INVERSE DYNAMICS OF A TRANSLATION-ROTATION HYBRID PARALLEL ROBOT

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Abstract. Matrix relations for dynamics analysis of a spatial two-module hybrid parallel manipulator are established in this paper. Knowing the relative motions of two serially connected moving platforms, the inverse dynamics problem is solved based on a set of recursive explicit equations of parallel robots dynamics. Finally, compact results and graphs of simulation for the input forces and powers of six actuators are obtained.

Key words: connectivity relations, hybrid parallel robot, dynamics, kinematics.

1. INTRODUCTION

Having closed-loop structures, the components of parallel manipulators are connected by spherical joints, universal joints, revolute joints or prismatic joints [1–3]. The mechanisms of parallel robots can be found in practical applications, in which it is desired to orient a rigid body in space at high velocities and accelerations [4, 5].

The hybrid robot is a combination of closed-chain spatial mechanisms or a sequence of parallel robots [6, 7]. From rigidity, accuracy and workspace point of view, this kind of hybrid architectures posses the advantages of both serial and parallel manipulators [8, 9]. Zhang and Song [10] analyzed the geometry and the position of a hybrid manipulator composed of two serially connected parallel robots. Shahinpoor [11] obtained and solved numerically a system of nonlinear equations for the inverse kinematics problem of a hybrid robotic system consisting of two connected parallel manipulators.

Considering the lower module as a positional mechanism and the upper module as an orientation device, a recursive method based on explicit equations of parallel robots dynamics is applied in the present paper to the inverse dynamics of a spatial hybrid manipulator, each of two different serially connected mechanisms having three degrees of freedom.

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2. KINEMATICS REVIEWS

The hybrid robot is made up of two different 3-DOF parallel modules, which are serially connected to a fixed base [12].

The positional lower module consists of a fixed circular base $A_1B_1C_1$ of radius *R*, a circular mobile platform $A_4B_4C_4$ of radius *r*, mass m_p and tensor of inertia $\hat{J}_p$ and three extensible legs with identical structure. Each leg connects the fixed base to the moving platform by two universal joints interconnected through a prismatic joint. First platform achieves only translation if along each leg the axis of lower revolute joint is parallel to that of the last one of upper joint and also, the two intermediate joint axes are parallel to one another.

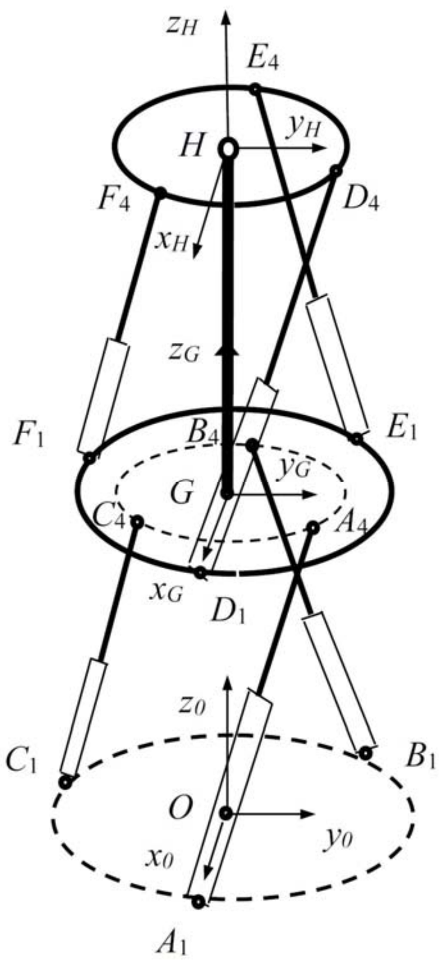

Fig. 1 – Hybrid parallel robot.

The architecture of the upper parallel module consists of a new identical mobile platform $D_4E_4F_4$ of radius *r*, mass m_p and inertia tensor J_p, connected to the lower moving base through a central spherical joint *H* and three identical extensible *UPS* legs, each comprising a prismatic actuator. The lower end of each leg is connected to the first moving platform by a universal joint situated on a circle of radius *R* (Fig. 1).

