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REACTION FORCES IN PARALLEL MECHANISM PAIRS

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Abstract. To calculate the links dimensions of a parallel mechanism, is necessary to determine the distribution of forces from its kinematic pairs. Also, the performance of such a mechanism can be evaluated after calculating the driving forces and reaction forces as well as the reaction moments from its kinematic pairs. To determine the force and moment distributions within the mechanism pairs, as well as the critical positions of the actual areas, the virtual mechanical work principle (J. Bernoulli, 1717) is used. For this, the links' positions must be determined, meaning the sizes of the pair variables of all the mechanism kinematic pairs. The Denavit - Hartenberg transformation matrix method is used in this paper for both kinematic and kinetostatic analysis of the mechanism. The proposed method is applied to a multi-loop multi-degree-of-freedom linkage of the *Stewart* platform.

Keywords: parallel linkages, reaction forces, kinematic pairs, Denavit-Hartenberg transformation matrices

LIST OF SYMBOLS

$\mathbf{A}_{ij}$ – the Denavit-Hartenberg transformation matrix of the coordinates of a point of the link with number j, belonging to i loop of linkage;

Z_{ij} – the axis of the pair with number j, belonging to i loop;

X_{ij} – the common normal between the axes $Z_{i,j-1}$ and Z_{ij};

$\mathbf{F}$ – the force whose components on the axes $O_{11}X_{11}Y_{11}Z_{11}$ are $F_{11X}, F_{11Y}, F_{11Z}$;

Δa_{Xij} – the virtual displacement of the coordinate system $O_{ij}X_{ij}Y_{ij}Z_{ij}$ with respect to $O_{i,j+1}X_{i,j+1}Y_{i,j+1}Z_{i,j+1}$;

R_{xij}, R_{yij}, R_{zij}, M_{xij}, M_{yij}, M_{zij} – the components of the reaction wrench in the pair with number j, belonging to i loop of linkage, caused by the force $\mathbf{F}$;

$a_{1,12}$ – the distance between the axes $Z_{1,12}$ and Z_{11}; this lengths represents one of the Denavit-Hartenberg parameters. The other parameters shown in Fig. 2a are the distance s_{91} and the angles α_{12} and θ_{11}.

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1. REACTION WRENCHES IN PAIRS OF PARALLEL ROBOTS. INTRODUCTION

The analysis of forces in parallel linkages is one of the important problems for mechanical design and control. A few authors have treated this subject proposing different methods. A new approach using screw theory which reduces the number of the unknowns has been proposed by Y. Zhao, J.F. Liu and Z. Huang for a 3-*RPS* parallel mechanism [11]. A method of reaction force and moment calculation for a 3-*RSS* pure translational *Delta*-type parallel link robot is presented in [4]. Reaction forces are identified in [8] for a quadruped robot having a parallel-serial architecture. W. Chen investigates in his thesis the force capabilities of robots based on parallel structures [1]. V. Kumar and C.J. Waldron came with a method to determine the forces distribution in closed kinematic chains [5].

The *Stewart* platform is a six degree of freedom parallel mechanism well known in literature, proposed by D. Stewart in 1965 [7]. Because of the parallel structure that has a high load capacity and distributes the payload on the kinematic chains, along with other interesting properties, this parallel robot has been studied extensively in the literature.

The Denavit-Hartenberg method used in this paper for static forces analysis into the linkages is a general one. The calculation requires only the matrix multiplications and the solving of a linear equation system.

To design the components of a linkage in general, of a parallel robot in particular, the determination of the force distribution from its kinematic pairs needs to be determined. The performance of any mechanism can be assessed only after calculating the driving forces or moments and the reaction forces and moments from the kinematic pairs. The principle of virtual mechanic work (J. Bernoulli, 1717) is used to determine the distribution of the forces and moments in the linkage, as well as the areas where critical positions are likely to appear. Hence, the link positions, i.e. the variables sizes of all the kinematic pairs of the linkage must be determined. The Denavit-Hartenberg transformation matrices method [3] is used for both kinematic and kinetostatic analysis of the mechanism.

The design of a linkage is made in two stages. The kinematic dimensions of the links are calculated first and then the forces and the moments are determined. The second stage is organological dimensioning, that depends on the distribution of forces in the mechanism. Further, only aspects of the static distribution of forces in parallel mechanisms are analysed.

