ASYMMETRICAL COLD STRIP ROLLING. A NEW ANALYTICAL APPROACH

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In this paper is given a solution of asymmetrical strip rolling problem using a Bingham type constitutive equation and the perturbation method. Rolling pressure distribution, rolling force, rolling torque, front tension, position of neutral points, which are affected by various rolling conditions are analyzed. The influence of the rolling speed on the whole process was studied using a Bingham type constitutive equation.

1. INTRODUCTION

A uniform plastic model of symmetrical strip rolling in which peripheral velocity and radius of the upper roll are equal to those of the lower roll was proposed by Orowan [1]. Recently, the symmetrical strip rolling using a Bingham type constitutive equation and the perturbation method was studied by N. Şandru and G. Camenschi [2, 3].

Asymmetrical rolling process was used to manufacture plates and sheets. The asymmetry is due to the rolls with different speeds or with different diameters. This process improves productivity of the rolling operation-the rolling force, pressure, torque is reducing, and the properties of the strip surface are improved.

An analytical solution for asymmetrical strip rolling process was proposed by Y. M. Hwang and G.Y. Tzou, using the slab method [4] and the rolling pressure, rolling force and torque were obtained.

In this paper is given a solution of asymmetrical strip rolling problem using the method presented in [2, 3].

2. SOLUTION OF THE PROBLEM

In order to solve the problem of asymmetrical strip rolling the following assumptions were made:

a) the roll is rigid; the strip being rolled is Bingham type viscoplastic, incompressible material;

b) deformation is plain strain;

c) the process takes place with high speed;

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d) the frictional factor between the roll and the material is constant over the arc of contact;

e) the flow direction of the strip at the entrance and the exit of the roll bite are horizontal.

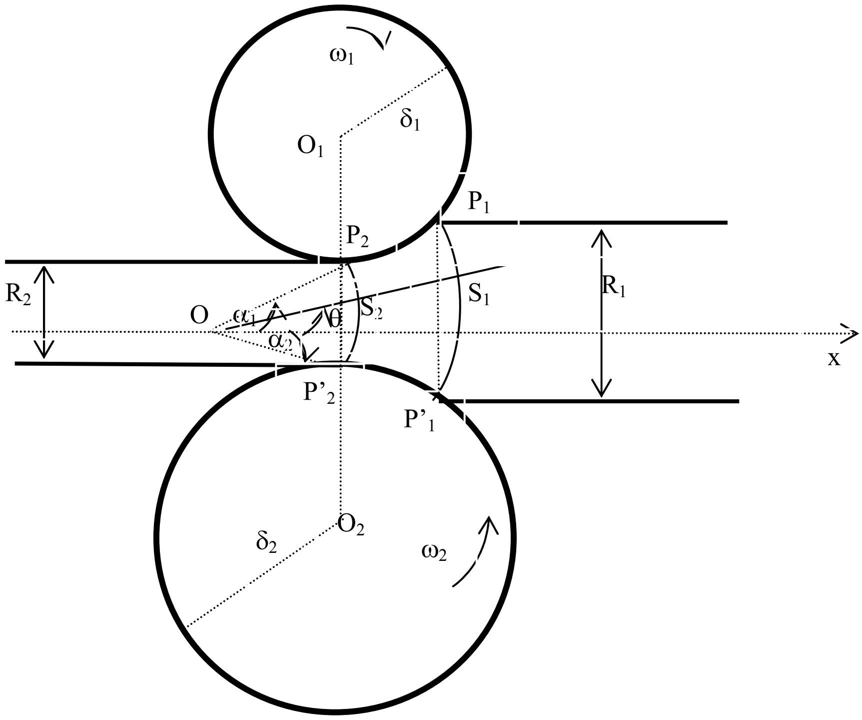

Fig. 1 – The geometry of asymmetrical cold rolling.

Fig. 1 is a schematic illustration of the asymmetrical strip rolling process. The viscoplastic deformation region is bounded by two singular surfaces S_1 and S_2. This zone is divided in three regions by the surfaces S_{N_1} and S_{N_2} passing through the neutral points N_1 for the upper roll, and N_2 for the lower roll. The position of the neutral points N_1, and N_2 and the equation of the surfaces S_{N_1} and S_{N_2} will be determined.

Assuming that the angle of strip and the upper roll between the contact point at the entrance in the deformation zone and the contact point at the exit from it (α_1), also the radius of the upper roll (δ_1) are known, we get the following conditions in order that the process to take place

$$\delta_2 = \frac{\left(R_1 - R_2 - 2\delta_1 \sin^2 \alpha_1\right)^2 + \delta_1^2 \sin^2 2\alpha_1}{2\left(R_1 - R_2 - 2\delta_1 \sin^2 \alpha_1\right)}, \tag{1}$$

$$\alpha_2 = \frac{1}{2}\arcsin\left(\frac{\delta_1}{\delta_2}\sin 2\alpha_1\right),$$

which we have from the geometry of the figure.