As is presented in our published paper [12], we assign successively the systems of coordinates $Q\,x_Q y_Q z_Q$ $(Q = O, G)$ and two mobile frames $G\,x_G y_G z_G$ and $H\,x_H y_H z_H$ on the moving platforms, with the centers $K = (G, H)$ initially located at identical relative elevation $OG = GH = h$, above the base.

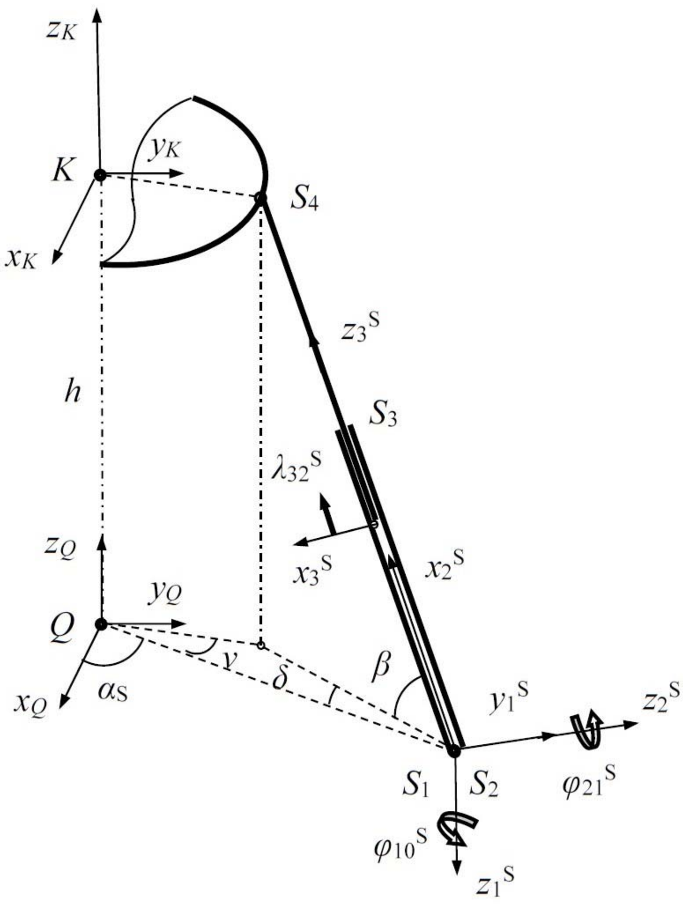

Fig. 2 – Kinematical scheme of the arbitrary leg S of the hybrid robot.

The active arbitrary leg $S = (A, B, C, D, E, F)$ of the hybrid parallel robot consists of a little cross $S_1 x_1^S y_1^S z_1^S$ of mass m_1 of a universal joint, which rotate with an angle φ_{10}^S and connects a moving cylinder $S_2 x_2^S y_2^S z_2^S$ of length l_2, mass m_2 and inertia tensor $\hat{J}_2$, having a relative rotation with the angle φ_{21}^S. An actuated piston of length l_3, mass m_3 and inertia tensor $\hat{J}_3$, linked to $S_3 x_3^S y_3^S z_3^S$, has a relative displacement λ_{32}^S. The six pistons of two modules are initially starting from the same position $l_1 = h/\sin\beta - l_3$ and the angles of orientation of universal joints are given by

$$\alpha_A = \alpha_D = 0, \quad \alpha_B = \alpha_E = -\alpha_C = -\alpha_F = \frac{2\pi}{3}, \quad \nu = \frac{\pi}{6} \tag{1}$$
$$(R - r\cos\nu)\tan\delta = r\sin\nu, \qquad r\sin\nu\tan\beta = h\sin\delta,$$

where the angle ν is defined as a chosen *twist* angle of the parallel robot and δ, β are two constant angles of rotation around axes z_1^S and z_2^S, respectively (Fig. 2).

