ON THE MODELING OF THE HUMAN KNEE JOINT*

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This paper describes the three-dimensional dynamic response of the tibio-femoral joint when subjected to sudden external pulsing loads utilizing an anatomical dynamic knee model. The model consists of two body segments in contact (the femur and the tibia) executing a general three-dimensional dynamic motion within the constraints of the ligamentous structures. Each of the articular surfaces at the tibio-femoral joint is represented by a separate mathematical function. The joint ligaments are modeled as nonlinear elastic springs. The six-degree-of-freedom joint motions are characterized using six kinematic parameters and ligamenous forces are expressed in terms of these six parameters.

1. INTRODUCTION

Most of the dynamic anatomical models of the knee available in the literature are two-dimensional, considering only motions in the sagittal plane. These models are described by Moeinzadeh *et al*. [3, 4, 5], [6, 7], Engin and Moeinzadeh [8], Wongchaisuwat *et al.* [9], Tumer *et al.* [10, 11], Abdel-Raham and Hefzy [12, 13, 14] and Ling *et al*. [15].

In summary, since Wongchaisuwat *et al.*'s model [9] is more a control strategy (the authors considered the tibia as a pendulum that swings about the femur) which causes and maintains contact between the femur and the tibia, is not considered a real mathematical model that predicts knee response under dynamic loading. Most of the remaining dynamic models can be perceived as different versions of a single dynamic model. Such a model is comprised of two rigid bodies: a fixed femur and a moving tibia connected by ligamentous elements and having contact at a single point. In this paper, the three-dimensional version of this dynamic model proposed by Hefzy M.S. and Abdel-Raham E.M. [2] is presented.

2. KINEMATIC ANALYSIS

The femur and tibia are modeled as two rigid bodies. Cartilage deformation is assumed relatively small compared to joint motions and not to affect relative motions and forces within the tibio-femoral joint. Furthermore, friction forces will

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be neglected because of the extremely low coefficients of friction of the articular surfaces. Hence, in this model, the resistance to motion is essentially due to the ligamentous structures and the contact forces. Nonlinear spring elements are used to simulate the ligamentous structures whose functional ranges are determined by finding how their lengths change during motion. The menisci were not taken into consideration in the present model.

Six quantities are used to fully describe the relative motions between moving and fixed rigid bodies: three rotations and three translations. These rotations and translations are the components of the rotation and translation vectors, respectively. The three rotation components describe the orientation of the moving system of axes (attached to the moving rigid body) with respect to the fixed system of axes (attached to the fixed rigid body). The three translation components describe the location of the origin of the moving system of axes with respect to the fixed one.

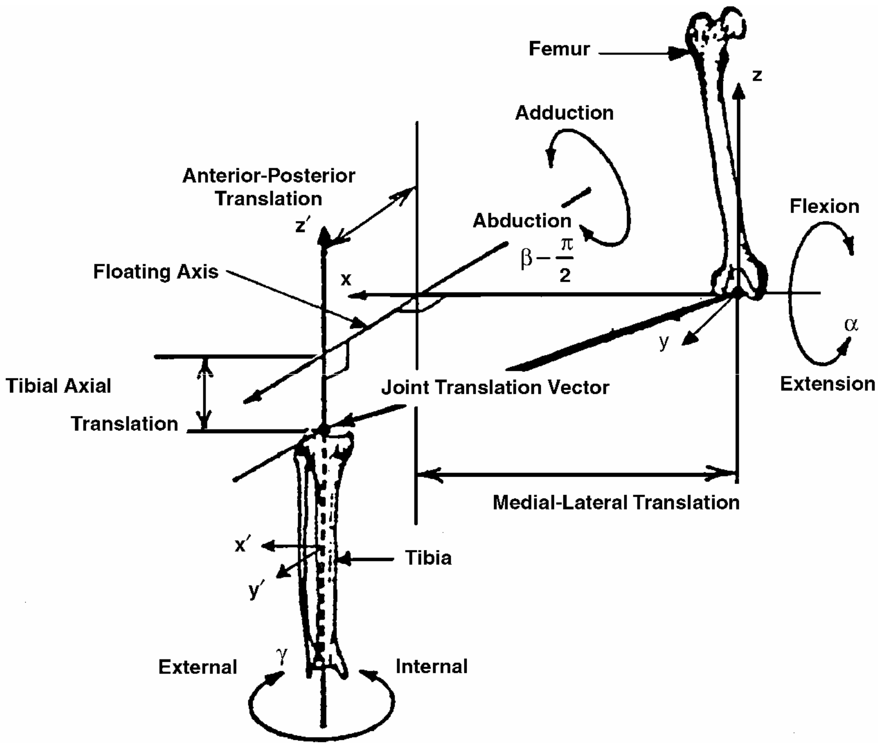

Fig. 1 – Tibio-femoral joint coordinate system.