Equations of the problem are

a) Cauchy equation

$$\operatorname{div}\mathbf{t} = \rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}, \tag{2}$$

b) continuity equation

$$\mathrm{I}_{\mathbf{d}} = 0, \tag{3}$$

c) deformation rate tensor equation

$$\mathbf{d} = \frac{1}{2}\left(\nabla\mathbf{v} + \left(\nabla\mathbf{v}\right)^{\mathrm{T}}\right), \tag{4}$$

d) Bingham type constitutive equation

$$\mathbf{t} = -p\mathbf{I} + \left(2\eta + \frac{k}{\sqrt{\mathrm{II}_{\mathbf{d}}}}\right)\mathbf{d},\ \mathrm{II}_{\mathbf{t}'} > k^2, \tag{5}$$

In (2)–(5), **t** is Cauchy stress tensor, p pressure, η, k – material constants, **d** – deformation rate tensor, $\mathrm{II}_{\mathbf{d}}$ – the second invariant of deformation rate tensor, $\mathrm{II}_{\mathbf{t}'}$ – the second invariant of tension tensor deviator, **v** – velocity vector.

Writing equations (2)–(5) in polar coordinates and turning them into non dimensional form by means of the relations

$$r = r^0 R_2,\ v_r = v_r^0 v_2,\ v_\theta = v_\theta^0 v_2,\quad p = p^0\frac{\eta v_2}{R_2}. \tag{6}$$

The following dimensionless combinations are put into evidence Bingham´s number – $\mathrm{Bg} = \dfrac{kR_2}{\eta v_2}$, and $\mathrm{Re} = \dfrac{\rho v_2 R_2}{\eta}$ – Reynolds number.

We shall suppose that Re << 1, therefore the inertial terms are neglected and extending the method used in [2] for the symmetrical strip rolling, we get

$$\Psi_i\left(r,\theta\right) = R_2 v_2 \Psi_{0i}^0 + \frac{k}{\eta} r^2 \varphi_i\left(\theta\right) + O\left(\mathrm{Bg}^2\right), \tag{7}$$

where

$$\Psi_{0i}^0\left(\theta\right) = a_i\theta + \frac{b_i}{2}\sin 2\theta + d_i + \frac{e_i}{2}\cos 2\theta,$$

$$\varphi_i(\theta) = A_i\theta + \frac{B_i}{2}\sin 2\theta + D_i + \frac{E_i}{2}\cos 2\theta + K_{1i}(\theta)\cos 2\theta + K_{2i}(\theta)\sin 2\theta$$

and

$$\frac{\mathrm{d}K_1(\theta)}{\mathrm{d}\theta} = -\frac{1}{2} f_{\mathrm{i}}(\theta)\sin 2\theta\,,$$
$$\frac{\mathrm{d}K_2(\theta)}{\mathrm{d}\theta} = \frac{1}{2} f_{\mathrm{i}}(\theta)\cos 2\theta\,, \tag{9}$$

with

$$\frac{\mathrm{d}f_i(\theta)}{\mathrm{d}\theta} = -\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\dfrac{\mathrm{d}^2\Psi_{0i}^0}{\mathrm{d}\theta^2}}{2\sqrt{\left(\dfrac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta}\right)^2 + \dfrac{1}{4}\left(\dfrac{\mathrm{d}^2\Psi_{0i}^0}{\mathrm{d}\theta^2}\right)^2}}\right) + 2\frac{\dfrac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta}}{\sqrt{\left(\dfrac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta}\right)^2 + \dfrac{1}{4}\left(\dfrac{\mathrm{d}^2\Psi_{0i}^0}{\mathrm{d}\theta^2}\right)^2}}\,. \tag{10}$$

Velocity vector components are given by

$$v_r^i = \frac{R_2 v_2}{r}\frac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta} + \frac{k}{\eta} r^2 \varphi_i'(\theta) + O\left(\mathrm{Bg}^2\right),$$
$$v_\theta^i = -\frac{2k}{\eta} r\varphi_i(\theta) + O\left(\mathrm{Bg}^2\right). \tag{11}$$

The physical components of the stress tensor are

$$t_{rr}^i = -\frac{2R_2 v_2 \eta}{r^2}\left(2\frac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta} - a_i\right) - \frac{\eta v_2}{R_2} c_i -$$

$$-2k\left(\frac{\dfrac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta}}{\sqrt{F_i(\theta)}} + \frac{\sqrt{F_i(\theta)}}{2a_i} - \varphi_i'(\theta) + \frac{C_i}{2} + 2A_i \ln\frac{r}{R_2}\right) + O\left(\mathrm{Bg}^2\right), \tag{12}$$

$$t_{\theta\theta}^i = \frac{2R_2 v_2 \eta}{r^2} a_i - \frac{\eta v_2}{R_2} c_i - 2k\left(\frac{\sqrt{F_i(\theta)}}{2a_i} + 2A_i \ln\frac{r}{R_2} + \varphi_i'(\theta) + \frac{C_i}{2}\right) + O\left(\mathrm{Bg}^2\right),$$

$$t_{r\theta}^{i}=\frac{R_2 v_2 \eta}{r^2}\frac{\mathrm{d}^2\Psi_{0i}^0}{\mathrm{d}\theta^2}+k\left(\frac{1}{2}\frac{\frac{\mathrm{d}^2\Psi_{0i}^0}{\mathrm{d}\theta^2}}{\sqrt{F_i(\theta)}}+\varphi_i''(\theta)\right)+O\left(\mathrm{Bg}^2\right),$$

with

$$F_i(\theta)=\left(\frac{\mathrm{d}\Psi_{0i}^0}{\mathrm{d}\theta}\right)^2+\frac{1}{4}\left(\frac{\mathrm{d}^2\Psi_{0i}^0}{\mathrm{d}\theta^2}\right)^2. \tag{13}$$