Starting from O, G and pursuing independent legs $OA_1A_2A_3A_4$, $OB_1B_2B_3B_4$, $OC_1C_2C_3C_4$, $GD_1D_2D_3D_4$, $GE_1E_2E_3E_4$ and $GF_1F_2F_3F_4$, we obtain the matrices of transformation

$$\sigma_{10} = \sigma_{10}^{\varphi} a_\delta \theta_3 a_\alpha^S, \;\; \sigma_{21} = \sigma_{21}^{\varphi} a_\beta \theta_1^{\mathrm{T}}, \;\; \sigma_{32} = \theta_2, \;\; \sigma_{20} = \sigma_{21}\sigma_{10}, \;\; \sigma_{30} = \sigma_{32}\sigma_{20} \tag{2}$$
$$S = (i, j) \;\; (i = A, B, C) \;\; (j = D, E, F), \quad \sigma = (p, q) \;\; (p = a, b, c) \;\; (q = d, e, f),$$

where we denote the matrices [13]:

$$a_\alpha^S = \mathrm{rot}(z, \alpha_S), \quad a_\delta = \mathrm{rot}(z, \delta), \quad a_\beta = \mathrm{rot}(z, \beta), \quad a_\nu = \mathrm{rot}(z, \nu)$$
$$\sigma_{\tau,\tau-1}^{\varphi} = \mathrm{rot}(z, \varphi_{\tau,\tau-1}^S), \quad (\tau = 1, 2) \tag{3}$$
$$\theta_1 = \mathrm{rot}(x, \pi/2), \quad \theta_2 = \mathrm{rot}(y, \pi/2), \quad \theta_3 = \mathrm{rot}(y, \pi).$$

Since all rotations of angles $\alpha_1, \alpha_2, \alpha_3$ take place successively, the rotation matrix $h_{30} = h_{32} h_{21} h_{10}$ of *second* moving platform is obtained by multiplying some relative basic matrices

$$h_{10} = \mathrm{rot}(x, \alpha_1), \quad h_{21} = \mathrm{rot}(y, \alpha_2), \quad h_{32} = \mathrm{rot}(z, \alpha_3). \tag{4}$$

We consider that during $3s$ the coordinates of G and the angles α_l can describe relative motions of two platforms through following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{h - z_0^G}{z_0^{G*}} = \frac{\alpha_l}{\alpha_l^*} = 1 - \cos\frac{\pi}{3}t \quad (l = 1, 2, 3). \tag{5}$$

Pursuing the kinematical modelling developed in our published paper [12], a set of 18 independent variables $\varphi^i_{\tau,\tau-1}$, λ^i_{32}, $\varphi^j_{\tau,\tau-1}$, λ^j_{32} characterising the kinematics of two modules are determined from 18 analytical equations

$$\begin{aligned}
&-(l_1+l_3+\lambda^i_{32})\cos(\varphi^i_{10}+\delta-\alpha_i)\cos(\varphi^i_{21}+\beta)=x^G_0+r\cos(\alpha_i+\nu)-R\cos\alpha_i\\
&(l_1+l_3+\lambda^i_{32})\sin(\varphi^i_{10}+\delta-\alpha_i)\cos(\varphi^i_{21}+\beta)=y^G_0+r\sin(\alpha_i+\nu)-R\sin\alpha_i\\
&(l_1+l_3+\lambda^i_{32})\sin(\varphi^i_{21}+\beta)=z^G_0\\
&-(l_1+l_3+\lambda^j_{32})\cos(\varphi^j_{10}+\delta-\alpha_j)\cos(\varphi^j_{21}+\beta)=r\vec{u}^{\,\mathrm{T}}_1h^{\mathrm{T}}_{30}a^{j\,\mathrm{T}}_{\alpha}a^{\mathrm{T}}_{\nu}\vec{u}_1-R\cos\alpha_j\\
&(l_1+l_3+\lambda^j_{32})\sin(\varphi^j_{10}+\delta-\alpha_j)\cos(\varphi^j_{21}+\beta)=r\vec{u}^{\,\mathrm{T}}_2h^{\mathrm{T}}_{30}a^{j\,\mathrm{T}}_{\alpha}a^{\mathrm{T}}_{\nu}\vec{u}_1-R\sin\alpha_j\\
&(l_1+l_3+\lambda^j_{32})\sin(\varphi^j_{21}+\beta)=h+r\vec{u}^{\,\mathrm{T}}_3h^{\mathrm{T}}_{30}a^{j\,\mathrm{T}}_{\alpha}a^{\mathrm{T}}_{\nu}\vec{u}_1\,. \qquad (6)
\end{aligned}$$

