NON-LINEAR ATTENUATION EFFECTS ON SOILS DYNAMIC RESPONSE EVALUATION*

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The object of this paper is to estimate the structural attenuation in geological site materials with the aid of the non-linear magnification functions maximum values decreasing. Using a non-linear one-degree-of-freedom model (NKV model) with material functions in terms of the strain levels, by numerical non-linear simulation one put into evidence the attenuation capacity of the site geological materials neglected by linear calculus.

1. INTRODUCTION

The strong dependence of the soils dynamic properties on strain or stress level produced by external loads is very well known. In the previous author's papers [1], [2], [3] this nonlinear behaviour was modeled assuming that the geological materials are nonlinear viscoelastic materials. This model describes the nonlinearity by the dependence of the material mechanical parameters: shear modulus G and damping ratio D in terms of shear strain invariant γ: $G = G(\gamma)$, $D = D(\gamma)$, or in terms of displacements x: $G = G(x), D = D(x)$ or twisting angle θ: $G = G(\theta)$, $D = D(\theta)$.

It is experimental observed that when the external loads are increasing the rigidity is reduced, due to the dynamic degradation effect [6], and the material damping increases. Thereby, $G = G(\theta)$ is an increasing function and $D = D(\theta)$ is decreasing function. These contradictory material evolutions have contradictory effects on dynamic structural response, which can be, amplify or diminish with respect to loading inputs.

Due to these two dynamic functions – one for material strength modeling and the other including material damping – this model can be regarded as an extension in the non-linear domain for the non-viscous linear Kelvin-Voigt model [10], and for this reason, in the next, we will use as denomination – *the nonlinear Kelvin-Voigt* model (*NKV model*). Using this NKV model, in a previous papers [4], [5]

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was presented some aspects concerning non-linear response of the site materials and differences between non-linear and linear evaluations.

In this paper, some numerical simulations using the NKV model will perform in order to evaluate, qualitative and quantitative, the ratio between dynamic imput and dynamic response for some site materials as cohesive soils (clay, marl), cohesionless soils (sand, gravel) and rocks (limestone and gritstone).

As can see in the next, the non-linear calculus proves that the dynamic response is decreasing while imput level is increasing, thus all the geological site materials have obvious attenuation capability which can be smaller or larger in terms of their strength and damping properties.

2. SOME NON-LINEAR MATERIAL FUNCTIONS

In order to exemplify the non-linear behaviour of the usual site materials, in Figs. 2.1, 2.2 and 2.3 some non-linear material functions obtained from resonant column test performed upon clay, marl, sand, gravel, limestone and gritstone sample are given.

