VIBRATION AMPLIFICATION IN OSCILLATING SYSTEMS WITH DEGRADING CHARACTERISTICS*

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The paper is an analysis of the dynamic response of an oscillating system, considering the modifications of the stiffness and damping characteristics, due to the vibration – generated damages. The study is focused on the difference in the system behavior, function of the way the system is initially situated relative to the harmonic input from the spectral point of view: above resonance, at resonance or below resonance. Experimental and simulation results are presented for simplified mechanical models of systems with stiffness degradation. As a convenient measure of the effect of duration and severity of the system vibration, the total energy dissipated through hysteresis is considered. It is shown that the most damaging situation occurs when the forcing frequency is slightly less than the system natural frequency. In this case, in a very short time interval from the start of the input action, the system displays a severe vibration output, which can lead to structural deterioration.

1. INTRODUCTION

The acceptance of plastic hinges occurrence in a building according to the seismic design standards [1–3] leads to a degradation of the structural restoring force and to an increase of the structural damping. The first effect could be beneficial if the natural vibration periods of the building are longer than those of the main spectral components of the ground motion. In this case, by structural stiffness degradation the building is "pulled" from the resonance regime resulting in a reduction of seismic response. On the other hand, if the main spectral components of the seismic ground motion are longer than the building natural periods, then the structure could be "dragged" to resonance with a significant increase of the seismic response, which can result in important building damages or even in collapse. The increase of the building structural damping capacity due to the occurrence of plastic hinges is beneficial in both cases as more of the kinetic energy injected to the building by the seismic action is consumed as the structure experiences repeated stress reversals. However, this increase of structural damping is not so important such as to reduce dramatically the vibration amplification within the resonance range [4].

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The aim of this paper is to illustrate the change in dynamic behavior of structures associated with system degradation. Simple mechanical models with harmonic inputs are used for both experimental and analytical approach of this problem.

2. ANALYTICAL MODEL

Although an multiple-storey building is envisaged, the model adopted (Fig. 1) employs only one mass, the aim being to approximate the vibration of the building in the range of its lowest mode. Only lateral motion is considered, the building being treated as a shear structure.

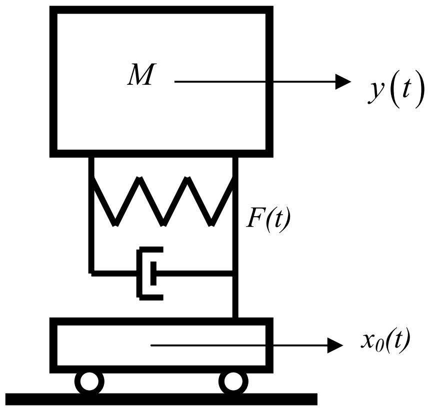

Fig. 1 – Schematic of mechanical system.

The sprung mass, M , is connected to the system base by an element of Kelvin-Voigt type, generating a hysteretic force, $F(t)$, given by

$$F(t) = kx(t) + c\dot{x}(t), \tag{1}$$

where $x(t) = y(t) - x_0(t)$ is the relative displacement between the top level and the system base. Degradation of the restoring force gradually increases as the structure experiences repeated stress reversals. The degradation due to the occurrence of plastic hinges, leads to a certain increase of energy dissipation by the internal damping mechanism. The viscous damping force term in (1) is view as an equivalent damping of the internal energy dissipation.

The parameters in any hysteretic model must become time dependent, if these de gradation effects are to be accounted for. The degradation mechanism can be modeled by allowing the parameters k and c to vary as functions of the response duration and severity [5, 6]. As a convenient measure of the combined effect of duration and severity is the total energy dissipated through hysteresis over the time interval $[0,\ t]$. Taking into account that in the considered hysteretic model the energy dissipation is of viscous type, a simplified model of degradation can be expressed by the following functional relationships for the instantaneous values of the system damping and stiffness coefficients

$$c(t)=c_0(1+a\int_0^t \dot{x}^2 \mathrm{d}t),\ k(t)=k_0(1-b\int_0^t \dot{x}^2 \mathrm{d}t), \tag{2}$$

where c_0 and k_0 are the initial values of the undamaged structure and a, b are non-negative parameters.