Using the skew-symmetric matrix

$$\tilde{\omega}^H_{30}=\dot{\alpha}_1h_{32}h_{21}\tilde{u}_1h^{\mathrm{T}}_{21}h^{\mathrm{T}}_{32}+\dot{\alpha}_2h_{32}\tilde{u}_2h^{\mathrm{T}}_{32}+\dot{\alpha}_3\tilde{u}_3\,,$$

which is associated with the angular velocity $\vec{\omega}^H_{30}$ of the *upper* moving platform, we obtain the *matrix conditions of connectivity* [14]:

$$\begin{aligned}
\vec{V}_i&=[\omega^i_{10}\;\;\omega^i_{21}\;\;v^i_{32}]^{\mathrm{T}}=[N_i]^{-1}\vec{P}_i\\
\vec{V}_j&=[\omega^j_{10}\;\;\omega^j_{21}\;\;v^j_{32}]^{\mathrm{T}}=[N_j]^{-1}\vec{P}_j\,, \qquad (7)
\end{aligned}$$

where following terms determines the contents of 3×3 invertible square matrices $[N_i],[N_j]$ and the column matrices $\vec{P}_i\,,\vec{P}_j$:

$$\begin{aligned}
&n^i_{m1}=u^{\mathrm{T}}_mp^{\mathrm{T}}_{10}\tilde{u}_3p^{\mathrm{T}}_{21}\vec{r}^{\,i}_{42}\,,\;\; n^i_{m2}=\vec{u}^{\,\mathrm{T}}_mp^{\mathrm{T}}_{20}\tilde{u}_3\vec{r}^{\,i}_{42}\,,\;\; n^i_{m3}=\vec{u}^{\,\mathrm{T}}_mp^{\mathrm{T}}_{20}\vec{u}_1\,,\;\; \vec{r}^{\,i}_{42}=\vec{r}^{\,i}_{32}+p^{\mathrm{T}}_{32}\vec{r}^{\,i}_{43}\\
&n^j_{m1}=u^{\mathrm{T}}_mq^{\mathrm{T}}_{10}\tilde{u}_3q^{\mathrm{T}}_{21}\vec{r}^{\,j}_{42}\,,\;\; n^j_{m2}=\vec{u}^{\,\mathrm{T}}_mq^{\mathrm{T}}_{20}\tilde{u}_3\vec{r}^{\,j}_{42}\,,\;\; n^j_{m3}=\vec{u}^{\,\mathrm{T}}_mq^{\mathrm{T}}_{20}\vec{u}_1\,,\;\; \vec{r}^{\,j}_{42}=\vec{r}^{\,j}_{32}+p^{\mathrm{T}}_{32}\vec{r}^{\,j}_{43}\\
&\qquad p^i_m=\vec{u}^{\,\mathrm{T}}_m\dot{\vec{r}}^{\,G}_0\,,\quad p^j_m=\vec{u}^{\,\mathrm{T}}_mh^{\mathrm{T}}_{30}\tilde{\omega}^H_{30}\vec{r}^{\,j_4}_H\quad (m=1,2,3)\,. \qquad (8)
\end{aligned}$$

Expressions of relative accelerations $\vec{\Gamma}_i=[\varepsilon^i_{10}\;\;\varepsilon^i_{21}\;\;\gamma^i_{32}]^{\mathrm{T}}$, $\vec{\Gamma}_j=[\varepsilon^j_{10}\;\;\varepsilon^j_{21}\;\;\gamma^j_{32}]^{\mathrm{T}}$ are obtained from the column matrices $\vec{\Gamma}_i=[N_i]^{-1}\vec{S}_i$, $\vec{\Gamma}_j=[N_j]^{-1}\vec{S}_j$, with the column matrices $\vec{S}_i=\dot{\vec{P}}_i-[\dot{N}_i]\vec{V}_i$, $\vec{S}_j=\dot{\vec{P}}_j-[\dot{N}_j]\vec{V}_j$.

3. INVERSE DYNAMICS MODEL

The relevant objective of the inverse dynamics of parallel robots is to determine the input forces or powers, which must be exerted by the actuators in order to produce a given trajectory of the end-effectors.

