geometric constraints. The matrices **K** and **C** are embedding the matrices of stiffness and damping coefficients of the suspension and tires and $\mathbf{K}_0, \mathbf{C}_0$ are embedding the stiffness and damping coefficients of the tires.

2. APPLICATION OF CANONICAL REPRESENTATION METHOD USING THE TRANSFER FUNCTIONS METHOD

We will use the canonical representations of the excitation and of the generalized coordinates of the system respectively:

$$\mathbf{x}_0^v(t) = \begin{bmatrix} \sum_{l=0}^{N} X_{1l}^0 \mathrm{e}^{\mathrm{i}\omega_l t} \\ \sum_{l=0}^{N} X_{2l}^0 \mathrm{e}^{\mathrm{i}\omega_l t} \\ \cdots\cdots\cdots \\ \sum_{l=0}^{N} X_{ml}^0 \mathrm{e}^{\mathrm{i}\omega_l t} \end{bmatrix}, \qquad \mathbf{q}(t) = \begin{bmatrix} \sum_{l=0}^{N} Q_{1l} \mathrm{e}^{\mathrm{i}\omega_l t} \\ \sum_{l=0}^{N} Q_{2l} \mathrm{e}^{\mathrm{i}\omega_l t} \\ \cdots\cdots\cdots \\ \sum_{l=0}^{N} Q_{nl} \mathrm{e}^{\mathrm{i}\omega_l t} \end{bmatrix}. \tag{5}$$

Introducing these expansions in relation (4), and equalizing the coefficients of the linear independent functions $\exp(\mathrm{i}\omega_l t)$ for the same values of the l one obtains the solution:

$$\begin{bmatrix} Q_{1l} \\ Q_{2l} \\ \cdots\cdots \\ Q_{nl} \end{bmatrix} = \mathbf{H}_{\mathbf{q}\,\mathbf{x}_0}(\omega_l) \begin{bmatrix} X_{1l}^0 \\ X_{2l}^0 \\ \cdots\cdots \\ X_{ml}^0 \end{bmatrix} \; ; \quad l = 1, N \tag{6}$$

where $\mathbf{H}_{\mathbf{q}\,\mathbf{x}_0}(\omega_l)$ is the transfer matrix:

$$\mathbf{H}_{\mathbf{q}\,\mathbf{x}_0}(\omega_l) = -\left[-\omega_l{}^2\mathbf{M} + \mathrm{i}\omega_l\mathbf{A}^{\mathrm{T}}\mathbf{C}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{K}\mathbf{A}\right]^{-1}\left[\mathrm{i}\omega_l\mathbf{A}^{\mathrm{T}}\mathbf{C}_0 + \mathbf{A}^{\mathrm{T}}\mathbf{K}_0\right]. \tag{7}$$

The input random variables $X_{1l}^0, X_{2l}^0, \ldots. X_{ml}^0$ are independent Gaussian random variables with zero mean and known dispersions

$$\mathrm{E}\left[\left|X_{k\,l}^0\right|^2\right] = D_{k\,l}^0; \quad k = 1,\ldots,m; \;\; l = 1,\ldots,N\,. \tag{8}$$

The coefficients $D_{k\,l}^0$ can be considered the coefficients of the canonical representations of the covariance functions of the excitation:

$$c_{\mathrm{x}_h^0 \mathrm{x}_h^0}(\tau) = \sum_{l=0}^{N} D_{h\,l}^0 \mathrm{e}^{\mathrm{i}\omega_l \tau}; \quad h = 1,2,\ldots,m\,. \tag{9}$$

So that, starting from a set of de realizations of the random process which represents the canonical expansion of the system input, one can determine the set of realizations of the independent variables $Q_{k\,l}$, $k = 1,\ldots,m;\; l = 1,\ldots,N$ from relation (6). From relation $(5)_2$ we find the set of realization of the random process which is the canonical representation of the system output.

3. CANONICAL REPRESENTATION METOD FOR TWO DEGREE OF FREEDOM QUARTER CAR MODEL

The equations of motion of the two degree of freedom system used for a quarter car model are:

$$\begin{aligned} &m_{\mathrm{s}}\ddot{x}_1 + c_{\mathrm{s}}(\dot{x}_1 - \dot{x}_2) + k_{\mathrm{s}}(x_1 - x_2) = 0 \\ &m_{\mathrm{u}}\ddot{x}_2 - c_{\mathrm{s}}(\dot{x}_1 - \dot{x}_2) - k_{\mathrm{s}}(x_1 - x_2) + k_{\mathrm{t}}(x_2 - x_0) = 0, \end{aligned} \tag{10}$$

where x_1 and x_2 are the vertical displacements of sprung and unsprung masses m_{s} and m_{u} measured with respect to their static equilibrium position considered when the tire-road contact point is placed on road profile reference axis.

