USE OF GENETIC ALGORITHMS FOR FITTING THE BOUC-WEN MODEL TO EXPERIMENTAL HYSTERETIC CURVES

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In this paper a Genetic Algorithms (GA) method is developed to identify the Bouc-Wen model parameters from the experimental data of periodic loading tests. It is considered an extension of classical model in order to increase its capacity to approximate experimental loops. From Bouc-Wen equation are derived integral conditions that characterize the coordinates of hysteresis curve. The objective function of GA is defined as a sum of relative errors obtained for a set of indices computed on the predicted and experimental loops.

1. INTRODUCTION

The Bouc-Wen model, widely used in structural and mechanical engineering, gives an analytical description of a smooth hysteretic behavior. It was introduced by Bouc [1] and extended by Wen [2], who demonstrated its versatility by producing a variety of hysteretic characteristics. The hysteretic behavior of materials, structural elements or vibration isolators is treated in a unified manner by a single nonlinear differential equation with no need to distinguish different phases of the applied loading pattern. In practice, the Bouc-Wen model is mostly used within the following inverse problem approach: given a set of experimental input–output data, how to adjust the Bouc-Wen model parameters so that the output of the model matches the experimental data. Once an identification method has been applied to tune the Bouc-Wen model parameters, the resulting model is considered as a "good" approximation of the true hysteresis when the error between the experimental data and the output of the model is small enough from practical point of view. Usually, the experimental data are obtained by imposing cyclic relative motions between the mounting ends on the testing rig of a sample material, structural element or vibration isolator and by recording the evolution of the developed force versus the imposed displacement. Once the hysteresis model was identified for a specific input, it should be validated for different types of inputs that can be applied on the testing rig, such as to simulate as close as possible the expected real inputs. Then this model can be used to study the dynamic

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behavior of different systems containing the tested structural elements or devices under different excitations.

Various methods where developed to identify the model parameters from the experimental data of periodic vibration tests. A frequency domain method was employed to model the hysteretic behavior of wire-cable isolators [3], iterative procedures were proposed for the parametric identification of a smoothed hysteretic model with slip [4], of a modified Bouc-Wen model to portray the dynamic behavior of magnetorhological dampers [5], of a generalized model for highly asymmetric hysteresis obtained in laboratory experiments for flexible connectors [6], etc. The Genetic Algorithms were widely used for curve fitting the Bouc-Wen model to experimentally obtained hysteresis loops for composite materials [7], nonlinear degrading structures [8] or magnetorheological fluid dampers [9-11].

In the present work, our primary focus is to use analytical relationships derived from the Bouc-Wen differential equation to determine the parameters of the model such as the predicted hysteresis curves and the experimental loops to have same absolute values of the maximum force, same coordinates of the loop-axes crossing points, and same slopes of the tangent at these key points of hysteretic loops. As the measured data have a certain degree of imprecision, one of the most suitable methods for approximating the model parameter is the genetic algorithms (GA) approach.

2. MATHEMATICAL MODEL

In this paper is assumed that the experimental hysteretic characteristic is a symmetric loop $-F_{\mathrm{m}} \le F(x) \le F_{\mathrm{m}}$, obtained for a periodic motion $-x_m(t) \le x(t) \le x_m(t)$, imposed between the mounting ends of the tested element. In the xOz system of coordinates, the loop-axes crossing points are denoted by $(0,\, F_0)$, $(x_0,\, 0)$, $(0,\, -F_0)$ and $(-x_0,\, 0)$. Next, the following dimensionless variables are considered

$$\begin{aligned} &\tau = t/T,\ \xi(\tau) = x(\tau T)/x_u,\ \xi'(\tau) = \mathrm{d}\xi/\mathrm{d}\tau,\ z(\xi) = F(x_u \xi)/F_u, \\ &\xi_m = \max\left|\xi(\tau)\right|,\ z_m = \max\left|z(\xi)\right|,\ \ \xi_0 = x_0/x_u,\ z_0 = F_0/F_u, \end{aligned} \tag{1}$$

where T is the period of the imposed cyclic motion and x_u, F_u are displacement and force reference units such as $\xi_m \le 1,\ z_m \le 1$. Then, a generic plot of the symmetric hysteresis loop $z(\xi)$ can be represented as shown in Fig. 1.