Dasgupta and Mruthyunjaya [15] used the Newton-Euler approach to develop closed-form dynamics equations of Stewart platform, considering all dynamic and gravity effects and the viscous friction at joints.

Knowing the kinematics state of each link as well as the external forces acting on the hybrid parallel manipulator, we apply explicit recursive matrix equations for the inverse dynamic problem, from the principle of virtual work, in order to obtain the input forces and powers required by six prismatic actuators. Independent hydraulic systems generate the forces $\vec{f}_{32}^{\,i} = f_{32}^{i}\vec{u}_3$, $\vec{f}_{32}^{\,j} = f_{32}^{j}\vec{u}_3$ and control the motion of six moving pistons of the legs.

The serial-parallel hybrid robot can artificially be transformed in a set of *seven open trees* by imaginary *cutting* each joint for two moving platforms and taking its effect into account by introducing the corresponding constraint conditions (Fig. 3).

The inertia force and the moment of inertia forces on an arbitrary rigid body T_k^S $(S = A, B, C, D, E, F)$ of the tree S

$$\vec{f}_k^{\,in\,S} = -m_k^S[\vec{\gamma}_{k0}^S + (\tilde{\omega}_{k0}^S\tilde{\omega}_{k0}^S + \tilde{\varepsilon}_{k0}^S)\vec{r}_{k0}^{\,S}],$$
$$\vec{m}_k^{\,in\,S} = -(m_k^S\tilde{r}_k^{CS}\vec{\gamma}_{k0}^S + \hat{J}_k^S\vec{\varepsilon}_{k0}^S + \tilde{\omega}_{k0}^S\hat{J}_k^S\vec{\omega}_{k0}^S), \quad (9)$$

are determined with respect to the centre of joint S_k. On the other hand, the wrench of two vectors $\vec{f}_k^{\,*S} = m_k^S g\,\sigma_{k0}\vec{u}_3$, $\vec{m}_k^{\,*S} = m_k^S g\,\tilde{r}_k^{CS}\sigma_{k0}\vec{u}_3$ $(\sigma = a, b, c, d, e, f)$ give the influence of the weight $m_k^S\vec{g}$ acting on same element T_k^S of the manipulator.

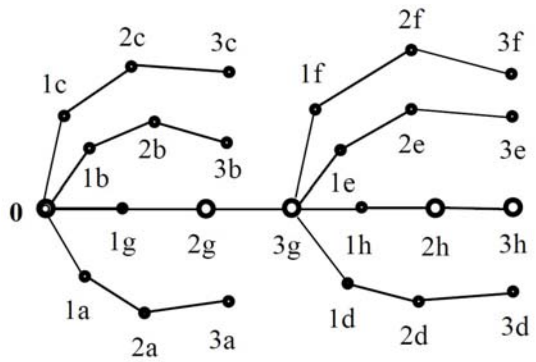

Fig. 3 – Seven open trees of the hybrid parallel robot.

Pursuing the leg S, two *significant recursive relations* generate the vectors

$$\vec{F}_k^S = \vec{F}_{k0}^S + \sigma_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^S, \quad \vec{M}_k^S = \vec{M}_{k0}^S + \sigma_{k+1,k}^{\mathrm{T}}\vec{M}_{k+1}^S + \tilde{r}_{k+1,k}^S\sigma_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^S, \quad (10)$$

with the notations $\vec{F}_{k0}^{S} = -\vec{f}_{k}^{\,in S} - \vec{f}_{k}^{\,*S}$, $\vec{M}_{k0}^{S} = -\vec{m}_{k}^{\,in S} - \vec{m}_{k}^{\,*S}$.

Starting from (10), we develop a set of six recursive relations for each leg *S*:

$$\vec{F}_3^S = \vec{F}_{30}^S, \quad \vec{F}_2^S = \vec{F}_{20}^S + \sigma_{32}^{\mathrm{T}}\vec{F}_3^S, \quad \vec{F}_1^S = \vec{F}_{10}^S + \sigma_{21}^{\mathrm{T}}\vec{F}_2^S$$