The variation of the degrading hysteretic loops $F(x_{\mathrm{L}}, N_{\mathrm{L}})$, for an imposed cyclic motion

$$x_{\mathrm{L}}(t)=X_{\mathrm{L}}\sin\omega_{\mathrm{L}}t, \tag{3}$$

applied over a time interval $T_{\mathrm{L}}=2\pi N_{\mathrm{L}}/\omega_{\mathrm{L}}$, where N_{L} is the number of repeated loading cycles, is given by

$$F(x_{\mathrm{L}}, N_{\mathrm{L}})=c_0\left(1+a\pi N_{\mathrm{L}}\omega_{\mathrm{L}}X_{\mathrm{L}}^2\right)\sqrt{X_{\mathrm{L}}^2-x_{\mathrm{L}}^2}++k_0\left(1-b\pi N_{\mathrm{L}}\omega_{\mathrm{L}}X_{\mathrm{L}}^2\right)x_{\mathrm{L}}. \tag{4}$$

In order to assess the values of degradation parameters a and b, one has to make assumption of the stiffness degradation and damping appreciation rates, $\delta_{\mathrm{k}}=k(T_{\mathrm{L}})/k_0$, and $\delta_{\mathrm{c}}=c(T_{\mathrm{L}})/c_0$, when the structure experiences N_{L} stress reversals produce by the cyclic loading (3):

$$a=\frac{\delta_{\mathrm{c}}-1}{\pi N_{\mathrm{L}}\omega_{\mathrm{L}}X_{\mathrm{L}}^2},\ b=\frac{1-\delta_{\mathrm{k}}}{\pi N_{\mathrm{L}}\omega_{\mathrm{L}}X_{\mathrm{L}}^2}. \tag{5}$$

For harmonic excitation, the equation of motion of mass M is

$$\ddot{x}+2\varsigma_0(1+a\int_0^t \dot{x}^2\mathrm{d}t)\sqrt{\omega_0(1-b\int_0^t \dot{x}^2\mathrm{d}t)}\,\dot{x}+\omega_0^2(1-b\int_0^t \dot{x}^2\mathrm{d}t)x=-\ddot{x}_0, \tag{6}$$

where $\varsigma_0=c_0/2\sqrt{k_0M}$, $\omega_0^2=k_0/M$ and $x_0(t)=X_0\sin\omega t$.

Introducing the dimensionless parameters

$$\begin{gathered}\tau=\omega_0 t,\ \xi(\tau)=\frac{x(\tau/\omega_0)}{X_0},\ \nu=\frac{\omega}{\omega_0},\ \alpha=\omega_0X_0^2a,\ \beta=\omega_0X_0^2b,\\ \nu_{\mathrm{L}}=\frac{\omega_L}{\omega_0},\ \mu_{\mathrm{L}}=\frac{X_{\mathrm{L}}}{X_0},\ \xi_{\mathrm{L}}(\tau)=\frac{x_{\mathrm{L}}(\tau/\omega_0)}{X_0},\ \Phi(\xi_{\mathrm{L}}, N_{\mathrm{L}})=\frac{F(x_{\mathrm{L}}, N_{\mathrm{L}})}{M\omega_0^2X_0},\end{gathered} \tag{7}$$

the dimensionless forms of equations (5) and (6) are

$$\alpha = \frac{\delta_c - 1}{\pi N_L \nu_L \mu_L^2}, \ \beta = \frac{1 - \delta_k}{\pi N_L \nu_L \mu_L^2}, \tag{9}$$

$$\xi'' + 2\varsigma_0 (1 + \alpha \int_0^\tau \xi'^2 \mathrm{d}t) \sqrt{(1 - \beta \int_0^\tau \xi'^2 \mathrm{d}t)}\, \xi' + (1 - \beta \int_0^\tau \xi'^2 \mathrm{d}t) \xi = -\sin \nu\tau . \tag{10}$$