2. KINETOSTATIC ANALYSIS OF PARALLEL LINKAGES

To illustrate the proposed calculation method, it is considered a *Stewart* parallel robot mechanism with six degrees of freedom [7] (Fig. 1). This multi-loop mechanism consist of a fixed platform (**1**, **12**) and a mobile platform (**1**, **6**). Between these two platforms, six connecting kinematic chains are mounted. The connecting kinematic chains are identical, both of type *2RTS*, each comprising three links.

The links of this parallel mechanism can be grouped into five independent loops (Fig .2a). In this figure, only the links of the first loop are numbered as ((**1*i***), $i=\overline{1,\ 12}$). The numbering is made, according to Denavit-Hartenberg convention, with two indices. The first index indicates the loops' number and the second, the link's number in the loop. Similarly, the axes of the Denavit-Hartenberg coordinate systems and the kinematic pairs are numbered in the above mentioned way.

The spherical pair is replaced with a kinematic chain composed of two fictional links and three revolute pairs. For the first loop, the axes of the revolute pairs are denoted Z_{14}, Z_{15}, Z_{16}, respectively Z_{17}, Z_{18}, Z_{19} (Fig. 2) and are perpendicular two by two [2, 9, 10].

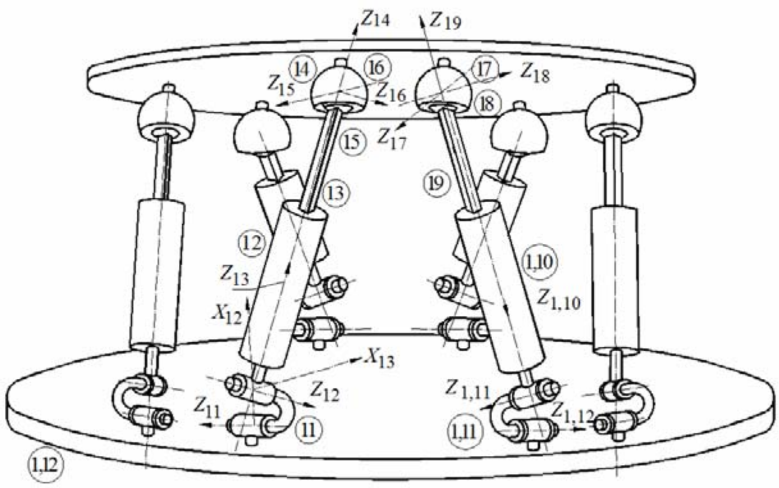

Fig. 1 – The Denavit-Hartenberg axis
of a parallel linkage with six degrees of freedom.

The kinematic pairs 1.3, 1.10, 2.10, 3.10, 4.10, and 5.10 are driving and their variables are the generalized coordinates of this linkage.

For every loop, the Denavit - Hartenberg [1, 2] matrix equations are:

$$\begin{gathered}\mathbf{A}_{i1}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{15}(q_{15})\mathbf{A}_{i6}(q_{i6})...\\ \mathbf{A}_{i,10}(q_{i,10})\mathbf{A}_{i,11}(q_{i,11})\mathbf{A}_{i,12}(q_{i,12)}=\mathbf{I},\ \ i=\overline{1,\ 5}.\end{gathered}\tag{1}$$

where q_{kl} is the θ_{kl} variable of the revolute pair ***l*** or the s_{kl} variable of the prismatic pair ***l***, belonging to loop ***k***.

F is a force applied to the link (**1**, **1**), whose components on the axes of the $O_{11}X_{11}Y_{11}Z_{11}$ system, attached to the fixed platform (**11**) are $F_{11X}, F_{11Y}, F_{11Z}$:

$$\mathbf{F} = F_{11X}\,\overline{i_{11}} + F_{11Y}\,\overline{j_{11}} + F_{11Z}\,\overline{k_{11}}\,. \tag{2}$$

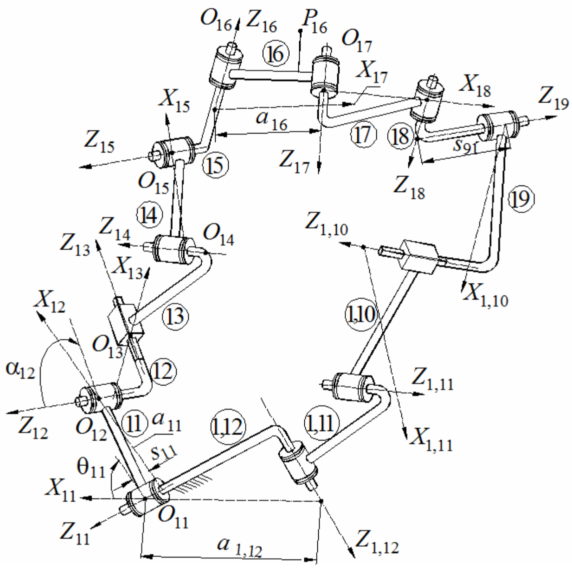

Fig. 2a – The first loop of the *Stewart* parallel linkage in undeformed state.