Introducing the following notations:

$$\omega_{\mathrm{s}}=\sqrt{\frac{k_{\mathrm{s}}}{m_{\mathrm{s}}}},\ \ \omega_{\mathrm{t}}=\sqrt{\frac{k_{\mathrm{t}}}{m_{\mathrm{u}}}},\ \ \zeta_{\mathrm{s}}=\frac{c_s}{2\sqrt{k_{\mathrm{s}}m_{\mathrm{s}}}},\ \ \eta=\frac{m_{\mathrm{u}}}{m_{\mathrm{s}}}, \tag{11}$$

equations (10) can be rewritten as:

$$\begin{aligned}&\ddot{x}_1+2\zeta_{\mathrm{s}}\omega_{\mathrm{s}}(\dot{x}_1-\dot{x}_2)+\omega_{\mathrm{s}}^2(x_1-x_2)=0\\&\ddot{x}_2-2\eta\zeta_{\mathrm{s}}\omega_{\mathrm{s}}(\dot{x}_1-\dot{x}_2)-\eta\omega_{\mathrm{s}}^2(x_1-x_2)+\omega_{\mathrm{t}}^2x_2=\omega_{\mathrm{t}}^2x_0\end{aligned} \tag{12}$$

Considering the canonical representations of the random input process $x_0(t)$ and the output process $x_1(t), x_2(t)$ of the oscillating system described by the equations (10), we can write [1],[3]:

$$x_0(t)=\sum_{n=-\infty}^{\infty}X_{0n}\mathrm{e}^{\mathrm{i}\omega_n t},\ \ x_1(t)=\sum_{n=-\infty}^{\infty}X_{1n}\mathrm{e}^{\mathrm{i}\omega_n t},\ \ x_2(t)=\sum_{n=-\infty}^{\infty}X_{2n}\mathrm{e}^{\mathrm{i}\omega_n t}, \tag{13}$$

where X_{0n} are independent random variables with zero mean and dispersions $\mathrm{E}\left\{|X_{0n}|^2\right\}=\sigma_{\mathrm{x}_0}^2D_{0n}$. The random variables X_{1n}, X_{2n} will be determined introducing relations (4) in (1) and identifying the coefficients of the same spectral components. It is considered that the random process $x_0(t)$ is a Gaussian stationary random process in the broad sense with zero mean and the covariance function $c_{x_0x_0}(\tau)$. The dispersions $\sigma_{x_1\mathrm{e}}, \sigma_{x_2\mathrm{e}}$ of the exact solution are compared with the dispersions $\sigma_{x_1}, \sigma_{x_2}$ of the approximate solution, obtained by the canonical representations method, for different expressions of the covariance function used in practice [4].

4. QUARTER CAR RESPONSE TO A STATIONARY RANDOM INPUT WITH EXPONENTIAL COSINE COVARIANCE FUNCTION

In this case the covariance function, $c_{x_0x_0}(\tau)$, and the one sided spectral density, obtained by applying the Fourier transform of $c_{x_0x_0}(\tau)$ are:

$$c_{x_0x_0}(\tau)=e^{-\alpha|\tau|}\cos(\beta\tau),\ \ G_{x_0}(\omega)=\frac{\alpha}{\pi}\left[\frac{1}{\alpha^2+(\omega+\beta)^2}+\frac{1}{\alpha^2+(\omega-\beta)^2}\right],\ \omega\geq 0. \tag{14}$$

Figure 1 illustrates typical plots of exponential covariance function and its one sided spectral density.

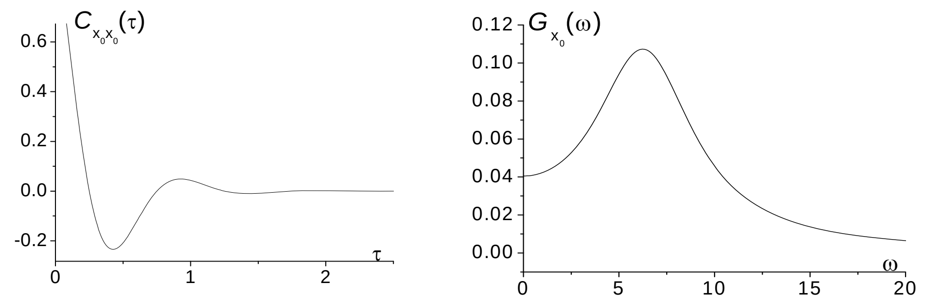


Fig. 1 – Typical plots for the exponential cosine covariance function and its one sided spectral density.