$$\vec{M}_3^S = \vec{M}_{30}^S, \quad \vec{M}_2^S = \vec{M}_{20}^S + \sigma_{32}^{\mathrm{T}}\vec{M}_3^S + \tilde{r}_{32}^S\sigma_{32}^{\mathrm{T}}\vec{F}_3^S, \quad \vec{M}_1^S = \vec{M}_{10}^S + \sigma_{21}^{\mathrm{T}}\vec{M}_2^S + \tilde{r}_{21}^S\sigma_{21}^{\mathrm{T}}\vec{F}_2^S$$

$$(S = A, B, C, D, E, F) \quad (\sigma = a, b, c, d, e, f)\,. \tag{11}$$

Specially, for the two moving platforms, we have

$$\vec{F}_3^H = m_p(g h_{30}\vec{u}_3 + \vec{\gamma}_{30}^H), \quad \vec{F}_2^H = h_{32}^{\mathrm{T}}\vec{F}_3^H, \quad \vec{F}_1^H = h_{21}^{\mathrm{T}}\vec{F}_2^H$$

$$\vec{M}_3^H = \hat{J}_p\vec{\varepsilon}_{30}^H, \quad \vec{M}_2^H = h_{32}^{\mathrm{T}}\vec{M}_3^H, \quad \vec{M}_1^H = h_{21}^{\mathrm{T}}\vec{M}_2^H\,. \tag{12}$$

$$\vec{F}_1^G = \vec{F}_2^G = \vec{F}_3^G = m_p(g\vec{u}_3 + \vec{\gamma}_{30}^G) + h_{10}^{\mathrm{T}}\vec{F}_1^H + d_{10}^{\mathrm{T}}\vec{F}_1^D + e_{10}^{\mathrm{T}}F_1^E + f_{10}^{\mathrm{T}}\vec{F}_1^F\,.$$

The *virtual velocities* are expressed as functions of the pose of the hybrid robot by the matrix kinematical equations (7) and (8). Considering some *independent virtual motions* of the spatial mechanism, virtual velocities should be compatible with the virtual motions imposed by all kinematical constraints at any instant in time. Let us assume that the robot has successively six independent virtual motions determined by following sets of velocities:

$$v_{32a}^{Av} = 1, \; v_{32a}^{Sv} = 0 \; (S \neq A); \quad v_{32b}^{Bv} = 1, \; v_{32b}^{Sv} = 0 \; (S \neq B);$$

$$v_{32c}^{Cv} = 1, \; v_{32c}^{Sv} = 0 \; (S \neq C); \quad v_{32d}^{Dv} = 1, \; v_{32d}^{Sv} = 0 \; (S \neq D); \tag{13}$$

$$v_{32e}^{Ev} = 1, \; v_{32e}^{Sv} = 0 \; (S \neq E); \quad v_{32f}^{Fv} = 1, \; v_{32f}^{Sv} = 0 \; (S \neq F)\,.$$

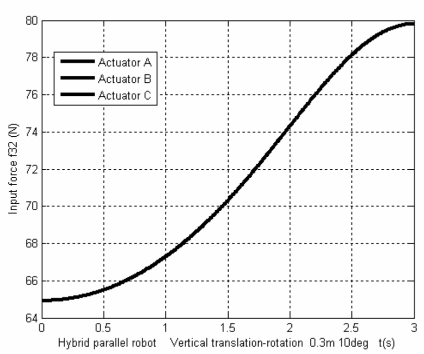

Fig. 4 – Input forces f_{32}^{i} of three actuators.

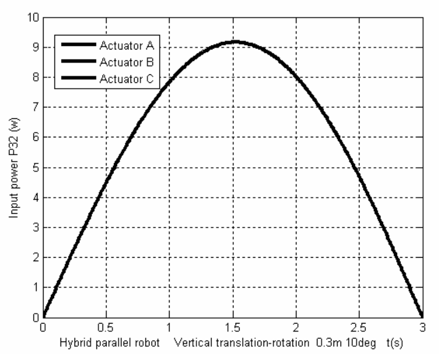

Fig. 5 – Input powers P_{32}^{i} of three actuators.

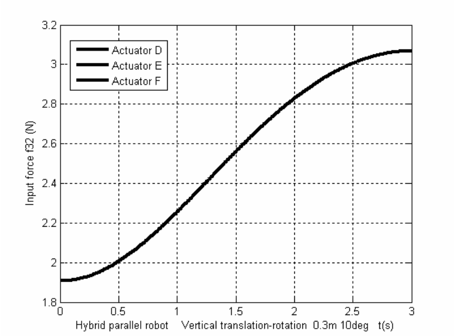

Fig. 6 – Input forces f_{32}^{j} of three actuators.

