ON THE YOUNG'S MODULUS FOR COMPOSITES BASED ON AUXETIC MATERIALS

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In this paper, the dynamic behavior of auxetic materials are interpreted in the light of Cosserat elasticity which admits degrees of freedom not present in classical elasticity: the rotation of points in the material, and a couple per unit area or the couple stress. The prediction of the Young'modulus is developed for a composite made up from aluminum and an auxetic material.

1. INTRODUCTION

Auxetic materials are gaining practical interest for their unusual and sometimes extreme mechanical response. The process of modeling these materials so far has highlighted a number of microstructural properties that are key to these materials (Munteanu *et al.* [1], Lakes [2–4]). The classical mechanics fail in describing the mechanical behavior of the auxetic material under deformation. The Cosserat, the micropolar and classical theories of elasticity are continuum theories, which make no reference to atoms or other structural features of the material, which is described. This is way, this paper focuses on the applying the Cosserat theory to describe the deformation of auxetic materials (Cosserat [5], Kröner [6], Hlavacek [7], [8], Adomeit [9], Berglund [10]). The estimation of the macroscopic Young modulus for a composite made from aluminum and an auxetic material is developed next. The Cosserat theory is coupled with the Bécus homogenization technique (Bécus [11]) and an inversion technique of experimental data. Details about the size effects in materials with negative Poisson's ratios and their properties can be found in Rosakis, Ruina, Lakes [12], Donescu *et al.* [13], [14] and Chiroiu *et al*. [15].

2. THE THEORY

Consider a chiral Cosserat medium, in a Cartesian coordinates system (x, y, z). The equations of motion for the case without body forces and body couples are (Eringen [16, 17], Mindlin [18, 19])

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$$\sigma_{kl,k} - \rho \ddot{u}_l = 0 , \quad m_{rk,r} + \varepsilon_{klr}\sigma_{lr} - \rho j \ddot{\varphi}_k = 0 . \tag{2.1}$$

Here σ_{kl} is the stress tensor, m_{kl} is the couple stress tensor, u is the displacement vector, φ_k is the microrotation vector which in Cosserat elasticity is cinematically distinct from the macrorotation vector $r_k = 1/2\varepsilon_{klm}u_{m,l}$, and ε_{klm} is the permutation symbol. We remember that φ_k refers to the rotation of points themselves, while r_k refers to the rotation associated with movement of nearby points. In (2.1) ρ is the mass density and j the microinertia. The constitutive equations are

$$\sigma_{kl} = \lambda e_{rr}\delta_{kl} + (2\mu + \kappa)e_{kl} + \kappa\varepsilon_{klm}(r_m - \varphi_m) + C_1\varphi_{r,r}\delta_{kl} + C_2\varphi_{k,l} + C_3\varphi_{l,k} , \tag{2.2a}$$

$$m_{kl} = \alpha\varphi_{r,r}\delta_{kl} + \beta\varphi_{k,l} + \gamma\varphi_{l,k} + C_1 e_{rr}\delta_{kl} + (C_2 + C_3)e_{kl} + (C_3 - C_2)\varepsilon_{klm}(r_m - \varphi_m), \tag{2.2b}$$

where $e_{kl} = 1/2(u_{k.l} + u_{l,k})$ is the macrostrain vector. λ, and μ are Lamé elastic constants,. κ is the Cosseraţ rotation modulus, α,β,γ, the Cosseraţ rotation gradient moduli, and C_i, $i = 1,2,3$ are the chiral elastic constants associated with noncentrosymmetry. For $C_i = 0$ the equations of isotropic micropolar elasticity are recovered. For $\alpha = \beta = \gamma = \kappa = 0$, (2.1) reduces to the constitutive equations of classical isotropic linear elasticity theory (Gauthier [20], Teodorescu *et al.* [21, 22]). The initial conditions are

$$u_i(x,y,z,0) = u_i^0(x,y,z) , \ \varphi_i(x,y,z,0) = 0 , \ i = 1,2,3 , \tag{2.3a}$$

$$m_{ij}(x,y,z,0) = 0 , \ \sigma_{ij}(x,y,z,0) = 0 , \quad i = j \neq 3 . \tag{2.3b}$$

Definition 1. Let $\mathbb{F} = \{\sigma_{kl}, m_{kl}, u_k, \varphi_k, \quad k,l = 1,2,3\}$, be a set composed of the asymmetric tensors σ_{kl}, m_{kl}, e_{kl}, $k,l = 1,2,3$, and the vectors u_k, φ_k. We call $\mathbb{F}$ an elastodynamic state on the bounded medium, if it satisfies (2.1)–(2.3).