Considering a unit value for σ_{x_0} with the physical dimension of the random process $x_0(t)$, it can be written:

$$c_{x_0x_0}(\tau)=\sum_{n=-\infty}^{\infty} D_{0n}\,\mathrm{e}^{\mathrm{i}\omega_n\tau},\qquad \omega_n=\frac{n\pi}{2T},\ \ n=0,\quad \pm1,\quad \pm2,... \tag{15}$$

The coefficients D_{0n} are given by:

$$\begin{aligned} D_{0n} &= \frac{1}{4T}\int_{-2T}^{2T} \mathrm{e}^{-\alpha|\tau|}\cos(\beta\tau)\,\mathrm{e}^{-\mathrm{i}\omega_n\tau}\mathrm{d}\tau = \\ &= \frac{1}{4T}\left\{\frac{\alpha+(-1)^n\mathrm{e}^{-2T\alpha}[-\alpha\cos(2\beta T)+(\beta-\omega_n)\sin(2\beta T)]}{\alpha^2+(\beta-\omega_n)^2}+\right. \\ &\quad\left.+\frac{\alpha+(-1)^n\mathrm{e}^{-2T\alpha}[-\alpha\cos(2\beta T)+(\beta+\omega_n)\sin(2\beta T)]}{\alpha^2+(\beta+\omega_n)^2}\right\}. \end{aligned} \tag{16}$$

Introducing the canonical representations (13) in equations (12) and identifying the coefficients of the same spectral components, yields the following systems of linear algebraic equations:

$$\begin{aligned} &\left(\omega_n^2-\omega_{\mathrm{s}}^2-2\mathrm{i}\zeta_{\mathrm{s}}\omega_{\mathrm{s}}\omega_n\right)X_{1n}+\omega_{\mathrm{s}}(\omega_{\mathrm{s}}+2\mathrm{i}\zeta_{\mathrm{s}}\omega_n)X_{2n}=0 \\ &\eta\omega_{\mathrm{s}}(\omega_{\mathrm{s}}+2\mathrm{i}\zeta_{\mathrm{s}}\omega_n)X_{1n}+\left(\omega_n^2-\omega_{\mathrm{t}}^2-\eta\omega_{\mathrm{s}}^2-2\mathrm{i}\eta\zeta_{\mathrm{s}}\omega_{\mathrm{s}}\omega_n\right)X_{2n}=-\omega_{\mathrm{t}}^2X_{0n}. \end{aligned} \tag{17}$$

By solving these systems one obtains:

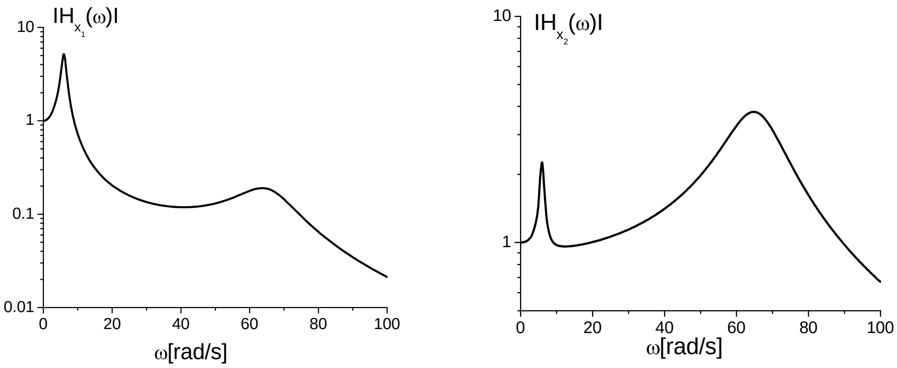
$$X_{1n} = H_{x_1}(\omega_n)X_{0n}, \quad X_{2n} = H_{x_2}(\omega_n)X_{0n}, \tag{18}$$

where $H_{x_1}(\omega)$ and $H_{x_2}(\omega)$ are the frequency response functions of system outputs $x_1(t)$ and $x_2(t)$, corresponding to the system input $x_0(t)$:

$$\begin{aligned} H_{x_1}(\omega) &= \frac{\omega_t^2\omega_s(\omega_s + 2i\zeta_s\omega)}{\left[\left(\omega^2-\omega_s^2\right)\left(\omega^2-\omega_t^2\right)-\eta\omega_t^2\omega^2\right]-i\zeta_s\omega_s\omega\left[\omega^2(1+\eta)-\omega_t^2\right]} \\ H_{x_2}(\omega) &= \frac{-\omega_t^2\left(\omega^2-\omega_s^2-2i\zeta_s\omega_s\omega\right)}{\left[\left(\omega^2-\omega_s^2\right)\left(\omega^2-\omega_t^2\right)-\eta\omega_t^2\omega^2\right]-i\zeta_s\omega_s\omega\left[\omega^2(1+\eta)-\omega_t^2\right]} \end{aligned} \tag{19}$$

Figure 2 shows the amplification factors of absolute displacements $x_1(t), x_2(t)$ for a set of typical values of vehicle suspension parameters:

$$\omega_s = 2\pi \text{ rad/s}, \; \omega_t = 20\pi \text{ rad/s}, \; \zeta_s = 0.25, \; \eta = 10. \tag{20}$$



a b

Fig. 2 – Amplification factor of: a) sprung mass; b) amplification factor of unsprung mass.

The discret components of the output one sided spectral densities can be calculated for canonical representations of system output (13), by using the estimates [5,6]:

$$G^*_{x_{kn}}(\omega_n) = 2\sigma_{x_0}^2\frac{D_{kn}}{\Delta\omega}, \; \Delta\omega = \omega_{n+1} - \omega_n = \frac{\pi}{2T}, \quad n = 1, \; 2, ..., N, \quad k = 1, 2. \tag{21}$$

The exact one sided spectral densities of system output are given by:

$$G_{x_k}(\omega) = \left|H_{x_k}(\omega)\right|^2 G_{x_0}(\omega), \quad k = 1, 2. \tag{22}$$

The mean square values of approximate solution are:

$$\sigma_{x_k}^2 = \sigma_{x_0}^2 \sum_{n=-\infty}^{\infty} D_{kn} \cong 2\sigma_{x_0}^2 \sum_{n=0}^{N} D_{kn}, \quad k = 1, 2 . \tag{23}$$

The mean square values of the exact solution are given by:

$$\sigma_{x_k \mathrm{e}}^2 = \int_0^\infty G_{x_k}(\omega)\mathrm{d}\omega = \int_0^\infty \left|H_{x_k}(\omega)\right|^2 G_{x_0}(\omega)\mathrm{d}\omega \cong \int_0^{\omega_{\max}} \left|H_{x_k}(\omega)\right|^2 G_{x_0}(\omega)\mathrm{d}\omega, \ k = 1, 2 \tag{24}$$

or

$$\sigma_{x_1 \mathrm{e}}^2 = \int_0^\infty G_{x_1}(\omega)\,\mathrm{d}\omega = \frac{\alpha\omega_\mathrm{t}^4\omega_\mathrm{s}^2\sigma_{x_0}^2}{\pi}\int_0^\infty \left(\frac{1}{\alpha^2+(\omega+\beta)^2}+\frac{1}{\alpha^2+(\omega-\beta)^2}\right)\cdot$$
$$\cdot\frac{\left(\omega_\mathrm{s}^2+4\zeta_\mathrm{s}^2\omega^2\right)\mathrm{d}\omega}{\left[\left(\omega^2-\omega_\mathrm{s}^2\right)\left(\omega^2-\omega_\mathrm{t}^2\right)-\eta\omega_\mathrm{s}^2\omega^2\right]^2+\zeta_\mathrm{s}^2\omega_\mathrm{s}^2\omega^2\left[\omega^2(1+\eta)-\omega_\mathrm{t}^2\right]^2},$$

$$\sigma_{x_2 \mathrm{e}}^2 = \int_0^\infty G_{x_2}(\omega)\,\mathrm{d}\omega = \frac{\alpha\omega_\mathrm{t}^4\sigma_{x_0}^2}{\pi}\int_0^\infty \left(\frac{1}{\alpha^2+(\omega+\beta)^2}+\frac{1}{\alpha^2+(\omega-\beta)^2}\right)\cdot \tag{25}$$
$$\cdot\frac{\left[\left(\omega^2-\omega_\mathrm{s}^2\right)^2+4\zeta_\mathrm{s}^2\omega_\mathrm{s}^2\omega^2\right]\mathrm{d}\omega}{\left[\left(\omega^2-\omega_\mathrm{s}^2\right)\left(\omega^2-\omega_\mathrm{t}^2\right)-\eta\omega_\mathrm{s}^2\omega^2\right]^2+\zeta_\mathrm{s}^2\omega_\mathrm{s}^2\omega^2\left[\omega^2(1+\eta)-\omega_\mathrm{t}^2\right]^2}.$$