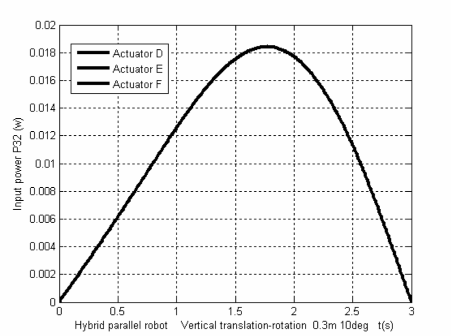

Fig. 7 – Input powers P_{32}^{j} of three actuators.

Using the *equations of parallel robots dynamics* [13, 14], following compact matrix relations results

$$f_{32}^{A} = \vec{u}_3^{\mathrm{T}}\{\vec{F}_3^{A} + \omega_{10a}^{Av}\vec{M}_1^{A} + \omega_{21a}^{Av}\vec{M}_2^{A} + \omega_{10a}^{Bv}\vec{M}_1^{B} + \omega_{21a}^{Bv}\vec{M}_2^{B} + \\ + \omega_{10a}^{Cv}\vec{M}_1^{C} + \omega_{21a}^{Cv}\vec{M}_2^{C}\} + v_{10a}^{Gv}\vec{u}_1^{T}\vec{F}_1^{G} + v_{21a}^{Gv}\vec{u}_2^{T}\vec{F}_2^{G} + v_{32a}^{Gv}\vec{u}_3^{T}\vec{F}_3^{G}, \quad (14)$$

for the *input force* required by the first active prismatic joint A_3 along the axis $A_3 z_3^{A}$ of lower module, and

$$f_{32}^{D} = \vec{u}_3^{T}\{\vec{F}_3^{D} + \omega_{10d}^{Dv}\vec{M}_1^{D} + \omega_{21d}^{Dv}\vec{M}_2^{D} + \omega_{10d}^{Ev}\vec{M}_1^{E} + \omega_{21d}^{Ev}\vec{M}_2^{E} + \\ + \omega_{10d}^{Fv}\vec{M}_1^{F} + \omega_{21d}^{Fv}\vec{M}_2^{F}\} + \omega_{10d}^{Hv}\vec{u}_1^{T}\vec{M}_1^{H} + \omega_{21d}^{Hv}\vec{u}_2^{T}\vec{M}_2^{H} + \omega_{32d}^{Gv}\vec{u}_3^{T}\vec{M}_3^{H} \quad (15)$$

for the *input force* required by the first active prismatic joint D_3 along the axis $D_3 z_3^{D}$ of upper module, for example.

The relations (10–15) represent the *inverse dynamics model* of the translational-rotational hybrid parallel robot.

As application let us consider same serial-parallel hybrid mechanism analysed in [12], which has the following mechanical and architectural characteristics

$$x_0^{G*} = 0.05\,\mathrm{m},\ y_0^{G*} = 0.05\,\mathrm{m},\ z_0^{G*} = 0.15\,\mathrm{m},\ \alpha_1^{*} = \pi/12,\ \alpha_2^{*} = \pi/18,\ \alpha_3^{*} = \pi/36$$

$$R = 0.6\,\mathrm{m},\ r = 0.2\,\mathrm{m},\ l_2 = 1\,\mathrm{m},\ l_3 = 0.6\,\mathrm{m},\ h = 0.8\,\mathrm{m}$$

$$m_2 = 1\,\mathrm{kg},\ m_3 = 0.75\,\mathrm{kg},\ m_p = 5\,\mathrm{kg},\ \Delta t = 3\,\mathrm{s},\ g = 9.81\,\mathrm{ms}^{-2}$$

$$\hat{J}_2 = \begin{bmatrix} 0.01 & & \\ & 0.325 & \\ & & 0.325 \end{bmatrix},\ \hat{J}_3 = \begin{bmatrix} 0.325 & & \\ & 0.325 & \\ & & 0.01 \end{bmatrix},\ \hat{J}_p = \begin{bmatrix} 00.6 & & \\ & 0.6 & \\ & & 1.2 \end{bmatrix}.$$

Assuming that the robot starts at rest from a central configuration, the MATLAB software for a computer program was developed to solve the inverse dynamics.