The tibio-femoral joint coordinate system introduced by Grood and Suntay [16] was used to define the rotation and translation vectors that describe the three-dimensional tibio-femoral motions. This joint coordinate system is shown in Fig. 1 and consists of an axis (x-axis) that is fixed on the femur ($\vec{i}$ is a unit vector parallel to the x-axis), an axis (z'-axis) that is fixed on the tibia ($\vec{k'}$ parallel to the z'-axis), and a floating axis perpendicular to these two fixed axes ($\vec{e_2}$ is a unit vector parallel to the floating axis). The three components of the rotation vector include flexion-extension, tibial internal-external, and varus-valgus rotations. Flexion-extension rotations, α, occur around the femoral fixed axis; internal-external tibial rotations, γ, occur about the tibial fixed axis; and varus-valgus rotations, β, (ad-abduction) occur about the floating axis. The rotation vector $\vec{\theta}$ is written as

$$\vec{\theta} = -\alpha\,\vec{i} - \beta\,\vec{e_2} - \gamma\,\vec{k'}\,, \tag{2.1}$$

The six parameters (three rotations and three translations) describing tibio-femoral motions are used to determine the transformation between the two coordinate systems

$$\vec{R} = \vec{R}_0 + \left[R\right]\vec{r}\,, \tag{2.2}$$

where $\vec{r}$ describes the position vector of a point with respect to the tibial coordinate system, and $\vec{R}$ describes the position vector of the same point with respect to the femoral coordinate system. The vector $\vec{R_0}$ is the position vector which locates the origin of the tibial coordinate system with respect to the femoral coordinate system, and $\left[R\right]$ is a (3×3) rotation matrix given by Grood and Suntay [16] as

$$\left[R\right] = \begin{bmatrix} \sin\beta\cos\gamma & \sin\beta\cos\gamma & \cos\beta \\ -\cos\alpha\sin\gamma - \sin\alpha\cos\beta\cos\gamma & \cos\alpha\cos\gamma - \sin\alpha\cos\beta\sin\gamma & \sin\alpha\sin\beta \\ \sin\alpha\sin\gamma - \cos\alpha\cos\beta\cos\gamma & -\sin\alpha\cos\gamma - \cos\alpha\cos\beta\sin\gamma & \cos\alpha\sin\beta \end{bmatrix}, \tag{2.3}$$

where α is the knee flexion angle, γ is the tibial external rotation angle, and β is ($\pi/2 \pm$ abduction); positive sign indicates a right knee and negative sign indicates a left knee.

3. CONTACT AND GEOMETRIC COMPATIBILITY CONDITIONS

In this model geometric primitives are employed. The coordinates of a sufficient number of points on the femoral condyles and tibial plateaus of several cadaveric knee specimens were obtained from related studies [17, 18]. A separate mathematical function was determined as an approximate representation for each of the medial femoral condyle, the lateral femoral condyle, the medial tibial plateau, and the lateral tibial plateau. The femoral articular surfaces are approximated as parts of spheres, while the tibial plateaus are considered as planar surfaces. The equation of the medial and lateral femoral spheres expressed in the femoral coordinate system of axes is written as

$$f\left(x,y\right)=-\sqrt{r^2-\left(x-h\right)^2-\left(y-k\right)^2}+1\,, \tag{3.1}$$

where values of parameters (*r, h, k*, and *l*) are obtained as 21, 23.75, 18.0, 12.0 mm and 20.0, 23.0, 16.0, 11.5 mm for the medial and lateral spheres, respectively. The equation of the medial and lateral tibial planes expressed in the tibial coordinate system of axes is written as

$$g\left(x^{'},y^{'}\right)=my^{'}=c\,, \tag{3.2}$$

where values of parameters (*m, c*) are obtained as 0.358, 213 mm and –0.341, 212.9 mm for the medial and lateral planes, respectively.