Introducing the notations:

$$\mu = \frac{\alpha}{\omega_\mathrm{t}}, \ \gamma = \frac{\omega_\mathrm{s}}{\omega_\mathrm{t}}, \ \lambda = \frac{\beta}{\alpha} = \frac{\beta}{\mu\omega_\mathrm{t}}, \ \xi = \frac{\omega}{\omega_\mathrm{t}}, \ \xi_n = \frac{\omega_n}{\omega_\mathrm{t}} = \frac{n\pi}{2\omega_\mathrm{t}T}, \ n = 0, 1, 2, ... \tag{26}$$

the relations (23) become:

$$\sigma_{x_1}^2 = 2\gamma^2\sigma_{x_0}^2 \sum_{n=0}^{\infty}\frac{\left(\gamma^2+4\zeta_\mathrm{s}^2\xi_n^2\right)D_{0n}}{\left[\left(\xi_n^2-\gamma^2\right)\left(\xi_n^2-1\right)-\eta\gamma^2\xi_n^2\right]^2+\gamma^2\zeta_\mathrm{s}^2\xi_n^2\left[\xi_n^2(1+\eta)-1\right]^2},$$

$$\sigma_{x_2}^2 = 2\sigma_{x_0}^2 \sum_{n=0}^{\infty}\frac{\left((\xi_n^2-\gamma^2)^2+4\zeta_\mathrm{s}^2\gamma^2\xi_n^2\right)D_{0n}}{\left[\left(\xi_n^2-\gamma^2\right)\left(\xi_n^2-1\right)-\eta\gamma^2\xi_n^2\right]^2+\zeta_\mathrm{s}^2\gamma^2\xi_n^2\left[\xi_n^2(1+\eta)-1\right]^2}, \tag{27}$$

where

$$D_{0n} \simeq \frac{\mu}{4N}\left[\frac{1}{\mu^2+(\lambda\mu-\xi_n)^2}+\frac{1}{\mu^2+(\lambda\mu+\xi_n)^2}\right]. \tag{28}$$

The exact solutions (25) are:

$$\sigma_{x_1\mathrm{e}}^2 = \frac{\mu}{\pi}\sigma_{x_0}^2\gamma^2 I_5, \quad \sigma_{x_2\mathrm{e}}^2 = \frac{\mu}{\pi}\sigma_{x_0}^2 I_6, \tag{29}$$

where

$$\begin{gathered} I_5 = \int_0^\infty\left[\frac{1}{\mu^2+(\lambda\mu-\xi\ )^2}+\frac{1}{\mu^2+(\lambda\mu+\xi\ )^2}\right]\cdot \\ \cdot\frac{\left(\gamma^2+4\zeta_\mathrm{s}^2\xi^2\right)\mathrm{d}\xi}{\left[\left(\xi^2-\gamma^2\right)\left(\xi^2-1\right)-\eta\gamma^2\xi^2\right]^2+\zeta_\mathrm{s}^2\gamma^2\xi^2\left[\xi^2(1+\eta)-1\right]^2}, \\ I_6 = \int_0^\infty\left[\frac{1}{\mu^2+(\lambda\mu-\xi\ )^2}+\frac{1}{\mu^2+(\lambda\mu+\xi\ )^2}\right]\cdot \\ \cdot\frac{\left[(\xi^2-\gamma^2)^2+4\zeta_\mathrm{s}^2\gamma^2\xi^2\right]\mathrm{d}\xi}{\left[\left(\xi^2-\gamma^2\right)\left(\xi^2-1\right)-\eta\gamma^2\xi^2\right]^2+\zeta_\mathrm{s}^2\gamma^2\xi^2\left[\xi^2(1+\eta)-1\right]^2}. \end{gathered} \tag{30}$$