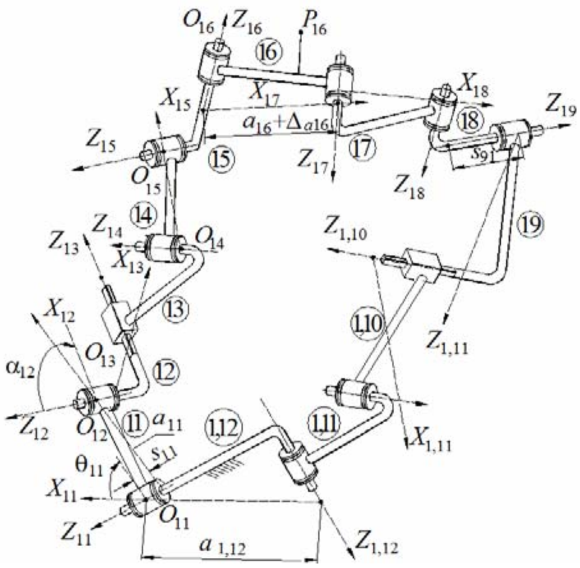

Fig. 2b – Virtual deformation of the first loop of *Stewart* parallel linkage.

The force **F** is applied to the *end-effector* (16) of the parallel linkage, at the point P of the mobile platform, with the coordinates X_{17P}, Y_{17P}, Z_{17P}. In each passive pair, the reaction wrench has five components. In the passive revolute pair (***i***, ***j***), the components are R_{xij}, R_{yij}, R_{zij}, M_{xij}, M_{yij}. In the passive prismatic pair (***i***, ***j***), the wrench components are R_{xij}, R_{yij}, M_{xij}, M_{yij}, M_{zij}. In each driving pair, the reaction wrench has all the components: R_{zij} is the driving force from the prismatic pair and M_{zij} is the driving moment from the revolute pair. In what follows, the frictional forces and moments between the links are neglected.

To calculate the sizes of the reaction wrench components caused by this force in the revolute pair **1*i***, reaction that acts on the link (**1**, ***i***) from the link (**1**, ***i*+1**), a virtual deformation of the linkage is assumed, thus of the first loop between link (**1**, ***i***) and link (**1**, ***i*+1**), and has as a result a change in the geometry of the linkage, without affecting the values of the independent variables from the driving pairs (Fig. 2b). This virtual deformation is being conveniently considered in each case, so that the calculation of the desired component of the reaction wrench is possible. The kinematic pairs numbered (**13**) and (**1**, **10**) of the loop **1** are driving pairs. The virtual deformation of the loop is expressed by a matrix $\mathbf{H}_i$ approximately equal to the unit matrix:

$$\mathbf{H}_i = \mathbf{I} + \Delta\mathbf{H}_i. \tag{3}$$

With that, the closing matrix equation of the first loop of the parallel linkage (Fig. 2b) becomes:

$$\begin{aligned}&\mathbf{A}_{11}(q_{11}+\Delta q_{11})\mathbf{A}_{12}(q_{12}+\Delta q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{1,i-1}(q_{1,i-1}+\Delta q_{1,i-1})\mathbf{H}_i\\ &\mathbf{A}_{1i}(q_{1i}+\Delta q_{1i})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{A}_{1,11}(q_{1,11+}\Delta q_{1,11})\mathbf{A}_{1,12}(q_{1,12+}\Delta q_{1,12}) = \mathbf{I}.\end{aligned} \tag{4}$$

Along a closed-loop, the product of the transformation matrices is equal with the unit matrix **I**. This unit matrix **I** denotes that the links are the component parts of the closed loops [2, 3, 9, 10].

