DYNAMICS OF A THREE-DEGREES-OF-FREEDOM SPATIAL PARALLEL MANIPULATOR

ŞTEFAN STAICU*, YU-WEN LI, XIN-JUN LIU, JINSONG WANG, LI-PING WANG**

Recursive matrix relations for the kinematics and dynamics of a spatial parallel manipulator, which has two translation degrees of freedom and one rotational degree of freedom, are established in this paper. Supposing that the position and the motion of the mobile platform are known, an inverse dynamic formulation is presented. Finally, some matrix relations and graphs for the forces of the three actuators are determined. Comparison of simulation results in ADAMS with the formulation by MATLAB program shows the validity of the new matrix approach.

LIST OF SYMBOLS

$a_{k,k-1}$ – orthogonal relative transformation matrix

$\varphi_{k,k-1}$ – relative rotation angle of T_k rigid body

$\vec{\omega}_{k,k-1}$ – relative angular velocity of T_k

$\vec{\omega}_{k0}$ – absolute angular velocity of T_k

$\tilde{\omega}_{k,k-1}$ – skew symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$

$\vec{\varepsilon}_{k,k-1}$ – relative angular acceleration of T_k

$\tilde{\varepsilon}_{k0}$ – absolute angular acceleration of T_k

$\tilde{\varepsilon}_{k,k-1}$ – skew symmetric matrix associated to the angular acceleration $\vec{\varepsilon}_{k,k-1}$

$\vec{r}^{\,A}_{k,k-1}$ – relative position vector of the centre A_k of joint

$\vec{v}^{\,A}_{k,k-1}$ – relative velocity of the centre A_k

$\vec{\gamma}^{\,A}_{k,k-1}$ – relative acceleration of the centre A_k

m_k – mass of T_k rigid body

$\hat{J}_k$ – symmetric matrix of tensor of inertia of T_k about the link-frame $A_k x_k y_k z_k$

$m_{q,q-1}$ – torque of the actuator T_{q-1}

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1. INTRODUCTION

Parallel manipulators are closed-loop mechanisms presenting very good performances in terms of accuracy, rigidity and stability to manipulate large loads. These mechanisms generally compose two platforms, which are connected by joints and legs acting in parallel. One of them is attached to the fixed reference base. The other one can have arbitrary motion in its workspace. Several mobile legs or limbs, made up as serial robots, connect a movable platform to the fixed frame. Typically, the number of actuators is equal to the number of degrees of freedom.

Compared with commonly known serial robots, parallel manipulators have some special characteristics: greater structural rigidity, potentially higher precision, stabile functioning, larger capacity and suitable position of the actuating systems. There has been great amount of research on the application of parallel manipulators, such as machine tools [23] and industrial robots [2].

In the last few years the parallel manipulators with less DOF have attracted the researchers and some of them have been used in the structure design of robotic manipulators. Parallel mechanisms can be found in many technical applications in which it is desired to orient a rigid body in space of high speed. Accuracy and precision in the execution of the task are essential since the robot is intended to operate on fragile objects; any errors in the positioning of the tool could lead to expensive damage. Recently, many efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators.

Many companies have developed them as high precision machine tools. The most known application is the aircraft simulator with six degrees of freedom, which is in fact the Stewart-Gough platform. A lot of works have focused on the dynamics of this parallel manipulator ([4, 8, 14, 22, 26, 27]). Geng [6] developed Lagrange's equations of motion under some simplifying assumptions regarding the geometry and inertia distribution of the manipulator. Dasgupta and Mruthyunjaya [4] used the Newton-Euler approach to develop closed-form dynamic equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints. They observed that the application of the Newton-Euler method is more economical in the case of parallel or hybrid manipulators than in serial robots. Pierrot and Company [15] presented a new family of parallel robots with 4–DOF. The parallel manipulator Star [7] and the parallel Delta robot ([1, 21, 24, 25, 30]) equipped with three engines, which have a parallel setting, train on the end-effector in a three degrees of freedom general translation motion.

In the present paper, a new matrix approach is adopted to derive the inverse dynamic equations of a spatial parallel manipulator [10], which has two translation degrees of freedom and one rotational degree of freedom (Fig. 1).

2. GEOMETRIC MODEL OF THE MANIPULATOR

The spatial mechanism consists of three kinematical closed chains, including three variable length limbs (Fig. 1). The motion of the platform is accomplished by the slide of three sliders of mass m_1 on the guide-ways.