This model accomodates two situations: a two-point contact and a single-point contact. Initially, a two-point contact situation is assumed with the femur and tibia in contact on both medial and lateral sides. In the calculations, if one contact force becomes negative, then the two bones within its compartment are assumed to be separated, and the single-point contact situation is introduced, thus maintaining contact in the other compartment. The contact condition requires that the position vectors of each contact point in the femoral and the tibial coordinate system, $\vec{R}_c$ and $\vec{r}_c$, respectively, satisfy eq.(2.2) as follows

$$\vec{R}_c=R_o+\left[R\right]\vec{r}_c\,, \tag{3.3}$$

where

$$\vec{R}_c=x_c\,\vec{i}+y_c\,\vec{j}+z_c\,\vec{k}\,, \tag{3.4a}$$

$$\vec{r}_c=x_c^{'}\,\vec{i}^{'}+y_c^{'}\,\vec{j}^{'}+z_c^{'}\,\vec{k}^{'}\,, \tag{3.4b}$$

where x_c, y_c, z_c and $x_c^{'}, y_c^{'}, z_c^{'}$ are the coordinates of the contact points in the femoral and tibial systems, respectively. Since contact occurs at points identifiable in both the femoral and tibial articulating surfaces, we can write at each contact point

$$z_c = f\left(x_c, y_c\right),\ z_c^{'} = g\left(x_c^{'}, y_c^{'}\right), \tag{3.5}$$

there $f\left(x_c, y_c\right)$ and $g\left(x_c^{'}, y_c^{'}\right)$ are given by eqs.(3.1) and (3.2), respectively. Eq. (3.3) can thus be rewritten as three scalar equations

$$x_c = x_0 + R_{11}x_c^{'} + R_{12}y_c^{'} + R_{13}g\left(x_c^{'}, y_c^{'}\right), \tag{3.6a}$$

$$x_c = x_0 + R_{11}x_c^{'} + R_{12}y_c^{'} + R_{13}g\left(x_c^{'}, y_c^{'}\right), \tag{3.6b}$$

$$f\left(x_c, y_c\right) = z_0 + R_{31}x_c^{'} + R_{32}y_c^{'} + R_{33}g\left(x_c^{'}, y_c^{'}\right), \tag{3.6c}$$

where R_{ij} is the ij-th component of the rotational transformation matrix $\left(R\right)$. Eqs. (3.6a) through (3.6c) constitute a mathematical definition for a contact point. Satisfying these equations at some given point will ensure that it is a contact point. Thus, in the two-point contact version of the model, eqs. (3.6a) through (3.6c) generate six scalar equations which represent the contact conditions. In the one-point contact version of the model, eqs. (3.6a) through (3.6c) produce three scalar equations which represent the contact conditions.

Also, the normals to the femoral and tibial surfaces at each contact point are always colinear, and their cross product must vanish. The position vector of the contact point in the femoral coordinate system (eq. 3.4a) is differentiated with respect to the local (x and y) coordinates to obtain two tangent vectors along these local directions. Cross product of these two tangent vectors is then employed to determine the unit vector normal to the femoral surface, $\vec{n}_f$, at the contact point. Using eq.(3.5) we have

$$\vec{n}_f = \frac{\left.\frac{\partial f}{\partial x}\right|_{\left(x_c, y_c\right)} \vec{i} + \left.\frac{\partial f}{\partial y}\right|_{\left(x_c, y_c\right)} \vec{j} - \vec{k}}{\sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}}, \qquad \left(x, y\right) = \left(x_c, y_c\right). \tag{3.7}$$

A similar analysis is performed to obtain the unit vector normal to the tibial surface, $\vec{n}_t^{'}$.

$$\vec{n}_t^{'} = \frac{\left.\frac{\partial g}{\partial x^{'}}\right|_{\left(x_c^{'}, y_c^{'}\right)} \vec{i}^{'} + \left.\frac{\partial g}{\partial y^{'}}\right|_{\left(x_c^{'}, y_c^{'}\right)} \vec{j}^{'} - \vec{k}^{'}}{\sqrt{1 + \left(\frac{\partial g}{\partial x^{'}}\right)^2 + \left(\frac{\partial g}{\partial y^{'}}\right)^2}}, \qquad \left(x^{'}, y^{'}\right) = \left(x_c^{'}, y_c^{'}\right). \tag{3.8}$$

Applying the rotational transformation matrix to eq. (3.8) yields the unit normal vector to the tibial surface, $\vec{n}_t$,expressed in the femoral coordinate system as

$$\vec{n}_t = \left(R_{11}n'_{tx'} + R_{12}n'_{ty'} - R_{13}n'_{tz'}\right)\vec{i} + \left(R_{21}n'_{tx'} + R_{22}n'_{ty'} - R_{23}n'_{tz'}\right)\vec{j} + \left(R_{31}n'_{tx'} + R_{32}n'_{ty'} - R_{33}n'_{tz'}\right)\vec{k} \quad (3.9)$$

.

Since the unit vectors normal to the surfaces of the femur and tibia are colinear, they are equal $\vec{n}_f = \vec{n}_t$. The scalar form of this vectorial equation represents the geometric compatibility conditions at each contact point.