Similarly, for the other four deformed independent closed-loops of the parallel linkage, the Denavit-Hartenberg matrix equations are:

$$\begin{aligned}&\mathbf{A}_{j1}(q_{j1}+\Delta q_{j1})\mathbf{A}_{12}(q_{12}+\Delta q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{j,i-1}(q_{j,i-1}+\Delta q_{j,i-1})\\ &\mathbf{A}_{ji}(q_{ji}+\Delta q_{ji})....\mathbf{A}_{j,10}(q_{j,10})\mathbf{A}_{j,11}(q_{j,11+}\Delta q_{j,11})\mathbf{A}_{j,12}(q_{j,12+}\Delta q_{j,12}) = \mathbf{I},\ j=\overline{2,5}.\end{aligned} \tag{5}$$

When the component R_{X1j} of the reaction force from the pair **1*i*** along the axis $O_{1j}X_{1j}$ is calculated, the virtual displacement Δa_{X1j} into the matrix $\mathbf{H}_j$ occurs in the following manner:

$$\mathbf{H}_j = \mathbf{H}_{xj} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ \Delta a_{x1j} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \tag{6}$$

or:

$$\mathbf{H}_j = \mathbf{I} + \mathbf{Q}_{Rx} \Delta a_{x1j}, \tag{7}$$

where the matrix operator $\mathbf{Q}_{Rx}$ has the form [6]:

$$\mathbf{Q}_{Rx} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}. \tag{8}$$

In the same manner the matrix operators for other virtual deformations are defined, which are necessary in all the reaction wrench components calculation [6]

$$\mathbf{Q}_{Ry} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}; \quad \mathbf{Q}_{Rz} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{Vmatrix}; \quad \mathbf{Q}_{Mx} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{Vmatrix};$$

$$\mathbf{Q}_{My} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{Vmatrix}; \quad \mathbf{Q}_{Mz} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}. \tag{9}$$

Note that all the matrix operators $\mathbf{Q}$ have the property expressed by the equality:

$$q_{ij} = -q_{ji},\ i = \overline{2,\ 4}, j = \overline{2,\ 4}\ .$$

By developing the matrix equations (4) and (5) it results a system of 30 equations [2, 3, 9, 10]:

$$
\begin{aligned}
&\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{1,i-1}(q_{1,i-1})\mathbf{A}_{1i}(q_{1i})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{A}_{1,11}(q_{1,11})\mathbf{A}_{1,12}(q_{1,12})+\\
&+\mathbf{Q}_q\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{1i}(q_{1i})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{A}_{1,11}(q_{1,11})\mathbf{A}_{1,12}(q_{1,12})\Delta q_{11}+\\
&+\mathbf{A}_{11}(q_{11})\mathbf{Q}_q\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{1i}(q_{1i})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{A}_{1,11}(q_{1,11})\mathbf{A}_{1,12}(q_{1,12})\Delta q_{12}+\\
&+\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\mathbf{Q}_q\mathbf{A}_{14}(q_{14})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{A}_{1,11}(q_{1,11})\mathbf{A}_{1,12}(q_{1,12})\Delta q_{14}+\\
&+...+\\
&+\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\mathbf{A}_{14}(q_{14})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{Q}_q\mathbf{A}_{1,11}(q_{1,11})\mathbf{A}_{1,12}(q_{1,12})\Delta q_{1,11}+\\
&+\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\mathbf{A}_{14}(q_{14})....\mathbf{A}_{1,10}(q_{1,10})\mathbf{A}_{1,11}(q_{1,11})\mathbf{Q}_q\mathbf{A}_{1,12}(q_{1,12})\Delta q_{1,12}=\\
&=\mathbf{I}-\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})...\mathbf{A}_{1,i-1}(q_{1,i-1})\mathbf{Q}_{Rx}\mathbf{A}_{1i}(q_{1i})...\mathbf{A}_{1,11}(q_{1,11})\mathbf{A}_{1,12}(q_{1,12})\Delta x_{1i};\\
&\mathbf{A}_{j1}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{15}(q_{15})\mathbf{A}_{j6}(q_{j6})....\mathbf{A}_{j,10}(q_{j,10})\mathbf{A}_{j,11}(q_{j,11})\mathbf{A}_{j,12}(q_{j,12})+\\
&+\mathbf{Q}_q\mathbf{A}_{j1}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{ji}(q_{ji})....\mathbf{A}_{j,10}(q_{j,10})\mathbf{A}_{j,11}(q_{j,11})\mathbf{A}_{j,12}(q_{j,12})\Delta q_{11}+\\
&+\mathbf{A}_{j1}(q_{11})\mathbf{Q}_q\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{ji}(q_{ji})....\mathbf{A}_{j,10}(q_{j,10})\mathbf{A}_{j,11}(q_{j,11})\mathbf{A}_{j,12}(q_{j,12})\Delta q_{12}+\\
&+\mathbf{A}_{j1}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\mathbf{Q}_q\mathbf{A}_{14}(q_{14})....\mathbf{A}_{j,10}(q_{j,10})\mathbf{A}_{j,11}(q_{j,11})\mathbf{A}_{j,12}(q_{j,12})\Delta q_{14}+\\
&+...+\\
&+\mathbf{A}_{j1}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})....\mathbf{A}_{j,9}(q_{1,9})\mathbf{A}_{j,10}(q_{1,10})\mathbf{Q}_q\mathbf{A}_{j,11}(q_{j,11})\mathbf{A}_{j,12}(q_{j,12})\Delta q_{j,11}+\\
&+\mathbf{A}_{j1}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\mathbf{A}_{14}(q_{14})....\mathbf{A}_{j,10}(q_{1,10})\mathbf{A}_{j,11}(q_{j,11})\mathbf{Q}_q\mathbf{A}_{j,12}(q_{j,12})\Delta q_{j,12}=\mathbf{I}.
\end{aligned} \tag{10}
$$