Theorem 1 (The one-by-one transformation). *The theory is based on the following theorem:*

$$\hat{u}_1 = K_{10}^2(u_1 + u_2 - u_3) , \ \hat{u}_2 = K_{11}^2(u_2 + u_3 - u_1) , \ \hat{u}_3 = K_{12}^2(u_3 + u_1 - u_2) ,$$

$$\hat{\varphi}_1 = K_{10}^2(\varphi_1 + \varphi_2 - \varphi_3) , \ \hat{\varphi}_2 = K_{11}^2(\varphi_2 + \varphi_3 - \varphi_1) , \ \hat{\varphi}_3 = K_{12}^2(\varphi_3 + \varphi_1 - \varphi_2) ,$$

with

$$K_{10}^2 = \frac{(C_2 + C_3)^2}{4(2\mu + \kappa)(\beta + \gamma)} , \ K_{11}^2 = \frac{(C_2 - C_3)^2}{4(2\mu + \kappa)(\gamma - \beta)} , \ K_{12}^2 = \frac{(3C_1 + C_2 + C_3)^2}{4(3\lambda + 2\mu + \kappa)(3\alpha + \beta + \gamma)} ,$$

transforms the elastodynamic state $\mathbb{F}$ *into another elastodynamic state* $\hat{\mathbb{F}} = \{\hat{\sigma}_{kl}, \hat{m}_{kl}, \hat{u}_k, \hat{\varphi}_k, k,l = 1,2,3\}$*, composed by the symmetric tensors* $\hat{\sigma}_{kl}$, $\hat{m}_{kl}$, $\hat{e}_{kl}$,

$k,l=1,2,3$, *and the vectors* $\hat{u}_k$, $\hat{\varphi}_k$, *that satisfies* (2.1)–(2.3). *The state* $\hat{\mathbb{F}}$ *can be decomposited in the form* $\hat{\mathbb{F}}=\hat{\mathbb{F}}_1+\hat{\mathbb{F}}_2$, *where* $\hat{\mathbb{F}}_1=\{\hat{\sigma}_{11},\hat{\sigma}_{13},\hat{\sigma}_{33},\hat{m}_{22},\hat{u}_1,\hat{u}_3,\hat{\varphi}_2\}$ *and* $\hat{\mathbb{F}}_2=\{\hat{\sigma}_{22},\hat{m}_{11},\hat{m}_{13},\hat{m}_{33},\hat{u}_2,\hat{\varphi}_1,\hat{\varphi}_3\}$.

After a proper combination of equations, the following equations in $\hat{u}=(\hat{u}_1,\hat{u}_2,\hat{u}_3)$ and $\hat{\varphi}=(\hat{\varphi}_1,\hat{\varphi}_2,\hat{\varphi}_3)$ are found.

$$(\lambda+2\mu+\kappa)\nabla\nabla\hat{u}-(\mu+\kappa)K_0^2\nabla\times\nabla\times\hat{u}+\kappa(1-K_0^2)\nabla\times\hat{\varphi}=\rho\ddot{\hat{u}}, \quad (2.4a)$$

$$(\alpha+\beta+\gamma)\nabla\nabla\hat{\varphi}-\gamma K_0^2\nabla\times\nabla\times\hat{\varphi}+\kappa(1-K_0^2)\nabla\times\hat{u}-2\kappa(1-K_0^2)\hat{\varphi}=\rho j\ddot{\hat{\varphi}}, \quad (2.4b)$$

with a coupling coefficient K_0 defined as

$$K_0^2=1+\frac{(C_1+C_2+C_3)^2}{(\lambda+2\mu+\kappa)(\alpha+\beta+\gamma)}. \quad (2.5)$$

We see that (2.4) are decoupled into two sets of equations in $\hat{\mathbb{F}}_1$, and $\hat{\mathbb{F}}_2$. Next, we will concentrate only to the set of equations corresponding to $\hat{\mathbb{F}}_1$, the other set being solved in a similar way.