The dimensionless form of the imposed relative cycle motion (3) is

$$\xi_L(\tau) = \mu_L \sin \nu_L \tau . \tag{11}$$

Assuming that the cyclic loading (11) is applied with the same amplitude as the system input ($\mu_L = 1$) and the same frequency as the system natural undamped frequency ($\upsilon_L = 1$), yields

$$\alpha = \frac{\delta_c - 1}{\pi N_L}, \ \beta = \frac{1 - \delta_k}{\pi N_L}. \tag{12}$$

After completion of N'_L loading cycles of this type, the system damping ratio and the natural undamped frequency become:

$$\varsigma_{N'_L} = \varsigma_0 \left[1 + (\delta_c - 1) \frac{N'_L}{N_L} \right], \ \nu_{N'_L} = 1 - (1 - \delta_k) \frac{N'_L}{N_L} \ , \tag{13}$$

and the resulting hysteretic loop is approximated by

$$\begin{gathered} \Phi(\xi_L, N'_L) = 2\zeta_{N'_L} \nu_{N'_L} \sqrt{1 - \xi_L^2} + \nu_{N'_L}^2 \xi_L , \\ \xi_L(\tau) = \sin \tau. \end{gathered} \tag{14}$$

3. NUMERICAL METHOD

For numerical solving of second order differential equations, portaying the motion of oscillating systems, an efficient method was proposed by Newmark [7]. In order to apply this method, equation (8) has to be written under the form

$$\xi'' = f(\xi', \xi, \tau) - \sin \nu\tau . \tag{13}$$

The Newmark discrete time method in five steps was applied [8], in order to obtain the approximate solution of equation (13) with the initial conditions:

$$\xi(0)=\xi_0 \quad ; \quad \xi'(0)=\xi'_0 . \tag{14}$$

At initial time $\tau_0 = 0$, the initial value of the acceleration $\xi''(0)=\xi''_0$ is evaluated from

$$\xi''_0 = f(\xi_0,\xi'_0,0) . \tag{15}$$

The principle of the method consists in the approximation of the discrete values $\xi_{n+1},\xi'_{n+1},\xi''_{n+1}$ by using the values obtained at moment $\tau_n = n\Delta\tau$. The steps of the method are:

1. Initialization of ξ''_{n+1} with an arbitrary value $\xi''_{n+1,\,i}$.
2. Evaluation of ξ'_{n+1} from:

$$\xi'_{n+1} = \xi'_n + (\xi''_n + \xi''_{n+1,i})\cdot\frac{\Delta\tau}{2} . \tag{16}$$

3. Approximation of ξ_{n+1} by:

$$\xi_{n+1} = \xi_n + \xi'_n\Delta t + (\xi''_n + \xi''_{n+1,i})\cdot\frac{(\Delta\tau)^2}{4} . \tag{17}$$

4. Introducing ξ_{n+1} and ξ'_{n+1} in (13), yields:

$$\xi''_{n+1,c} = f(\xi_{n+1},\xi'_{n+1},\tau_{n+1}) - \sin\nu\tau_{n+1} . \tag{18}$$

5. The values $\xi''_{n+1,\,c}$ and $\xi''_{n+1,\,i}$ are compared. If the difference is not sufficiently small, $\xi''_{n+1,\,i}$ is replaced by $\xi''_{n+1,\,c}$ and the algorithm is repeated from the step 2. Otherwise, a new iteration is initiated. Usually, for the initialization of the unknown acceleration value the preceding accepted value is used.

4. NUMERICAL RESULTS

To illustrate the dynamic behavior of the considered degrading system, suppose that after completion of $N_{\mathrm{L}} = 45$ loading cycles, given by (11) for $\mu_{\mathrm{L}} = \upsilon_{\mathrm{L}} = 1$, the stiffness coefficient decreases two times ($\delta_{\mathrm{k}} = 0.5$) while the damping ratio increases 3 times ($\delta_{\mathrm{c}} = 3$). For this case study, applying of relation (12) yields $\alpha = 0.014,\ \beta = 0.0035$.

The evolution of the degrading hysteretic loops versus the number of loading cycles is shown in Fig. 2, for the initial damping ratio $\varsigma_0 = 0.05$.