The value of T can be taken of the form $T = N/\omega_\mathrm{t}$, and writing:

$$\xi_n = \frac{n\pi}{2N}, \ \ \xi_{n+1}-\xi_n = \frac{\pi}{2N}, \tag{31}$$

one obtains:

$$\begin{gathered} \sigma_{x_1}^2 \approx \gamma^2\sigma_{x_0}^2\frac{\mu}{\pi}\int_0^{\pi/2}\left[\frac{1}{\mu^2+(\lambda\mu-\xi\ )^2}+\frac{1}{\mu^2+(\lambda\mu+\xi\ )^2}\right]\cdot \\ \cdot\frac{\left(\gamma^2+4\zeta_\mathrm{s}^2\xi^2\right)\mathrm{d}\xi}{\left[\left(\xi^2-\gamma^2\right)\left(\xi^2-1\right)-\eta\gamma^2\xi^2\right]^2+\zeta_\mathrm{s}^2\gamma^2\xi^2\left[\xi^2(1+\eta)-1\right]^2} \end{gathered} \tag{32}$$

and

$$\sigma_{x_2}^2 \approx \frac{\mu}{\pi}\sigma_{x_0}^2 \int_0^{\pi/2} \left[\frac{1}{\mu^2+(\lambda\mu-\xi\ )^2} + \frac{1}{\mu^2+(\lambda\mu+\xi\ )^2} \right] \cdot$$
$$\cdot \frac{\left[(\xi^2-\gamma^2)^2+4\zeta_s^2\gamma^2\xi^2\right]\mathrm{d}\xi}{\left[\left(\xi^2-\gamma^2\right)\left(\xi^2-1\right)-\eta\gamma^2\xi^2\right]^2+\zeta_s^2\gamma^2\xi^2\left[\xi^2(1+\eta)-1\right]^2}. \tag{33}$$

5. NUMERICAL RESULTS

Taking for example the values from (18), the normalized values $G^*_{x_{kn}} \big/ \sigma^2_{x_0}$ values from (21), obtained by canonical representations, can be plotted comparatively with the exact normalized spectral densities $G_{x_k}(\omega) \big/ \sigma^2_{x_0}$, calculated for the same parameter values (Fig. 3).

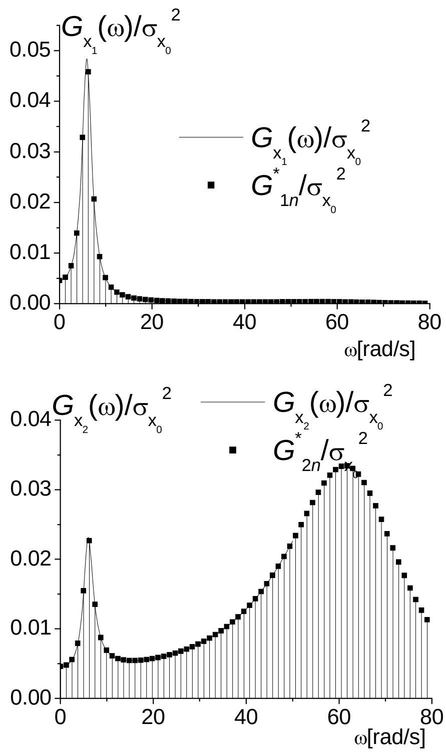


Fig. 3 – Spectral densities for the covariance function exponential-cosine.

The graphics of the relative errors $e_{\mathrm{r}} = \left|\sigma_{x_{\mathrm{k}}\mathrm{e}} - \sigma_{x_{\mathrm{k}}}\right| / \sigma_{x_{\mathrm{k}}\mathrm{e}}$, $k = 1, 2$ for different values of the damping coefficient ζ_{s} are given in Fig. 4.

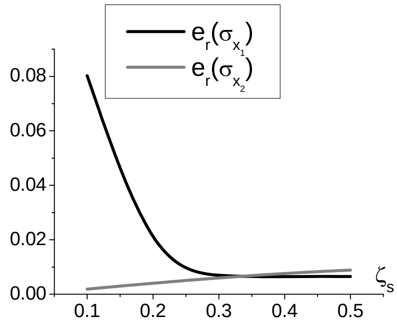


Fig. 4 – The relative errors of the response displacements dispersions.

6. CONCLUSIONS

The results obtained in this paper demonstrate the applicability of canonical representations method to analysis of linear random vibrations.

The canonical representations of the relative displacements of a quarter car model allow the assessment of optimum suspension damping with respect to a trade-off between comfort and road- holding criteria formulated in terms of sprung mass acceleration and dynamic road-tire contact force [4].

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