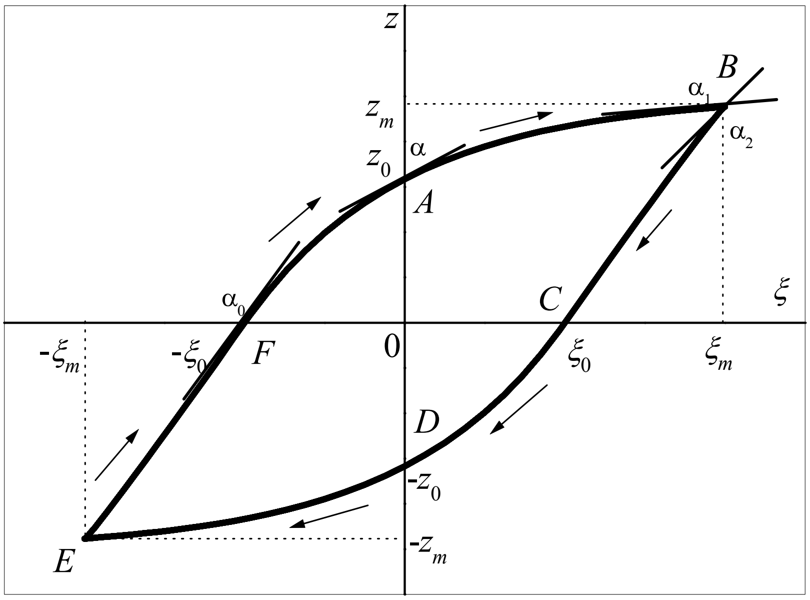

Fig. 1 – Generic experimental hysteretic loop.

With the previous notations, the Bouc-Wen model is described by the following non-linear differential equation

$$\frac{\mathrm{d}z}{A-|z|^n\left[\beta+\gamma\operatorname{sgn}\left(\xi' z\right)\right]}=\mathrm{d}\xi, \tag{2}$$

where A, β, γ, n are loop parameters controlling the shape and magnitude of the hysteresis loop $z(\xi)$. In the classical models the parameter n is assumed a natural number but, this constraint doesn't allow good approximation for some experimental loops [12]. Therefore, in this paper the exponent in Bouc-Wen model is not necessarily a natural number, being considered as a positive real number p to be determined within the fitting procedure of experimental data.

In order to characterize the hysteresis loop there are defined some integral and derivative conditions. These relations will be used for definition of genetic algorithm objective function. By taking the integral on the branches AB, BC and CD, one can write the following relations:

$$\int_{z_0}^{z_m}\frac{\mathrm{d}z}{A-z^p(\beta+\gamma)}=\int_0^{\xi_m}\mathrm{d}\xi=\xi_m, \tag{3}$$

$$\int_{0}^{z_m} \frac{\mathrm{d}z}{A - z^p(\beta-\gamma)} = \int_{\xi_0}^{\xi_m} \mathrm{d}\xi = \xi_m - \xi_0 , \tag{4}$$

$$\int_{-z_0}^{0} \frac{\mathrm{d}z}{A - |z|^p(\beta+\gamma)} = \int_{0}^{\xi_0} \mathrm{d}\xi = \xi_0 . \tag{5}$$

Next, there are denoted by α and α_0 the tangent slopes at the points $(-\xi_0,\ 0)$ and (0, z_0), respectively. Then, using (2), one can derive the equations:

$$\frac{\mathrm{d}z}{\mathrm{d}\xi}(0) = \alpha \Rightarrow \alpha = A - |z|^p(\beta+\gamma) , \tag{6}$$

$$\frac{\mathrm{d}z}{\mathrm{d}\xi}(\xi_0) = \alpha_0 \Rightarrow \alpha_0 = A . \tag{7}$$