The decrease of hysteretic loop slope indicates the stiffness degradation while the increase of their inner surface area show the appreciation of the damping capacity.

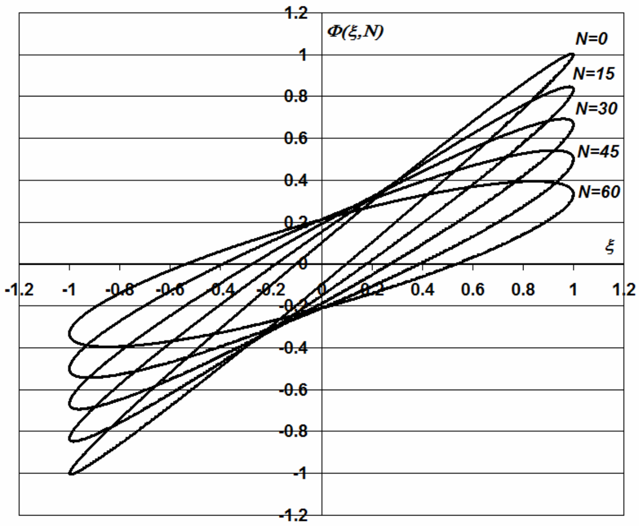

Fig. 2 – Evolution of degrading hysteretic loops versus the number of cyclic loading.

In order to study the dynamic behavior of the same degrading structure, excited by harmonic inputs with frequencies within the resonant range, the equation of motion (10) was numerically solved for:

$$\nu = 0.6, 0.75, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3 \text{ and } 1.4.$$

The time histories of elative displacement $\xi(\tau)$, obtained for each of these inputs, are shown in Figs. 3–11.