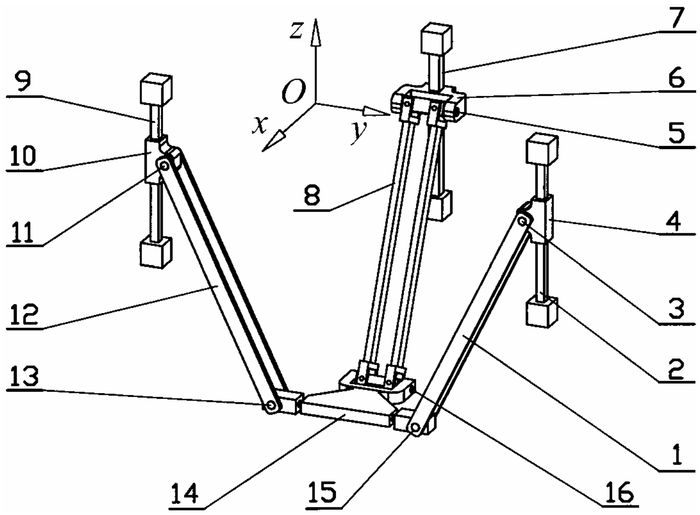

Fig. 1– Spatial parallel manipulator.

The movable platform is an isosceles triangle $A_3B_3C_5$, which is described by the sizes $A_3B_3 = 2r,\ \ A_3C_5 = B_3C_5 = r\sqrt{2}$. It has the mass m_3 and the tensor of inertia $\hat{J}_3$ (Fig. 2). This platform is connected to a fixed platform, which consists of a three guide-ways (2), (7) and (9), through the legs (1), (8) and (12). Parameter R denotes the size of the base platform, where $A_2B_2 = 2R,\ A_2C_2 = B_2C_2 = R$. The legs (1) and (12), have identical chains, consisting of a constant link which is connected to a universal joint at the bottom end and a passive revolute joint at the other. These legs have also the length L, same mass m_2 and same tensor of inertia $\hat{J}_2$. The revolute joint is then attached to an active slider, which is mounted on the guide-way (2) or (9). The third leg (8) consists of an active prismatic system, a revolute joint that is attached to a third slider, an intermediary mechanism with four revolute joints that connect a planar four-bar parallelogram. This classical mechanism is finally connected to the moving platform.

Let $Ox_0y_0z_0\,(T_0)$ be a global frame, located at the centre of the side A_2B_2 with the Oz_0 axis normal to the base platform and the Oy_0 axis along A_2B_2. The three-degrees-of-freedom manipulator is moving with respect to this Cartesian referential.

One of three active elements of the robot is the first body C_1 of the limb C, for example. This slider is mounted on the guide way (7) and effect a vertical translation with the displacement λ_{10}^{C}. A local leg frame $C_2 x_2^C y_2^C z_2^C$ is attached to a horizontal transmission bar with its origin at point C_2, the $C_2 z_2^C$ direction along the horizontal rotating axis of the revolute joint. This bar has a relative rotation with the angle φ_{21}^{C}. Further on, two identical bars with length L, same mass m_2 and same tensor of inertia $\hat{J}_2$, rotates about the T_2^C frame with the angle $\varphi_{32}^{C} = \varphi_{62}^{C}$. The parallelogram of the leg C is closed by an element T_4^C, which has the same dimensions with T_2^C.

The displacements λ_{10}^{A}, λ_{10}^{B}, λ_{10}^{C} of the three actuators A_1, B_1, C_1 are considered as parameters, which give the position of the mechanism. But, in the inverse geometric problem, one can suppose that the coordinates y_0^P, z_0^P of the point P and the rotational angle θ of the platform, give the position of the mechanism.

Pursuing the legs A, B, C, one obtains the following passing matrices

$$
\begin{aligned}
& a_{10} = a_1, \, a_{21} = a_{21}^{\varphi} a_{\alpha} a_4 \\
& b_{10} = a_2, \, b_{21} = b_{21}^{\varphi} a_{\alpha} a_4 \\
& c_{10} = a_3, \, c_{21} = c_{21}^{\varphi} a_{\alpha} a_4 \\
& c_{32} = c_{32}^{\varphi} a_4, \, c_{43} = c_{32}^{\varphi} a_5 \\
& c_{54} = c_{54}^{\varphi} a_{\alpha} a_6 \, , c_{62} = c_{32}^{\varphi} a_4 ,
\end{aligned} \tag{1}
$$

where one denoted ([17])

$$
a_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \, a_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
a_4 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \, a_5 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, a_6 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \tag{2}
$$

$$
a_{\alpha} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \, a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^{A} & \sin\varphi_{k,k-1}^{A} & 0 \\ -\sin\varphi_{k,k-1}^{A} & \cos\varphi_{k,k-1}^{A} & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,k-j} \ .$$

Rotation condition of the platform is given by following identity

$$c_{50}^{\circ\mathrm{T}} \mathrm{c}_{50} = a \tag{3}$$

and by the matrices

$$c_{50}^{\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \ a = R^{\mathrm{T}} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \tag{4}$$

where a is the rotation matrix around Py_A axis. From this relation, one obtains $\theta = \varphi_{54}^C - \varphi_{21}^C$.