4. LIGAMENTOUS FORCES

In this analysis, external loads are applied, and ligamentous and contact forces are then determined. The model includes 12 nonlinear spring elements that represent the different ligamentous structures and the capsular tissue posterior to the knee joint. These elements were assumed to carry load only when they are in tension, when their length is larger than their slack, unstrained length, L_0. Ligaments exhibit a region of nonlinear force-elongation relationship, the "toe" region, in the initial stage of ligament strain, then a linear force-elongation relationship in later stages [19]. A two-piece force-elongation relationship was thus used to evaluate the magnitudes of the ligamentous forces [20–24]. The magnitude of the force in the *j*th ligamentous element is thus expressed as

$$F_j = \begin{cases} 0; & \varepsilon_j \le 0 \\ K_{1j}\left(L_j - L_{0j}\right)^2; & 0 \le \varepsilon_j \le 2\varepsilon_1 \,, \\ K_{2j}\left(L_j - \left(1+\varepsilon_1\right)L_{0j}\right)^2; & \varepsilon_j \ge 2\varepsilon_1 \end{cases} \quad (4.1)$$

where K_{1j} and K_{2j} are the stiffness coefficients of the j*th* spring element for the parabolic and linear regions, respectively, and L_j and L_{0j} are its current and slack lengths, respectively. The strain in the j*th* ligamentous element, ε_j, is given by (4.2) and the linear range threshold is specified as $\varepsilon_1 = 0.03$.

$$\varepsilon_j = \frac{L_j - L_{0j}}{L_{0j}}. \quad (4.2)$$

The slack length of each spring element is obtained by assuming an extension ratio at full extension and using the following relation

$$\varepsilon_j = \frac{L_j\big|_{at\ full\ extension}}{L_{0j}}, \tag{4.3}$$

to evaluate the spring element's slack length, L_{0j}, from its length at full extension. The values of the different ligaments were specified according to the data available in the literature.

5. CONTACT FORCES AND EQUATIONS OF MOTION

Contact forces are induced at one or both contact points between the tibia and the femur. These forces are applied normal to the articular surface. Thus, the contact force applied to the tibia is expressed as $\vec{N}_i = N_i \vec{n}_i$. In the two-point contact situation, $i = 1, 2$ and in the single-point contact situation, $i = 1$. The equations governing the three-dimensional motion of the tibia with respect to the femur are the second order differential Newton's and Euler's equations of motion. Newton's equations are

$$F_{ex} + W_x + \sum_{i=1}^{2} N_{ix} + \sum_{j=1}^{12} F_{jx} = m\ddot{x}_0, \tag{5.1}$$

$$F_{ey} + W_y + \sum_{i=1}^{2} N_{iy} + \sum_{j=1}^{12} F_{jy} = m\ddot{y}_0, \tag{5.2}$$

$$F_{ez} + W_z + \sum_{i=1}^{2} N_{iz} + \sum_{j=1}^{12} F_{jz} = m\ddot{z}_0, \tag{5.3}$$

where m is the mass of the leg, $\ddot{x}_0, \ddot{y}_0$, and $\ddot{z}_0$ are the components of the linear acceleration of the center of mass of the leg (in the fixed femoral coordinate system); W_x, W_y, and W_z are the components of the weight of the leg; and F_{ex}, F_{ey}, and F_{ez} are the components of the external forcing pulse applied to the tibia. Euler's equations of motion are written with respect to the moving tibial system of axes which is the tibial centroidal principal system of axes $\left(x', y', z'\right)$. Thus, the angular velocity components $\left(\dot{\theta}_{x'}, \dot{\theta}_{y'}, \dot{\theta}_{z'}\right)$ and angular acceleration components $\left(\ddot{\theta}_{x'}, \ddot{\theta}_{y'}, \ddot{\theta}_{z'}\right)$, in the Euler equations are

$$\dot{\theta}_{x'} = -\dot{\alpha}\sin\beta\cos\gamma - \dot{\alpha}\beta\cos\beta\cos\gamma + \dot{\alpha}\gamma\sin\beta\sin\gamma + \dot{\beta}\sin\gamma + \dot{\beta}\gamma\cos\gamma\,, \tag{5.4}$$

$$\dot{\theta}_{y'} = -\dot{\alpha}\sin\beta\sin\gamma - \dot{\alpha}\beta\cos\beta\sin\gamma - \dot{\alpha}\gamma\sin\beta\cos\gamma - \dot{\beta}\cos\gamma + \dot{\beta}\gamma\sin\gamma\,, \tag{5.5}$$