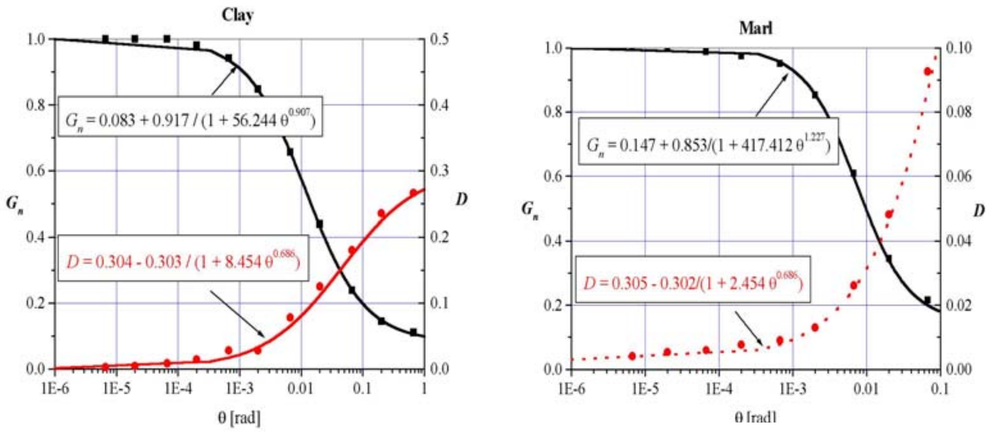
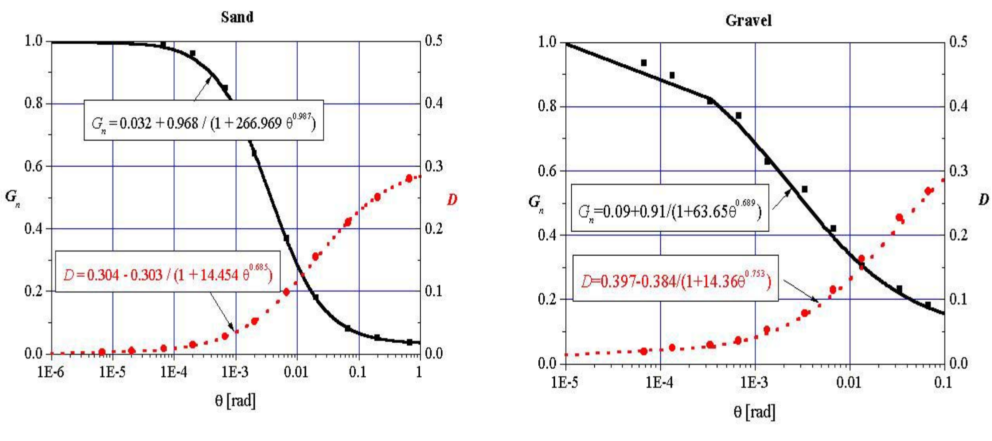


Fig. 2.1 – Dynamic functions for cohesive soils.



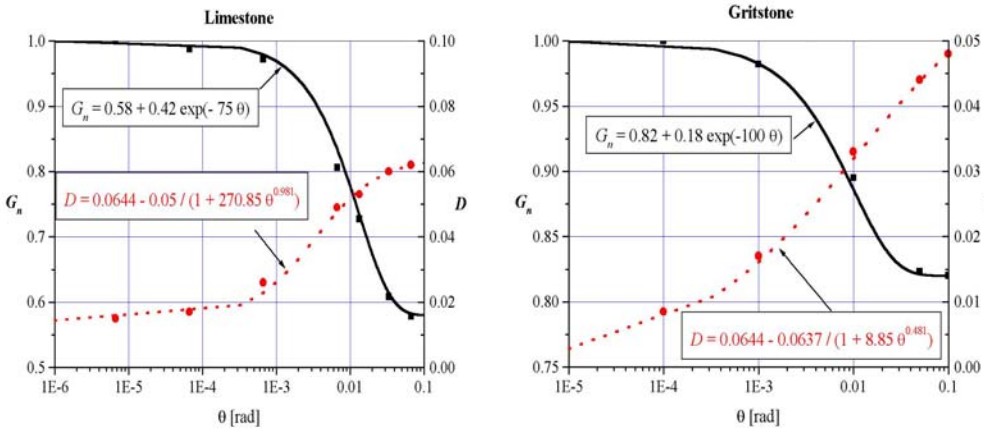


Fig. 2.3 – Dynamic functions for rocks.

3. NON-LINEAR KELVIN-VOIGT MODEL

In linear dynamics, a usual description of a solid single-degree-of-freedom behaviour is given by the Kelvin-Voigt model consisting of a spring (with a stiffness *k*) and a dashpot (with a viscosity *c*) connected in parallel. The governing equation of this system for torsional harmonic vibrations (usually resonant column system excitation) is:

$$J_0\ddot{\theta} + c \cdot \dot{\theta} + k \cdot \theta = M_0 \cdot \sin \omega t\,, \tag{3.1}$$

where θ is the system displacement (rotation, in this case), J_0 is the moment of inertia of the vibrator, M_0 are the excitation amplitude and ω the pulsation of the harmonic external impute.