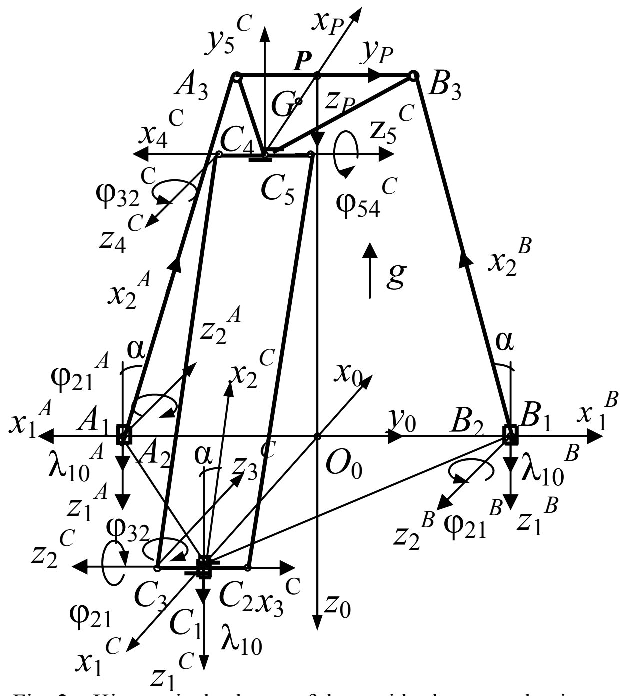

Fig. 2 – Kinematical scheme of the upside-down mechanism.

Considering that the platform moves for one second and the motion's trajectory of the movable platform is given as

$$\vec{r}_0^{\,\mathrm{P}} = [0 \quad y_0^P \quad z_0^P - h]^{\mathrm{T}}$$

$$\begin{aligned} y_0^P &= -t^2(t^2 - 1.3t + 0.3) \\ z_0^P &= 0.2t^2(2t^2 - t - 1) \\ \theta &= 0.4\sin(4\pi t), \end{aligned} \tag{5}$$

the displacements $\lambda_{10}^A, \lambda_{10}^B, \lambda_{10}^C$ and the angles $\varphi_{21}^A, \varphi_{21}^B$, $\varphi_{21}^C, \varphi_{32}^C$ are given by the following geometric constraint conditions

$$\vec{r}_{10}^{\,A} + a_{20}^T \vec{r}_{32}^{\,A} - a^T \vec{r}_{A_3} = \vec{r}_0^{\,P}, \; \vec{r}_{10}^{\,B} + b_{20}^T \vec{r}_{32}^{\,B} - a^T \vec{r}_{B_3} = \vec{r}_0^{\,P}, \tag{6}$$

$$\vec{r}_{10}^{\,C} + c_{20}^T \vec{r}_{43}^{\,C} + c_{50}^T \vec{r}_5^{\,P} = \vec{r}_0^{\,P},$$

where one denoted:

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{r}_{10}^{\,A} = \begin{bmatrix} 0 \\ -R \\ \lambda_{10}^A \end{bmatrix}, \vec{r}_{10}^{\,B} = \begin{bmatrix} 0 \\ R \\ \lambda_{10}^B \end{bmatrix}, \vec{r}_{10}^{\,C} = \begin{bmatrix} -R \\ 0 \\ \lambda_{10}^C \end{bmatrix} \tag{7}$$

$$\vec{r}_{32}^{\,A} = L\vec{u}_1, \; \vec{r}_{32}^{\,B} = L\vec{u}_1, \; \vec{r}_{43}^{\,C} = -L\vec{u}_2$$

$$\vec{r}_{A_3} = -r\vec{u}_2, \; \vec{r}_{B_3} = r\vec{u}_2, \; \vec{r}_5^{\,P} = r\vec{u}_1$$

$$h = \sqrt{L^2 - (R-r)^2}, \sin\alpha = \frac{R-r}{L}, \cos\alpha = \frac{h}{L}$$

$$z_1 = \lambda_{10}^A + h, \; z_2 = \lambda_{10}^B + h, \; z_3 = \lambda_{10}^C + h.$$

3. INVERSE KINEMATICAL ANALYSIS

The geometry and the motion capability of spatial manipulator has been analysed by Liu [11]. For inverse kinematics, the velocities and accelerations formulae are derived now in matrix closed form. Because the leg (1) and (2) have one rotational degree of freedom about the $A_2 z_2^A$ and $B_2 z_2^B$ axis, the angular velocities and angular accelerations can be written

$$\vec{\omega}_{20}^{\,A} = \dot{\varphi}_{21}^A \vec{u}_3, \; \vec{\omega}_{20}^{\,B} = \dot{\varphi}_{21}^B \vec{u}_3, \; \vec{\varepsilon}_{20}^{\,A} = \ddot{\varphi}_{21}^A \vec{u}_3, \; \vec{\varepsilon}_{20}^{\,B} = \ddot{\varphi}_{21}^B \vec{u}_3. \tag{8}$$