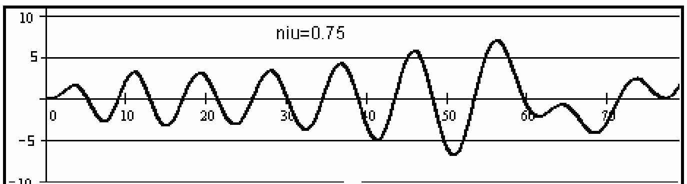

Fig. 4

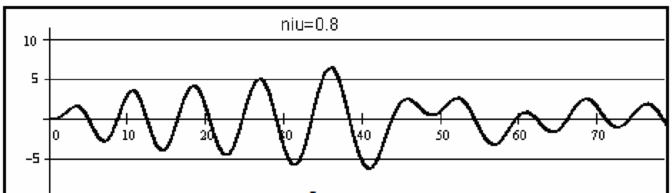

Fig. 5

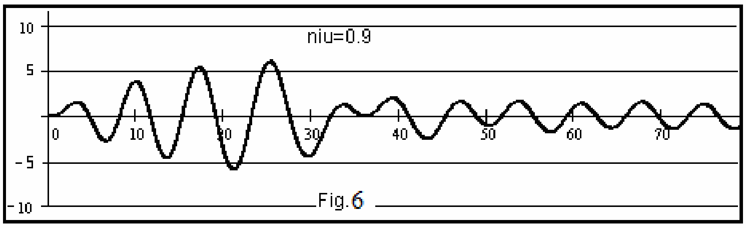

Fig. 6

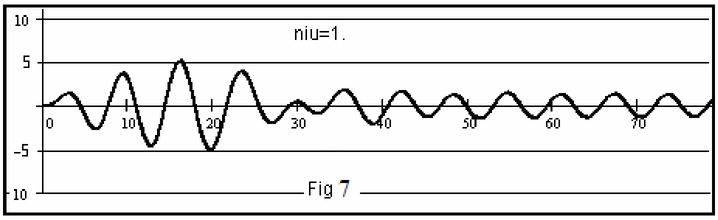

Fig 7

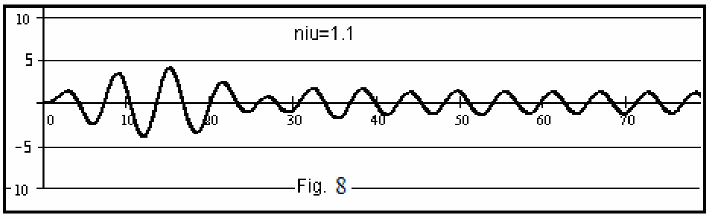

Fig. 8

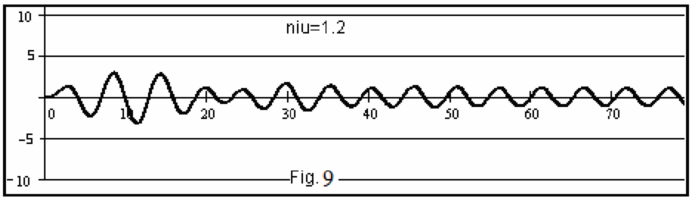

Fig. 9

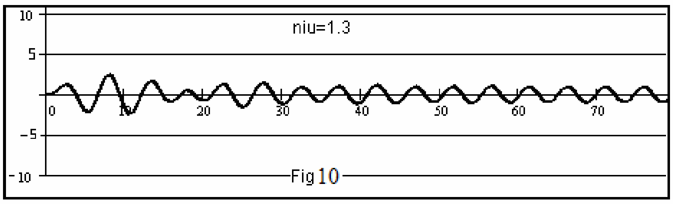

Fig 10

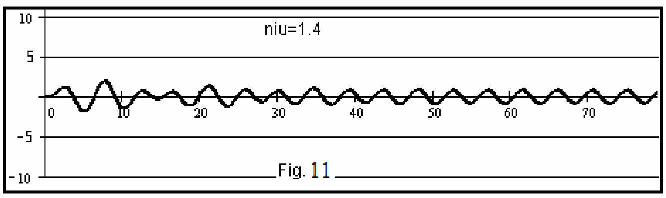

Fig. 11

The above results show that the structural degradation can lead, only after a few cycles, to an important output magnification (despite of the associated increase of system damping capacity), if the input frequency is at most 25% lower than the initial natural frequency of the oscillating system. On the other hand, the effect of structural degradation could be benefic if the forcing frequency is equal or at most 25% higher than the resonant frequency of the undamaged system. Outside this frequency range, the system degradation is not so important, due to the system filtering properties.

5. EXPERIMENTAL ANALYSIS

To illustrate the change of dynamic behavior of a structure, due to system degradation, an experiment was conducted on a simple mechanical structure. A cantilever beam from composite material with a concentrated mass on top was mounted vertically on an electro-dynamic shaker. From practical point of view, this experimental study is relevant only when the beam vibrates in the neighborhood of the first resonance vibration mode because in this case the dynamic structural output is strongly amplified by favored transfer of the kinetic energy from the base imposed motion to the structure and damaging shear forces can develop. The structural degradation was produced by gradually delaminating of the composite material, resulting in a decrease of bending stiffness and a corresponding increase of internal damping. The frequency of the first bending vibration mode, denoted by f, and the associated modal damping ratio (ς) were obtained from the free vibration records.

In Figs. 12–14 are presented the experimental results, showing the effect of structural degradation on the amplification factor $A_{\text{r.m.s}}$ (defined as the ratio of the r.m.s values of the top and base acceleration). As one can observe, the value of the amplification factor became 2.3 times greater as the bending stiffness degradation was gradually increased. This important increase of $A_{\text{r.m.s}}$ was obtained despite the fact that the damping ratio increased with 60%. During this test, the r.m.s value of the input acceleration and its fundamental frequency were maintained practically constant ($a_{\text{r.m.s.}} = 0.064g$, $f_{\text{in}} = 2.78\text{Hz}$).