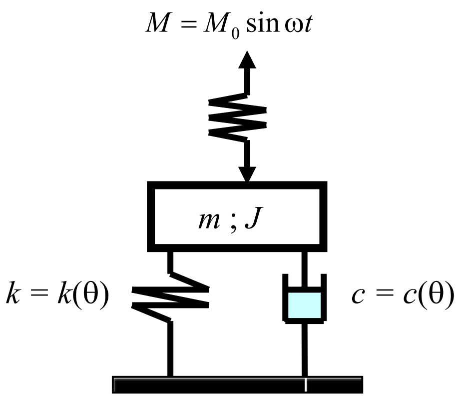


Fig. 3.1 – NKV model.

Using the same method that describes the nonlinearity by strain dependence of the material parameters, we assume that the damper viscosity c and the spring stiffness k are functions in terms of rotation θ:

$$c(\theta)=2J_0\omega_0\cdot D(\theta)=2J_0\omega_0 D_0 D_n(\theta)=c_0\cdot D_n(\theta),$$
$$k(\theta)=\frac{I_p}{h}\cdot G(\theta)=\frac{I_p}{h}G_0 G_n(\theta)=k_0\cdot G_n(\theta), \tag{3.2}$$

where ω_0 is the system undamped natural pulsation, $I_p = \pi\phi^4/32$ is the polar moment of the specimen, (ϕ, h) are the diameter and height of the cylindrical specimen and $D_n(\theta)$, $G_n(\theta)$ are the normalized forms of the damping function $D(\theta)$ and modulus function $G(\theta)$ in terms of theirs initial values D_0 and G_0.

Thus, the most expected form of the governing equation for the non-linear behaviour of this single-degree-of-freedom system is:

$$J_0\ddot{\theta}+c(\theta)\cdot\dot{\theta}+k(\theta)\cdot\theta=M_0\cdot\sin\omega t, \tag{3.3}$$

with the analogue model from Fig. 3.1.

By using the change of variable $\tau = \omega_0 t$ and by introducing a new "time" function $\varphi(\tau) = \theta(t) = \theta(\tau/\omega_0)$ [4], one obtains from eq. (3.3) a dimensionless form of the non-linear equation of motion:

$$\varphi''+C(\varphi)\cdot\varphi'+K(\varphi)\cdot\varphi=\mu\cdot\sin\upsilon\tau, \tag{3.4}$$

where the superscript accent denotes the time derivative with respect to τ, and:

$$C(\varphi)=C(\theta)=\frac{c(\theta)}{J_0\omega_0}=2D(\theta);\quad K(\varphi)=K(\theta)=\frac{k(\theta)}{J_0\omega_0^2}=\frac{k(\theta)}{k(0)}=\frac{G(\theta)}{G(0)}=G_n(\theta);$$
$$\mu=\frac{M_0}{J_0\omega_0^2}=\frac{M_0}{k(0)}=\theta_{st};\quad \upsilon=\frac{\omega}{\omega_0}. \tag{3.5}$$

For a given normalized amplitude μ and relative pulsation υ, the non-linear equation (3.4) can be numerically solved using a computer program based on the Newmark algorithm [11, 12] and a solution of the form $\varphi = \varphi(\tau;\mu,\upsilon)$ can be obtained in the form:

$$\varphi(\tau;\upsilon,\mu)=\mu\Phi(\upsilon;\mu)\sin(\upsilon\tau-\psi), \tag{3.6}$$

where $\Phi(\upsilon;\mu)$ is the non-linear magnification function.