Velocities and accelerations of sliders A_1, B_1 and joints A_2, B_2 can be obtained as

$$\vec{v}_{10}^{A} = \dot{\lambda}_{10}^{A}\vec{u}_3,\ \vec{v}_{10}^{B} = \dot{\lambda}_{10}^{B}\vec{u}_3,\ \vec{v}_{20}^{A} = a_{21}\vec{v}_{10}^{A},\ \vec{v}_{20}^{B} = b_{21}\vec{v}_{10}^{B}$$

$$\vec{\gamma}_{10}^{A} = \ddot{\lambda}_{10}^{A}\vec{u}_3,\ \vec{\gamma}_{10}^{B} = \ddot{\lambda}_{10}^{B}\vec{u}_3,\ \vec{\gamma}_{20}^{A} = a_{21}\vec{\gamma}_{10}^{A},\ \vec{\gamma}_{20}^{B} = b_{21}\vec{\gamma}_{10}^{B}. \tag{9}$$

On the other hand, leg (8) has two rotational degrees of freedom. The kinematics of components of chain $C_2C_3C_4C_5$ are characterised by the matrices ([18]):

$$\tilde{\omega}_{k0}^{C} = c_{k,k-1}\tilde{\omega}_{k-1,0}^{C}c_{k,k-1}^{T} + \omega_{k,k-1}^{C}\tilde{u}_3, \tag{10}$$

which are *associated* to absolute angular velocities given by recurrence relations

$$\vec{\omega}_{k0}^{C} = c_{k,k-1}\vec{\omega}_{k-1,0}^{C} + \omega_{k,k-1}^{C}\vec{u}_3,\ \omega_{k,k-1}^{C} = \dot{\varphi}_{k,k-1}^{C}. \tag{11}$$

Following relation give the absolute velocity $\vec{v}_{k0}^{C}$ of joint C_k:

$$\vec{v}_{k0}^{C} = c_{k,k-1}\{\vec{v}_{k-1,0}^{C} + \tilde{\omega}_{k-1,0}^{C}\vec{r}_{k,k-1}^{C}\},\ \vec{v}_{10}^{C} = \dot{\lambda}_{10}^{C}\vec{u}_3\ \ (k = 2,3,4,5). \tag{12}$$

Absolute angular accelerations $\vec{\varepsilon}_{k0}^{C}$ and accelerations $\vec{\gamma}_{k0}^{C}$ of joint C_k can be deduced from the time derivative of equations (10), (11), (12):

$$\begin{aligned}
&\vec{\varepsilon}_{k0}^{C} = c_{k,k-1}\vec{\varepsilon}_{k,k-1}^{C} + \varepsilon_{k,k-1}^{C}\vec{u}_3 + \omega_{k,k-1}^{C}c_{k,k-1}\tilde{\omega}_{k-1,0}^{C}c_{k,k-1}^{T}\vec{u}_3,\ \ \varepsilon_{k,k-1}^{C} = \ddot{\varphi}_{k,k-1}^{C},\\
&\tilde{\omega}_{k0}^{C}\tilde{\omega}_{k0}^{C} + \tilde{\varepsilon}_{k0}^{C} = \omega_{k,k-1}^{C}\omega_{k,k-1}^{C}\tilde{u}_3\tilde{u}_3 + \varepsilon_{k,k-1}^{C}\tilde{u}_3 +\\
&+c_{k,k-1}(\tilde{\omega}_{k-1,0}^{C}\tilde{\omega}_{k-1,0}^{C} + \tilde{\varepsilon}_{k-1,0}^{C})c_{k,k-1}^{T} + 2\omega_{k,k-1}^{C}c_{k,k-1}\tilde{\omega}_{k-1,0}^{C}c_{k,k-1}^{T}\tilde{u}_3,\\
&\vec{\gamma}_{k0}^{C} = c_{k,k-1}\vec{\gamma}_{k-1,0}^{C} + \gamma_{k,k-1}^{C} + c_{k,k-1}(\tilde{\omega}_{k-1,0}^{C}\tilde{\omega}_{k-1.0}^{C} + \tilde{\varepsilon}_{k-1,0}^{C})\vec{r}_{k,k-1}^{C},\ \ \vec{\gamma}_{10}^{C} = \ddot{\lambda}_{10}^{C}\vec{u}_3.
\end{aligned} \tag{13}$$