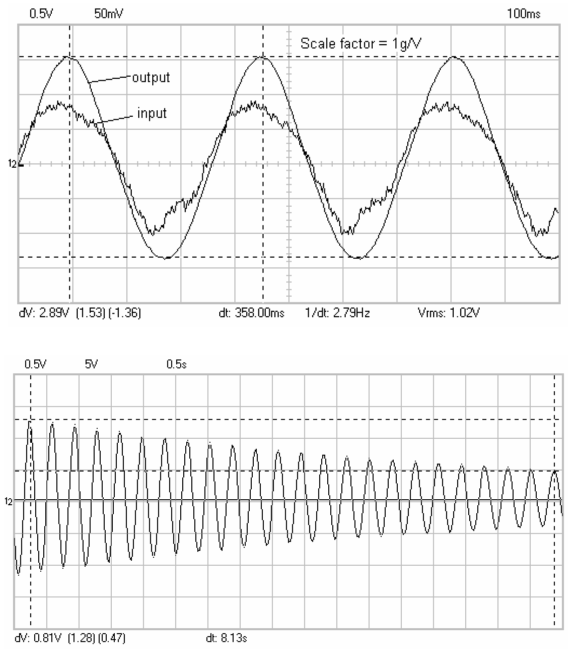

Fig. 12 – Forced (f_{in} = 2.78Hz, A_{rms}= 16.2) and free vibration of the undamaged beam (f = 2.83Hz, ζ = 0.0069).

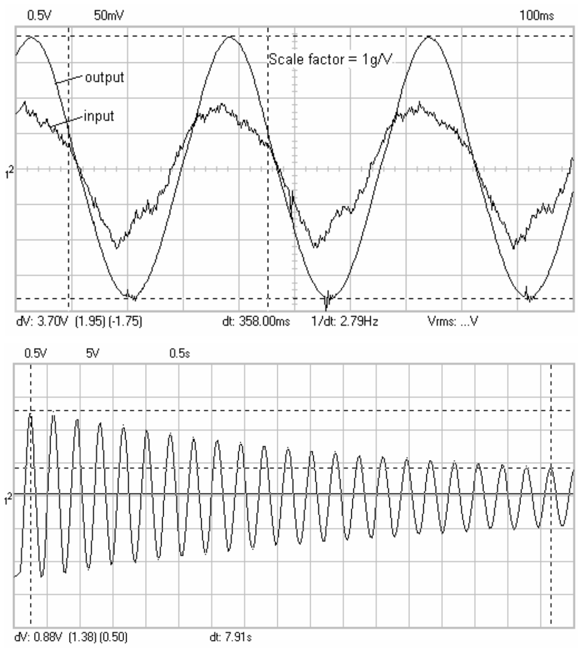

Fig.13 – Forced (f_{in} = 2.78Hz, A_{rms}= 20.1) and free vibration of the slightly damaged beam (f = 2.79Hz, ζ = 0.0073).

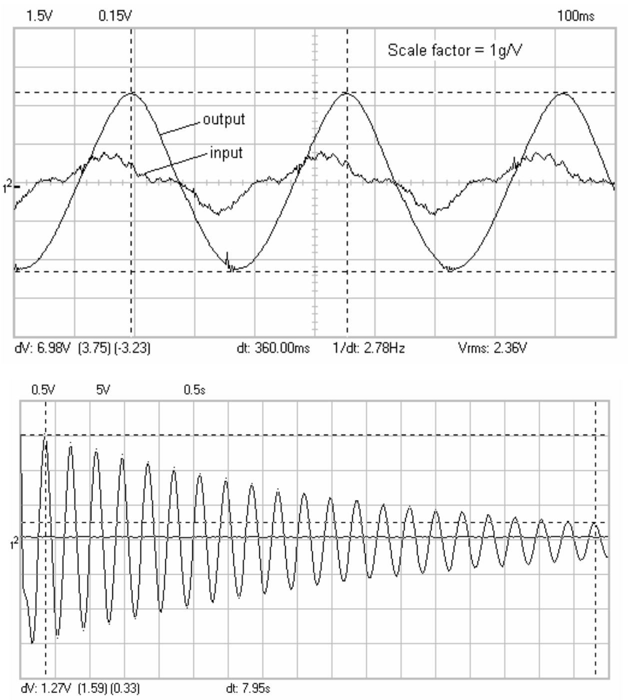

Fig.14 – Forced (f_{in} = 2.78Hz, A_{rms}= 36.7) and free vibration of the moderately damaged beam (f = 2.64Hz, ζ = 0.012).

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