4. NON-LINEAR MAGNIFICATION FUNCTION

From known harmonic excitations with $\mu=\text{ct.}$, the non-linear magnification functions $\Phi(\upsilon,\mu)$ can be obtained in terms of normalized pulsation υ only:

$$\Phi(\upsilon,\mu) = \Phi(\upsilon)\Big|_{\mu=ct.} \quad (4.1)$$

The non-linear magnification functions thus obtained can be used to evaluate the enlargement or reduction of the dynamic response. As usually, the magnification functions give a measure for "the magnification" of the dynamic response with respect to the static input. But, the same function can suggest a qualitative image for magnification or attenuation of the dynamic response provoked by a dynamic input.

In this paper, the magnification functions have been used to compare the magnification or attenuation of the dynamic response obtained by non-linear calculus with respect to the dynamic linear response. As one can see in the next, this comparison make apparently the non-linear softening effects due to the non-linear mechanical characteristics of the materials from site soil deposits.

These effects will be exposed with the aid of the numerical simulation results using the NKV model based by the single-degree-of freedom resonant column system. The excitation used in this simulation process is of harmonic type with the normalized amplitude values μ corresponding to the μ values give by peak ground acceleration observed during some earthquakes [7].

The simulation results are given in Figs. 4.1, 4.2 and 4.3. As one can see from this figures, the peak amplitude of the non-linear magnification functions depends on the excitation amplitude μ and the resonance peaks occurs at different normalized pulsation υ situated before the excitation pulsation (usually soils have a softening nonlinearity type). Also, in these figures the dependence of the peak amplitude $\Phi_{\max}$ in terms of normalized pulsation υ: $\Phi_{\max} = \Phi_{\max}(\upsilon)$ is given under "resonant curves" denomination.

One can remark from these results that all the geological site materials have obvious attenuation capability, which can be smaller or larger function of their strength and damping properties and in terms of loading dependence of these mechanical characteristics.

In order to compare the attenuation capability between the site materials which has been investigated (rocks, cohesionless soils and cohesive soils) and their loading dependence, the resonant curves can be converted in the form $\Phi_{\max} = \Phi_{\max}(\mu)$ and then in the normalized form with respect to their linear values.. Such resonant curves in terms of normalized amplitude μ are shown together in Fig. 4.4. As expected, from this comparison results that the rock materials have reduced attenuation properties followed by cohesionless soils and cohesive soils with more attenuation behaviour.

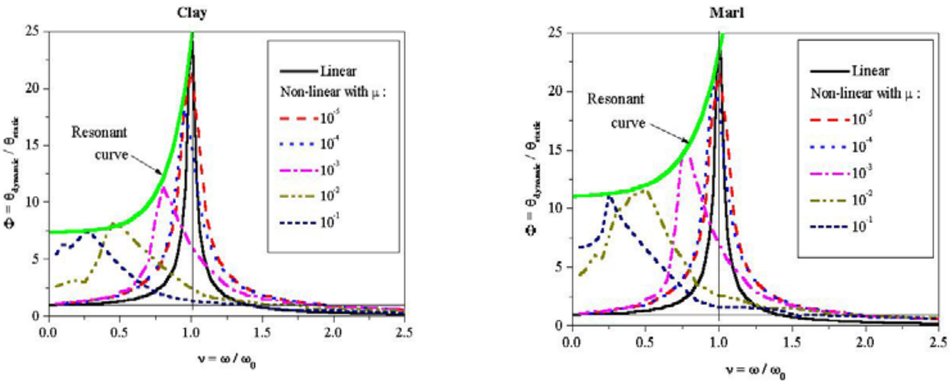


Fig. 4.1 – Magnification functions for cohesive soils.