Following matrix relations give *the kinematical conditions of connectivity* for the relative velocities:

$$\begin{aligned}
&v_{10}^{A}\vec{u}_i^{T}\vec{u}_3 + \omega_{21}^{A}L\vec{u}_i^{T}a_{20}^{T}\tilde{u}_3\vec{u}_1 = \dot{\vec{r}}_0^{P},\\
&v_{10}^{B}\vec{u}_i^{T}\vec{u}_3 + \omega_{21}^{B}L\vec{u}_i^{T}b_{20}^{T}\tilde{u}_3\vec{u}_1 = \dot{\vec{r}}_0^{P}\ (i = 2,3),\\
&v_{10}^{C}\vec{u}_j^{T}\vec{u}_3 - \omega_{21}^{C}\{L\vec{u}_j^{T}c_{20}^{T}\tilde{u}_3c_{32}^{T}\vec{u}_2 - r\vec{u}_j^{T}c_{20}^{T}\tilde{u}_3c_{32}^{T}\vec{u}_2\} -\\
&-\omega_{32}^{C}L\vec{u}_j^{T}c_{30}^{T}\tilde{u}_3\vec{u}_2 + \omega_{54}^{C}L\vec{u}_j^{T}c_{50}^{T}\tilde{u}_3\vec{u}_1 = \dot{\vec{r}}_0^{P}\ (j = 1,2,3),\\
&\omega_{54}^{C} = \omega_{21}^{C} + \dot{\theta}.
\end{aligned} \tag{14}$$

The relations (14) give the Jacobian of the mechanism. This matrix is an essential element for the analysis of the robot workspace. Also, these relations offer a matrix closed-form for the velocities $v_{10}^{A}, v_{10}^{B}, v_{10}^{C}, \omega_{21}^{A}, \omega_{21}^{B}, \omega_{21}^{C}, \omega_{32}^{C}, \omega_{54}^{C}$ of the parallel manipulator.

Let us assume that the manipulator has a virtual motion determined by the velocities $v_{10c}^{Cv} = 1,\ v_{10c}^{Av} = 0,\ v_{10c}^{Bv} = 0.$ Characteristic virtual velocities, expressed as function of robot's position are given by the connectivity conditions (14). Some other compatibility relations can be obtained if other two virtual movements are considered: $v_{10a}^{Av} = 1,\ v_{10a}^{Bv} = 0, v_{10a}^{Cv} = 0$ and $v_{10b}^{Bv} = 1,\ v_{10b}^{Cv} = 0,\ v_{10b}^{Av} = 0$.

Again, the relative accelerations $\gamma_{10}^{A}, \gamma_{10}^{B}, \gamma_{10}^{C}, \varepsilon_{21}^{A},\ \varepsilon_{21}^{B}, \varepsilon_{21}^{C}, \varepsilon_{32}^{C}, \varepsilon_{54}^{C}$ of the components of this robot are given by some new conditions of connectivity, which are obtained from the time derivative of (14) relations. The following relations results:

$$
\begin{aligned}
&\gamma_{10}^{A}\ \vec{u}_i^{T}\vec{u}_3 + \varepsilon_{21}^{A} L\ \vec{u}_i^{T} a_{20}^{T}\tilde{u}_3\vec{u}_1 = \ddot{\vec{r}}_0^{P} - \omega_{21}^{A}\omega_{21}^{A} L\ \vec{u}_i^{T} a_{20}^{T}\tilde{u}_3\tilde{u}_3\vec{u}_1\,,\\
&\gamma_{10}^{B}\ \vec{u}_i^{T}\vec{u}_3 + \varepsilon_{21}^{B} L\ \vec{u}_i^{T} b_{20}^{T}\tilde{u}_3\vec{u}_1 = \ddot{\vec{r}}_0^{P} - \omega_{21}^{B}\omega_{21}^{B} L\ b_{20}^{T}\tilde{u}_3\tilde{u}_3\vec{u}_1 \quad (i = 2,\ 3),\\
&\gamma_{10}^{C}\vec{u}_j^{T}\vec{u}_3 - \varepsilon_{21}^{C}\{L\vec{u}_j^{T} c_{20}^{T}\tilde{u}_3 c_{32}^{T}\vec{u}_2 - r\vec{u}_j^{T} c_{20}^{T}\tilde{u}_3 c_{32}^{T}\vec{u}_2\} - \varepsilon_{32}^{C} L\vec{u}_j^{T} c_{30}^{T}\tilde{u}_3\vec{u}_2 +\\
&+\varepsilon_{54}^{C} L\vec{u}_j^{T} c_{50}^{T}\tilde{u}_3\vec{u}_1 = \ddot{\vec{r}}_0^{P} + \omega_{21}^{C}\omega_{21}^{C}\{L\vec{u}_j^{T} c_{20}^{T}\tilde{u}_3\tilde{u}_3 c_{32}^{T}\vec{u}_2 - r\vec{u}_j^{T} c_{20}^{T}\tilde{u}_3\tilde{u}_3 c_{52}^{T}\vec{u}_1\} +\\
&+\omega_{32}^{C}\omega_{32}^{C} L\ \vec{u}_j^{T} c_{30}^{T}\tilde{u}_3\tilde{u}_3\vec{u}_2 - \omega_{54}^{C}\omega_{54}^{C} r\ \vec{u}_j^{T} c_{50}^{T}\tilde{u}_3\tilde{u}_3\vec{u}_1 + 2\omega_{21}^{C}\omega_{32}^{C} L\ \vec{u}_j^{T} c_{20}^{T}\tilde{u}_3\vec{u}_2 -\\
&-2\omega_{21}^{C}\omega_{54}^{C} r\ \vec{u}_j^{T} c_{20}^{T}\tilde{u}_3 c_{52}^{T}\tilde{u}_3\vec{u}_1 \quad (j = 1,\ 2,\ 3),\\
&\varepsilon_{54}^{C} = \varepsilon_{21}^{C} + \ddot{\theta}\ .
\end{aligned}
\tag{15}
$$