$$\dot{\theta}_{z'} = -\dot{\alpha}\cos\beta + \dot{\alpha}\beta\sin\beta - \dot{y}\,, \tag{5.6}$$

$$\begin{aligned}\ddot{\theta}_{x'} &= -\ddot{\alpha}\sin\beta\cos\gamma - \ddot{\alpha}\beta\cos\beta\cos\gamma + \ddot{\alpha}\gamma\sin\beta\sin\gamma - \dot{\alpha}^2\beta\sin\gamma - \dot{\alpha}^2\gamma\sin\beta\cos\beta\cos\gamma \\ &-2\dot{\alpha}\beta\cos\beta\cos\gamma + 2\dot{\alpha}\dot{\beta}\gamma\cos\beta\sin\gamma + 2\dot{\alpha}\gamma\sin\beta\sin\gamma + \beta\sin\gamma + \ddot{\beta}\gamma\cos\gamma + 2\ddot{\beta}\cos\gamma,\end{aligned} \tag{5.7}$$

$$\begin{aligned}\ddot{\theta}_{y'} &= -\ddot{\alpha}\sin\beta\sin\gamma - \ddot{\alpha}\beta\cos\beta\sin\gamma - \ddot{\alpha}\gamma\sin\beta\cos\gamma - \dot{\alpha}^2\beta\cos\gamma - \dot{\alpha}^2\gamma\sin\beta\cos\beta\cos\gamma \\ &-2\dot{\alpha}\beta\cos\beta\cos\gamma + 2\dot{\alpha}\dot{\beta}\gamma\cos\beta\cos\gamma - 2\dot{\alpha}\dot{\gamma}\sin\beta\cos\gamma - \ddot{\beta}\sin\gamma + \ddot{\beta}\gamma\sin\gamma + 2\ddot{\beta}\dot{\gamma}\sin\gamma,\end{aligned} \tag{5.8}$$

$$\ddot{\theta}_{z'} = \ddot{\alpha}\cos\beta + \ddot{\alpha}\beta\sin\beta + \dot{\alpha}^2\gamma\sin^2\beta + 2\dot{\alpha}\dot{\beta}\sin\beta + \dot{\beta}^2\gamma - \ddot{\gamma}\,. \tag{5.9}$$

Euler's equations are written in a scalar form as

$$\left(\sum M\right)_{y'} = I_{y'y'}\ddot{\theta}_{y'} + \left(I_{x'x'} - I_{z'z'}\right)\dot{\theta}_{x'}\dot{\theta}_{z'}\,, \tag{5.10}$$

$$\left(\sum M\right)_{y'} = I_{y'y'}\ddot{\theta}_{y'} + \left(I_{x'x'} - I_{z'z'}\right)\dot{\theta}_{x'}\dot{\theta}_{z'}\,, \tag{5.11}$$

$$\left(\sum M\right)_{z'} = I_{z'z'}\ddot{\theta}_{z'} + \left(I_{y'y'} - I_{x'x'}\right)\dot{\theta}_{x'}\dot{\theta}_{y'}\,, \tag{5.12}$$

where $\left(\sum M\right)_{x'}, \left(\sum M\right)_{y'}$ and $\left(\sum M\right)_{z'}$ are the sum of the moments of all forces acting on the tibia around the x', y' and z' axis, respectively, and $I_{x'x'}, I_{y'y'}$, and $I_{z'z'}$ are the principal moments of inertia of the leg about this centroidal principal axis. In this analysis, the mass of the leg is taken as m = 4.0kg. Also, the leg was assumed to be a right cylinder; mass moments of inertia were thus calculated as $I_{x'x'} = 0.0672\ \mathrm{kg \cdot m^2}$, $I_{y'y'} = 0.0672\ \mathrm{kg \cdot m^2}$, and $I_{z'z'} = 0.005334\ \mathrm{kg \cdot m^2}$.

6. CONCLUSION

This paper presents a three-dimensional anatomical dynamic model of the tibio-femoral joint that predicts its response under sudden impact loads. The model consists of two body segments in contact (the femur and the tibia) executing a general three-dimensional dynamic motion within the constraints of the ligamentous structures. The sixdegrees-of-freedom joint motions are characterized using six kinematic parameters and ligamentous forces are expressed in terms of these six parameters. The model suggests that the three-dimensional dynamic anatomical modeling of the human musculo-skeletal joints is a versatile tool for the study of the internal forces in these joints.

Future work includes incorporating the patella into the model and using it to determine knee response under different loading conditions and to predict the behavior of the joint following ligamentous injuries and different reconstruction procedures.

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