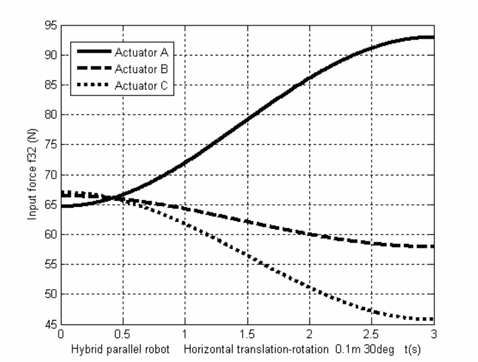

Fig. 8 – Input forces f_{32}^{i} of three actuators.

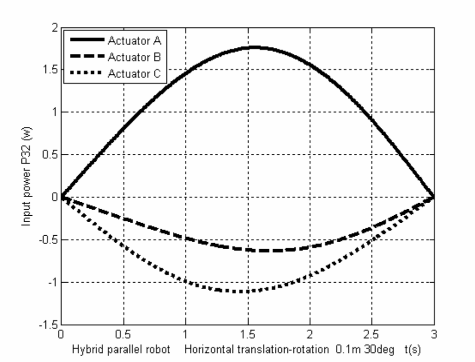

Fig. 9 – Input powers P_{32}^{i} of three actuators.

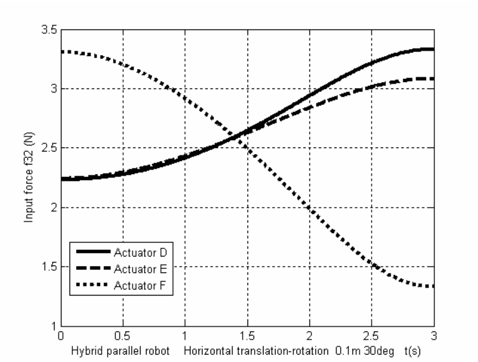

Fig. 10 – Input forces f_{32}^{j} of three actuators.

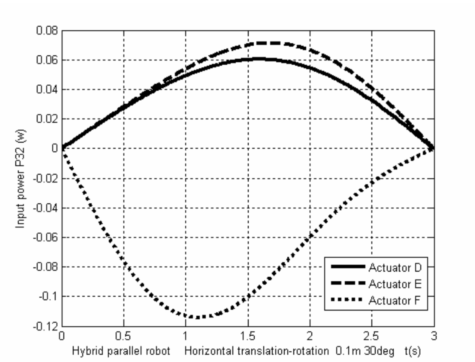

Fig. 11 – Input powers P_{32}^{j} of three actuators.

For the first example, the lower platform translates along the *vertical direction* z_0 while the upper platform rotates about same axis. Considering the weights of compounding elements as passive external forces acting on the spatial hybrid manipulator, the time-history evolution for the input forces f_{32}^{S} and input powers P_{32}^{S} (Figs. 4–7) of six prismatic actuators are calculated for a period of $\Delta t = 3$ seconds in terms of given equations (5).

As can be seen, it is proved to be true that all active forces and powers of lower module are permanently equal to one another but different to the forces and the powers of upper module.

In the case when the platforms make simultaneously a horizontal translation-rotation about x_0 axis, the graphs are plotted and illustrated in Figs. 8–11.

4. CONCLUSIONS

Using a set of explicit recursive matrix equations, already implemented in dynamics of parallel robots, a novel algorithm establishes the time-history evolution of input forces developed by the actuators of the serial-parallel hybrid manipulator, where the number of links of the mechanisms is increased and the value of total degrees-of-freedom is augmented.

The current matrix recursive formulation can easily be transformed in a model which is successfully expected to be deployed for automatic robotic control of hybrid parallel robots.

The concept and the procedure above developed can be immediately extended to analysis of a complex robotic system composed of a multitude serially arranged similar parallel modules, but are also applicable to study of hybrid robots that are composed of different structures of parallel modules.

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