The relations (14) and (15) represent *the inverse kinematical model* of parallel manipulator.

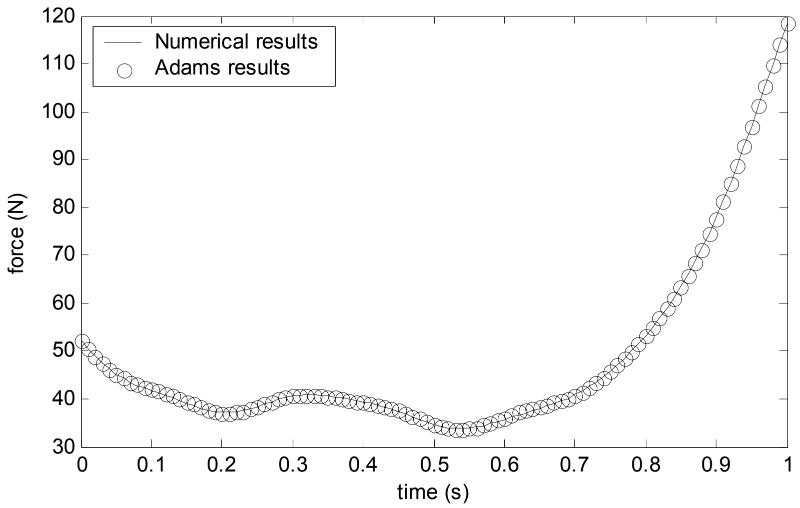

Fig. 3 – Force f_{10}^{A} of first actuator.

4. INVERSE DYNAMIC MODEL

Although the parallel manipulators with less DOF have been investigated to some extent, works on their dynamics are relatively few. Because of existence of multiple closed-loop chains, dynamic analysis of a parallel manipulator is quite complicated.

The motion of the movable platform is controlled by three independent pneumatic systems that generate three forces

$$\vec{f}_{10}^{A} = f_{10}^{A}\vec{u}_3, \ \vec{f}_{10}^{B} = f_{10}^{B}\vec{u}_3, \ \vec{f}_{10}^{C} = f_{10}^{C}\vec{u}_3, \tag{16}$$

which train the sliders on the guide ways $A_1 z_1^A$, $B_1 z_1^B$, Cz_1^C .

Let us consider that the motion of the platform is known. In these conditions, position, velocity and acceleration of each link and joint are fist determined. The force of inertia and the resultant moment of the forces of inertia of the T_k body are determined with respect to A_k joint's centre. On the other hand, the characteristic vectors $\vec{f}_k^{\,*}, \vec{m}_k^{\,*}$ evaluate the influence of the action of the weight $m_k \vec{g}$ and of other external and internal forces applied to the same element T_k. Then the forces and the moments that are acting each rigid body are definitively determined.

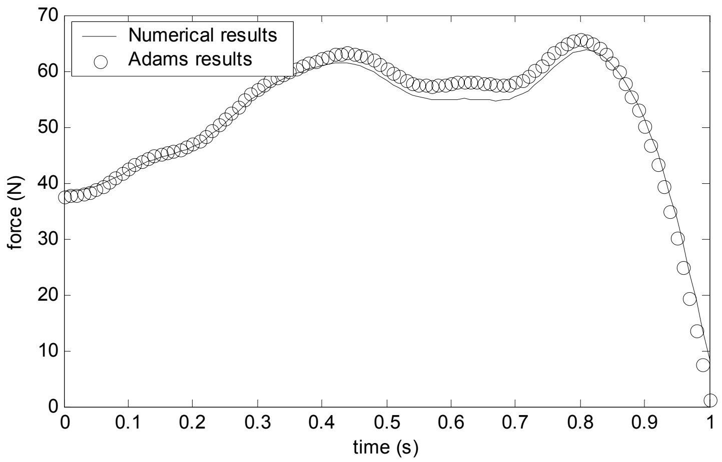

Fig. 4 – Force f_{10}^{B} of second actuator.

In the inverse dynamic problem, in the present paper, one applies the virtual powers method in order to establish some recursive matrix relations for the forces of the three actuators.