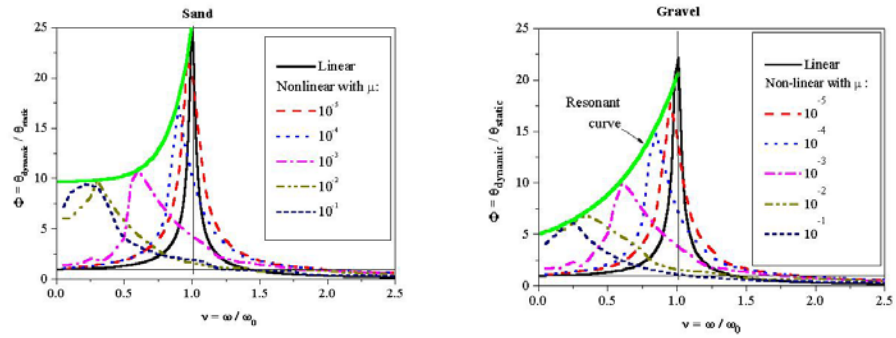


Fig. 4.2 – Magnification functions for cohesionless soils.

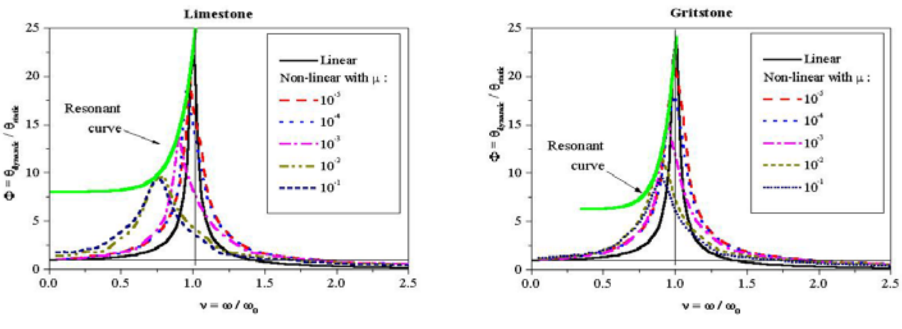


Fig. 4.3 – Magnification functions for rocks.

CONCLUDING REMARKS

The results illustrated in the above figures (Figs. 4.1, 4.2, 4.3 and 4.4) allow us to have the following remarks:

- The non-linear magnification functions are proper tools for the qualitative description of the attenuation capacity of the geological materials.
- The mechanical response of all site geological materials (soils and rocks) is amplified with respect to the same static loading but diminished with respect to the dynamic impute.
- The non-linear calculus proves that the dynamic response is decreasing while loading level is increasing.
- The resonance amplitudes of the non-linear magnification functions are inferior to the maximum amplitude of the corresponding linear magnification function, thus non-linear calculus put into evidence the attenuation capacity neglected by the linear calculus.
- Whereas the linear calculus leads to the unique resonance value, the non-linear calculus leads to the multiple resonance values in terms of excitation amplitudes.
- The resonance amplitude peaks of the soils and rocks are displaced towards low pulsations, which is a typical behaviour of the materials with softening stiffness.
- By comparison with rock materials, the soils have small initial mechanical strength and undergo more degradation damages, but the structural dynamic attenuation is more obvious due to their internal damping capabilities (see Fig. 4.4).

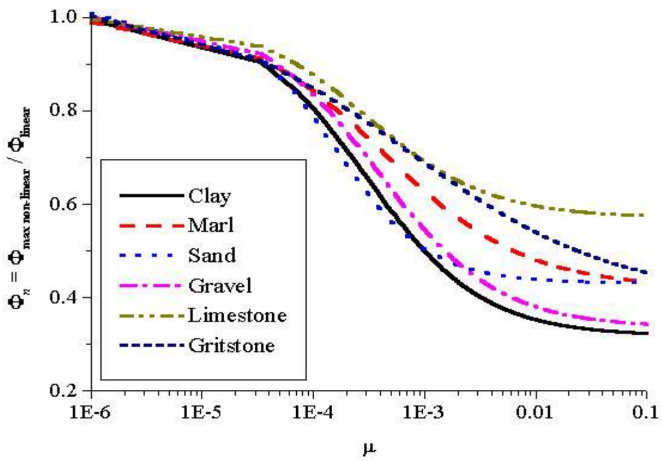


Fig. 4.4 – Resonant curves in normalized form.

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