This system, consisting of 30 equations, contains 31 unknown variables:

$$\Delta q_{11},\ \Delta q_{12},\ \Delta q_{14},\ \Delta q_{15},\ \Delta q_{16},\ \Delta q_{i7},\ \Delta q_{i8},\ \Delta q_{i9},\ \Delta q_{i,11},\ \Delta q_{i,12},\quad i=\overline{1,5},\quad \Delta a_{x1i}$$

and it's solved in relation to the ratios:

$$
\begin{aligned}
&u_1=\frac{\Delta q_{11}}{\Delta a_{x1i}};\ u_2=\frac{\Delta q_{12}}{\Delta a_{x1i}},\ u_3=\frac{\Delta q_{14}}{\Delta a_{x1i}},\ u_4=\frac{\Delta q_{15}}{\Delta a_{x1i}},\ u_5=\frac{\Delta q_{16}}{\Delta a_{x1i}},\\
&u_{5,i+1}=\frac{\Delta q_{i7}}{\Delta a_{x1i}},\ u_{5,i+2}=\frac{\Delta q_{i8}}{\Delta a_{x1i}},\ u_{5,i+3}=\frac{\Delta q_{i9}}{\Delta a_{x1i}},\ u_{5,i+4}=\frac{\Delta q_{i,11}}{\Delta a_{x1i}},\\
&u_{5,i+5}=\frac{\Delta q_{i,12}}{\Delta a_{x1i}},\ i=\overline{1,\ 5}.
\end{aligned} \tag{11}
$$

If the components of a reaction force from a kinematic pair belonging to the second loop are calculated, the closing matrix equations of the five independent loops will be:

$$
\begin{aligned}
&\mathbf{A}_{21}(q_{11}+\Delta q_{11})\mathbf{A}_{12}(q_{12}+\Delta q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{2,i-1}(q_{2,i-1}+\Delta q_{2,i-1})\mathbf{H}_i\\
&\mathbf{A}_{2i}(q_{2i}+\Delta q_{2i})....\mathbf{A}_{2,10}(q_{2,10})\mathbf{A}_{2,11}(q_{2,11+}\Delta q_{2,11})\mathbf{A}_{2,12}(q_{2,12+}\Delta q_{2,12})=\mathbf{I}.
\end{aligned} \tag{12}
$$

$$\mathbf{A}_{i1}(q_{i1}+\Delta q_{i1})\mathbf{A}_{12}(q_{12}+\Delta q_{12})\mathbf{A}_{13}(q_{13})\ldots\mathbf{A}_{i,j-1}(q_{i,j-1}+\Delta q_{i,j-1})$$
$$\mathbf{A}_{ij}(q_{ij}+\Delta q_{ij})\ldots.\mathbf{A}_{i,10}(q_{i,10})\mathbf{A}_{i,11}(q_{i,11+}\Delta q_{i,11})\mathbf{A}_{i,12}(q_{i,12+}\Delta q_{i,12})=\mathbf{I},\ i=\overline{1,3,5}. \quad (13)$$

The components R_{1i} of the reaction force along the axes $O_{1i}x_{1i}$, $O_{1i}y_{1i}$, $O_{1i}z_{1i}$, have the values:

$$R_{x1i}=\frac{\Delta q_{i7}}{\Delta a_{x1i}}P_{11x},\quad R_{y1i}=\frac{\Delta q_{i7}}{\Delta a_{x1i}}P_{11y},\quad R_{z1i}=\frac{\Delta q_{i7}}{\Delta a_{x1i}}P_{11z}. \quad (14)$$