As the virtual velocities method shows, the dynamic equilibrium condition of the mechanism is that the virtual power of the external, internal and inertia forces, which is developed during a general virtual displacement, must be null. Applying *the fundamental equations of the parallel robots dynamics* established in compact form by Stefan Staicu [16, 19], the following matrix relations give the forces exerted on the legs by the sliders along their directions:

$$\begin{aligned}
f_{10}^{A} &= \vec{u}_3^{\,T}\{\vec{F}_1^{A} + \omega_{21a}^{Av}\vec{M}_2^{A} + \omega_{21a}^{Bv}\vec{M}_2^{B} + \omega_{21a}^{Cv}\vec{M}_2^{C} + \\
&\quad + \omega_{32a}^{Cv}\vec{M}_3^{C} + \omega_{43a}^{Cv}\vec{M}_4^{C} + \omega_{54a}^{Cv}\vec{M}_5^{C} + \omega_{62a}^{Cv}\vec{M}_6^{C}\}, \\
f_{10}^{B} &= \vec{u}_3^{\,T}\{\vec{F}_1^{B} + \omega_{21b}^{Av}\vec{M}_2^{A} + \omega_{21b}^{Bv}\vec{M}_2^{B} + \omega_{21b}^{Cv}\vec{M}_2^{C} + \\
&\quad + \omega_{32b}^{Cv}\vec{M}_3^{C} + \omega_{43b}^{Cv}\vec{M}_4^{C} + \omega_{54b}^{Cv}\vec{M}_5^{C} + \omega_{62b}^{Cv}\vec{M}_6^{C}\}, \\
f_{10}^{C} &= \vec{u}_3^{\,T}\{\vec{F}_1^{C} + \omega_{21c}^{Av}\vec{M}_2^{A} + \omega_{21c}^{Bv}\vec{M}_2^{B} + \omega_{21c}^{Cv}\vec{M}_2^{C} + \\
&\quad + \omega_{32c}^{Cv}\vec{M}_3^{C} + \omega_{43c}^{Cv}\vec{M}_4^{C} + \omega_{54c}^{Cv}\vec{M}_5^{C} + \omega_{62c}^{Cv}\vec{M}_6^{C}\},
\end{aligned} \tag{17}$$

where one denote

$$\begin{aligned}
\vec{F}_k^{A} &= \vec{F}_{k0}^{A} + a_{k+1,k}^{T}\vec{F}_{k+1}^{A} \\
\vec{M}_k^{A} &= \vec{M}_{k0}^{A} + a_{k+1,k}^{T}\vec{M}_{k+1}^{A} + \tilde{r}_{k+1,k}^{A}a_{k+1,k}^{T}\vec{F}_{k+1}^{A},
\end{aligned} \tag{18}$$

$$\begin{aligned}
\vec{F}_{k0}^{A} &= m_k^{A}\left[\vec{\gamma}_{k0}^{A} + \left(\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A}\right)\ \vec{r}_k^{\,CA}\right] - \vec{f}_k^{*A}, \\
\vec{M}_{k0}^{A} &= m_k^{A}\tilde{r}_k^{CA}\vec{\gamma}_{k0}^{A} + \hat{J}_k^{A}\vec{\varepsilon}_{k0}^{A} + \tilde{\omega}_{k0}^{A}\hat{J}_k^{A}\vec{\omega}_{k0}^{A} - \vec{m}_k^{*A}, \\
\vec{f}_k^{*A} &= -9.806\, m_k^{A}\, a_{k0}\, \vec{u}_3, \\
\vec{m}_k^{*A} &= -9.806\, m_k^{A}\, \tilde{r}_k^{CA}\, a_{k0}\, \vec{u}_3.
\end{aligned}$$

The relations (17) and (18) represent *the inverse dynamic model* of the 3-DOF spatial parallel manipulator.

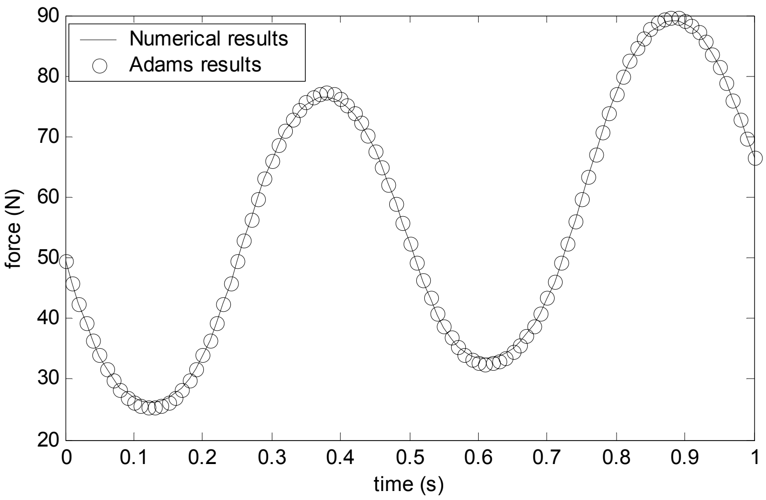

Fig. 5 – Force f_{10}^{C} of third actuator.