As $z_0 > 0$, then from (6) and (7) is obtained the relation:

$$\alpha - \alpha_0 + z_0{}^p(\beta+\gamma) = 0 . \tag{8}$$

In the same way, if α_1 and α_2 are the slopes of left hand and right hand tangents at the point $(\xi_m,\ z_m)$ then:

$$\left.\frac{\mathrm{d}z}{\mathrm{d}\xi}\right|_{\substack{\xi=\xi_m \\ \xi'>0}} = \alpha_1 \Rightarrow \alpha_1 = A - z_m{}^p(\beta+\gamma) , \tag{9}$$

$$\left.\frac{\mathrm{d}z}{\mathrm{d}\xi}\right|_{\substack{\xi=\xi_m \\ \xi'<0}} = \alpha_2 \Rightarrow \alpha_2 = A - z_m{}^p(\beta-\gamma) . \tag{10}$$

For many experimental curves, the coefficients α_1 and α_2 are difficult to be accurately estimated, so they are used only when the sample time interval of recorded force and displacement signals is sufficiently small.

3. GA OPTIMIZATION

The main goal of this paper is to find the values of parameters A, β, γ, p such that the obtained Bouc-Wen loop represents a good approximation of the experimental one. As the measured data have a certain degree of imprecision, one of the most suitable methods for approximating the coefficients A, β, γ, p is the genetic algorithms approach. Here is proposed to be used a real coded GA with four genes that are the codifications of parameters $A,\ \beta,\ \gamma,\ p$. The objective function is defined as a sum of five indexes, three of them having integral formulation and the other two obtained from (7) and (8). Hence, the partial performance indices are defined by

$$I_1 = \frac{1}{\xi_m}\left|\int_{z_0}^{z_m} \frac{\mathrm{d}z}{A - z^p(\beta+\gamma)} - \xi_m\right|,\ I_2 = \frac{1}{\xi_m - \xi_0}\left|\int_0^{z_m} \frac{\mathrm{d}z}{A - z^p(\beta-\gamma)} - (\xi_m - \xi_0)\right|,$$
$$I_3 = \frac{1}{\xi_0}\left|\int_{-z_0}^{0} \frac{\mathrm{d}z}{A - |z|^p(\beta+\gamma)} - \xi_0\right|, \qquad (11)$$

$$I_4 = |\alpha_0 - A|,\ I_5 = \left|\alpha - \alpha_0 + z_0{}^p(\beta+\gamma)\right|. \qquad (12)$$

Therefore, the global performance index is given by

$$I = w_1 I_1 + w_2 I_2 + w_3 I_3 + w_4 I_4 + w_5 I_5, \qquad (13)$$

where $w_i,\ i = \overline{1,\ 5}$ are weights that can be set such as the search is oriented to minimize the most significant indices.

The other GA parameters used in numerical simulations are:

- average crossover with probability 0.7;
- uniform mutation with probability $\frac{1}{N}$, where N is the number of genes;
- Monte Carlo selection.

4. APPLICATION OF GA FOR FITTING THE BOUC WEN MODEL TO EXPERIMENTAL HYSTERETIC LOOPS OF SEISMIC DEVICES

Base isolation and dissipative bracing of buildings are modern and efficient seismic protection strategies already implemented in many countries. While base isolation is a more appealing solution in the case of new buildings, the dissipative braces are use especially in the seismic retrofitting of the existing ones.

The force-displacement characteristic of most seismic protection devices is of hysteretic type. Usually, the experimental hysteretic loops are obtained by imposing cyclic relative motions between the device mounting ends on the testing rig and by recording the evolution of the developed force versus the imposed displacement. By fitting a Bouc-Wen model type to experimental data, one obtains a single non-linear first order equation which can describe the evolution of force developed by one device for almost any loading pattern (periodic, aperiodic or random). All these equations are then added to the system of equations which models the motion of the protected building. Thus, is obtained an enlarged system, which can portray the dynamic behavior of the protected structure with a better accuracy than it can be achieved by employing other methods (equivalent linearization, phase description of hysteretic loops by piece-wise continuous functions, etc).

In this paper, the proposed method for fitting the Bouc-Wen model to experimental hysteretic loops is illustrated for two seismic devices, manufactured by the Italian Company FIPP INDUSTRIALE:

– Lead Rubber Bearing (LRB), used for seismic protection by base isolation;

– Buckling-Restrained Axial Damper (BRAD), used for seismic protection by inter-storey dissipative bracing.