In an undeformed parallel linkage, the coordinates of the point P of the link (**1**, **6**) in relation to the fixed system $O_{11}X_{11}Y_{11}Z_{11}$ are calculated with the relation:

$$\begin{Vmatrix}1\\X_{11P}\\Y_{11P}\\Z_{11P}\end{Vmatrix}=\mathbf{A}_{11}(q_{11}).\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\ldots.\ \mathbf{A}_{16}(q_{16}).\begin{Vmatrix}1\\X_{17P}\\Y_{17P}\\Z_{17P}\end{Vmatrix}, \quad (15)$$

After the virtual deformation of the parallel linkage with the size Δq_x, the coordinates of the same point are:

$$\begin{Vmatrix}1\\X_{11P}+\Delta a_{X_{11P}}\\Y_{11P}+\Delta a_{Y_{11P}}\\Z_{11P}+\Delta a_{Z_{11P}}\end{Vmatrix}=\mathbf{A}_{11}(q_{11}+\Delta a_{X11})\mathbf{A}_{12}(q_{12}+\Delta a_{X12})\mathbf{A}_{13}(q_{13})\ldots$$
$$\ldots\mathbf{A}_{15}(q_{15}+\Delta a_{X15})\mathbf{A}_{i6}(q_{16}+\Delta q_{a_{X16}}).\begin{Vmatrix}1\\X_{17P}\\Y_{17P}\\Z_{17P}\end{Vmatrix}, \quad (16)$$

Hence the:

$$\begin{Vmatrix}1\\\Delta_{x11P}\\\Delta_{y11P}\\\Delta_{z11P}\end{Vmatrix}=(\mathbf{Q}_a\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\ldots\mathbf{A}_{16}(q_{16})\Delta q_{11}+$$
$$+\mathbf{A}_{11}(q_{11})\mathbf{Q}_a\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})\ldots\mathbf{A}_{16}(q_{16})\Delta q_{12}+$$
$$+\ldots+$$
$$+\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\ldots\mathbf{Q}_a\mathbf{A}_{15}(q_{15})\mathbf{A}_{16}(q_{16})\Delta q_{15}+$$

$$+\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{Q}_a\mathbf{A}_{16}(q_{16})\Delta q_{16})\begin{Vmatrix} 1 \\ x_{17P} \\ y_{17P} \\ z_{17P} \end{Vmatrix}\Delta_X \tag{17}$$

But:

$$q_{1k} = u_k \Delta q_{1k}\,, \tag{18}$$

and noting:

$$\mathbf{A}_{11}(q_{11})\mathbf{A}_{12}(q_{12})\mathbf{A}_{13}(q_{13})...\mathbf{A}_{1,k-1}(q_{1,j-1})\mathbf{Q}_a\mathbf{A}_{1k}(q_{1k})...\mathbf{A}_{16}(q_{16})\begin{Vmatrix} 1 \\ x_{11k} \\ y_{11k} \\ z_{11k} \end{Vmatrix}\mathbf{U}_k = \mathbf{L}_k\,. \tag{19}$$

and

$$\mathbf{L}_{12} + \mathbf{L}_{12} + ... + \mathbf{L}_{15} + \mathbf{L}_{16} = \mathbf{M}_6\,, \tag{20}$$

can be written:

$$\mathbf{M}_{16}\Delta x_{16} = \begin{Vmatrix} 0 \\ \Delta x_{11P} \\ \Delta y_{11P} \\ \Delta z_{11P} \end{Vmatrix}, \tag{21}$$

or:

$$m_{16,2}\Delta x_{11P} = \Delta x_{11P}\,; \quad m_{16,3}\Delta x_{11P} = \Delta y_{11P}\,; \quad m_{16,4}\Delta x_{11P} = \Delta z_{11P}\,. \tag{22}$$

In the same manner, the components of the wrench reaction are calculated on the other axis of the Denavit-Hartenberg coordinate system by replacing the matrix operator **Q**. Also, by changing the position of the matrix **H** in the matrix equation (4), the reaction wrench components of all the linkages' kinematic pairs can be calculated.

3. CONCLUSION

The proposed method for analysing the distribution of forces in the kinematic pairs of the parallel linkages using the Denavit-Hartenberg transformation matrices and the principle of virtual work is general and relatively easy to apply. This statement is taking into account the fact that the operations that have to be carried out are only 4×4 matrix multiplications, to which is added the solving of a system with 30 linear equations. The exhibited example is very intricate, in order to prove the boundless possibilities of application.

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