As applications let us consider a manipulator which has the following characteristics:

$$r = 0.1\,\text{m},\ R = 0.25\,\text{m},\ \ L = 0.4\,\text{m},$$
$$m_1 = 0.9751\,\text{kg},\ m_2 = 0.9803\,\text{kg},\ m_3 = 8.1573\ \text{kg},$$
$$\hat{J}_2 = \begin{bmatrix} 0.0006 & & \\ & 0.0524 & \\ & & 0.0524 \end{bmatrix},\ \hat{J}_3 = \begin{bmatrix} 0.0269 & & \\ & 0.0658 & \\ & & 0.0501 \end{bmatrix}.$$

Assuming that there is not external force and moment acting on the movable platform, the actuator forces f_{10}^A (Fig. 3), f_{10}^B (Fig. 4), f_{10}^C (Fig. 5) are shown by MATLAB code for one second of platform's evolution.

ADAMS is used to perform the kinematic and dynamic simulation for this manipulator. The kinematic and dynamic parameters of the legs, platform and sliders are the same as the numerical example. The revolute, universal and prismatic joints are defined to connect the rigid bodies according to geometric model. In ADAMS, the motion of a rigid body is driven by motion at a joint. Because the inverse kinematics can be presented in closed form, the motions at three prismatic joints between the sliders and the ground are defined and input according to relations (7), (8), (9). So the platform can move along the trajectory described as equations (5), (6). The simulation model in ADAMS is shown in Fig.6.

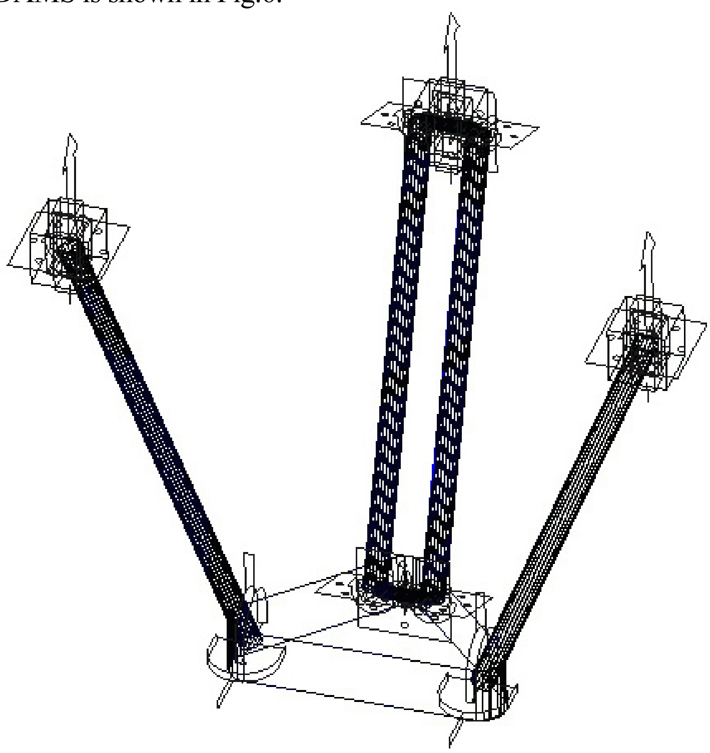

Fig. 6 – Simulation model in ADAMS.

The simulation results of the actuator forces are also shown in Figs.3, 4, 5. It can be shown that the simulation results are very close to the mathematical results. Moreover, other kinematic and dynamic analysis, such as velocity, acceleration and kinetic energy, can also be output by ADAMS. However, it is difficult for ADAMS to give the inherent dynamic performance of the mechanism and to optimise the structure of the robot. The simulation results can also hardly be used in the real-time control model.

5. CONCLUSIONS

Using the Newton-Euler classic method, which takes into account each of rigid bodies separate of the kinematical chains, a system of numerous equations of dynamic equilibrium for all moving body are written and must be solved. The solution of this system leads to all the forces and moments at the joints, including the actuator forces.

Analytical calculi involved in the Lagrange formalism are too long and they have risk of making errors. Also, the time for numerical calculus grows with the number of bodies of the device.

Within the inverse positional analysis some exact relations giving in real-time the position, velocity and acceleration of each element of a 3-DOF spatial parallel manipulator have been established in closed form in the present study.

Based on virtual work principle, the new approach described above is far more efficient and establishes a direct recursive determination of the variation in real-time of the forces of the three actuators. In a context of control, the iterative matrix relations (17) and (18), given by this simulation dynamic model, can be transformed in a model for automatic command of the spatial manipulator.

Comparison of simulation results by MATLAB program with the formulation in ADAMS procedure shows the validity of this new matrix model.

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