3. YOUNG' MODULUS OF THE COMPOSITE

Consider a 2D composite plate which occupies the region $x\in[0, L]$, $z\in[-c, c]$, and made up of two aluminium thin layers and a core between them, made by an auxetic material (Fig. 3.1). In this study, the auxetic material is the conventional open cell polyurethane with negative Poisson's ratio. The specimens are produced from conventional gray open-cells polyurethane foam with Poisson ratio –0.23, and 30-35 pores/inch and 0.0027 g/cm^3 density, by means of process which has been previously defined by Scarpa *et al*. [23], [24]. The constitutive law

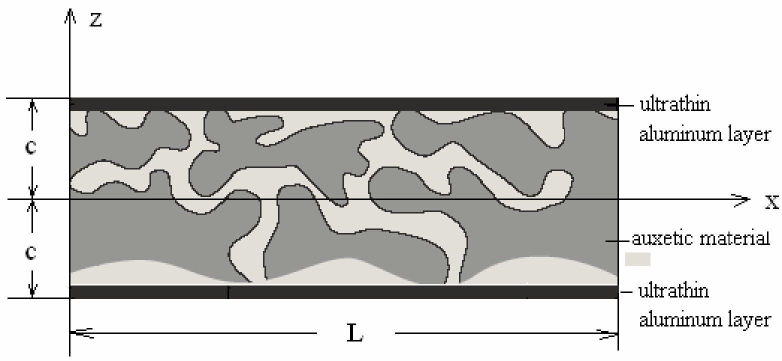

Fig. 3.1 – The composite plate.

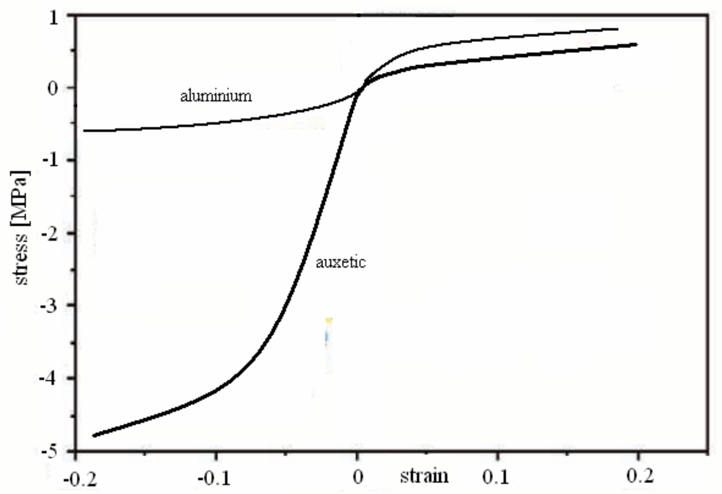

Fig. 3.2 – The constitutive law for aluminium and auxetic amaterial.

for aluminium and auxetic materials is determined by an inverse technique based on experimental data (Fig. 3.2).

The motion equations are given by (2.4) with K_0^2 given by (2.5). We suppose that all material constants are functions of x. The equations (2.4) are decoupled into two sets of equations in $\hat{\mathbb{F}}_1$, and $\hat{\mathbb{F}}_2$. So, we will concentrate only to the set of equations corresponding to $\hat{\mathbb{F}}_1$. To predict the Young' modulus from the Lamé elastic constants λ, μ, we have the formula $E_0 = \frac{(3\lambda + 2\mu)\mu}{\lambda + \mu}$. We are interested in knowing the influence of the Cosserat rotation modulus κ, the Cosserat rotation gradient moduli α, β, γ, and the chiral elastic constants C_i, $i = 1, 2, 3$, on the effective Young' modulus value of the laminated plate.

To evaluate the homogenized elastic Young's modulus, consider the boundary-value problem for the plate subjected to an axial tension in the *x*-direction. We find by using the Bécus homogenization [10]

$$E = F(\tilde{C}') + E', \quad E' = \frac{(2\mu' + \kappa')(3\lambda' + 2\mu' + \kappa_{aux})}{(2\lambda' + 2\mu' + \kappa_{aux})} + \frac{1}{2}\tilde{p}^2, \tag{3.1}$$

$$\tilde{p}^2 = \frac{2\kappa_{aux}}{(K_0'^2 - 1)}, \quad K_0'^2 = 1 + \frac{(C_{1aux} + C_{2aux} + C_{3aux})^2}{(\lambda' + 2\mu' + \kappa_{aux})(\alpha_{aux} + \beta_{aux} + \gamma_{aux})}, \tag{3.2}$$

where $\tilde{C}_{al}$ are the aluminum constants and $\tilde{C}_{aux}$, the auxetic constants.

In (3.1), the function $F(\tilde{C}')$ is numerically determined only. In the simulation, the Young's moduli of aluminum and respectively, of the auxetic material, are 109 GPa, and respectively, 115 kPa. From (3.1) and (3.2), the Young's modulus of the composite is calculated as 112 GPa.

4. CONCLUSIONS

In this paper, the prediction of the Young'modulus is developed for a composite made up from aluminium and an auxetic material. The auxetic material implies a stiffening effect leading to increased Young's elastic constants. The composite auxetic structures are used widely in a variety of engineering application, in aerospace and aircraft applications, because of their stronger and lighter weight.

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