The normalized force-displacement curves, obtained from those reported by the manufacturer in the product technical notes [13], [14], and hysteretic loops predicted by the developed fitting method, are shown comparatively in Figs. 2 and 3.

It should be mentioned that the force- displacement curve of LRB device was obtained by a shear test conducting on a column of two devices, under a static vertical load 1,500 KN and a cyclic horizontal load applied at the middle. The reference values chosen from experimental curves in this case were $x_u = 50$ mm and $F_u = 150$ kN. After about 5.000 generations, the following values of model parameters were obtained: $A = 0.504$, $\beta = -13.165$, $\gamma = 13.05$, $p = 0.74$.

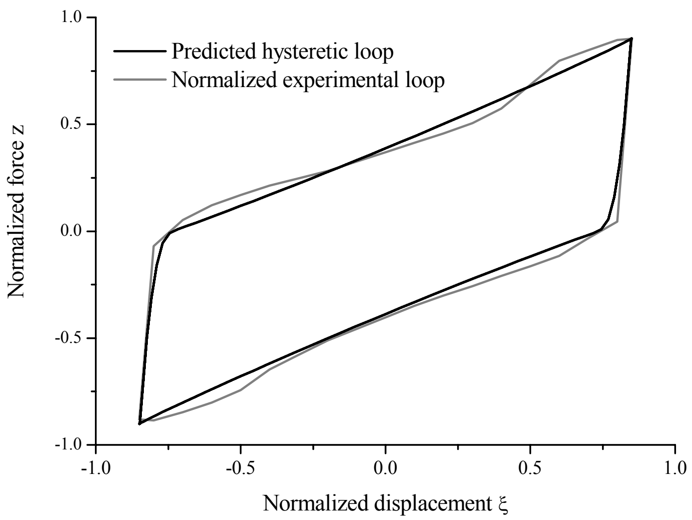

Fig. 2 – The experimental and predicted hysteretic loops for LRB devices.

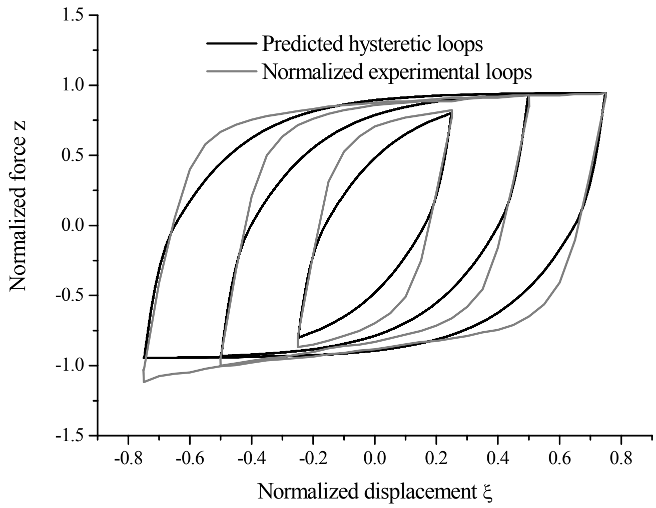

Fig. 3 – The experimental and predicted hysteretic loops for BRAD device.

For the BRAD experimental loops the reference units are x_u = 20 mm and F_u = 200 kN and Fig. 3 depicts the predicted and experimental loops for three different amplitudes of the imposed displacement, for the following set of model parameters $A = 4.1$, $\beta = -7.72$, $\gamma = 12.07$, $p = 1.054$. The fitting method was applied for experimental data corresponding to the maximum relative displacement allowed for this device.

5. CONCLUSIONS

The proposed method for fitting Bouc-Wen model to experimental hysteretic loops is based on analytical relationships. These relations are expressed in terms of integral and differential conditions which describe the essential properties of hysteretic loop. The objective function defined as a function of the analytical performance indices is suitable to be efficiently used within a genetic algorithms procedure. The obtained results show the extension of the classical Bouc-Wen model by taking the parameter n as a positive real number could be very useful in increasing the approximation accuracy of experimental loops.

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