Denoting by X, Y the stress resultants in directions Ox, Oy, acting on a surface with the equation $r=r(\theta)$, $\theta\in(-\alpha_2,\alpha_1)$, we have

$$X=\int_{-\alpha_2}^{\alpha_1}\left[\left(rt_{rr}-r't_{r\theta}\right)\cos\theta-\left(rt_{r\theta}-r't_{\theta\theta}\right)\sin\theta\right]\mathrm{d}\theta,$$

$$Y=\int_{-\alpha_2}^{\alpha_1}\left[\left(rt_{rr}-r't_{r\theta}\right)\sin\theta+\left(rt_{r\theta}-r't_{\theta\theta}\right)\cos\theta\right]\mathrm{d}\theta. \tag{14}$$

So, we get

$$\frac{X^j}{4\eta v_2}=\frac{R_2}{2}\left[-a_j\left(\frac{\sin\alpha_1}{r(\alpha_1)}+\frac{\sin\alpha_2}{r(\alpha_2)}\right)-b_j\left(\frac{\cos\alpha_1\sin 2\alpha_1}{r(\alpha_1)}+\frac{\cos\alpha_2\sin 2\alpha_2}{r(\alpha_2)}\right)-\right.$$

$$\left.-e_j\left(\frac{\cos\alpha_1\cos 2\alpha_1}{r(\alpha_1)}-\frac{\cos\alpha_2\cos 2\alpha_2}{r(\alpha_2)}\right)-c_j\frac{1}{2R_2^2}\left(r(\alpha_1)\sin\alpha_1+r(\alpha_2)\sin\alpha_2\right)\right]+$$

$$+\mathrm{Bg}\left[A_j\frac{1}{2R_2}\left(r(\alpha_1)\sin\alpha_1\left(1-\ln\frac{r^2(\alpha_1)}{R_2^2}\right)+r(\alpha_2)\sin\alpha_2\left(1-\ln\frac{r^2(\alpha_2)}{R_2^2}\right)\right)+\right. \tag{15}$$

$$+B_j\frac{1}{2R_2}\left(r(\alpha_1)\sin\alpha_1+r(\alpha_2)\sin\alpha_2\right)-C_j\frac{1}{4R_2}\left(r(\alpha_1)\sin\alpha_1+r(\alpha_2)\sin\alpha_2\right)+$$

$$+E_j\frac{1}{2R_2}\left(r(\alpha_1)\cos\alpha_1-r(\alpha_2)\cos\alpha_2\right)-\frac{r(\alpha_1)}{2R_2}\cos\alpha_1 I_{1j}-$$

$$-\frac{1}{2a_j}\frac{1}{2R_2}\left(r(\alpha_1)\sin\alpha_1\sqrt{F_j(\alpha_1)}+r(\alpha_2)\sin\alpha_2\sqrt{F_j(-\alpha_2)}\right)+$$

$$+\frac{1}{R_2}r(\alpha_1)\left(K_{1j}\cos\alpha_1+K_{2j}\sin\alpha_1\right)\Bigg],$$

where

$$I_{1j}=\int_{-\alpha_2}^{\alpha_1}\frac{a_j+b_j\cos 2t\text{-}e_j\sin 2t}{\sqrt{F_j(t)}}\mathrm{d}t\,, \tag{16}$$

with $j = 1, 2, 3$ placed in regions 1, 2, 3 of the deformation zone and the vertical stress resultants is

$$\frac{Y^j}{4\eta v_2}=\frac{R_2}{2}\Bigg[a_j\left(\frac{\cos\alpha_1}{r(\alpha_1)}-\frac{\cos\alpha_2}{r(\alpha_2)}\right)+b_j\left(-\frac{\sin\alpha_1\sin 2\alpha_1}{r(\alpha_1)}+\frac{\sin\alpha_2\sin 2\alpha_2}{r(\alpha_2)}\right)+$$

$$+e_j\left(\frac{-\sin\alpha_1\cos 2\alpha_1}{r(\alpha_1)}-\frac{\sin\alpha_2\cos 2\alpha_2}{r(\alpha_2)}\right)+c_j\frac{1}{2R_2^2}\left(r(\alpha_1)\cos\alpha_1-r(\alpha_2)\cos\alpha_2\right)\Bigg]+$$

$$+\mathrm{Bg}\Bigg[\text{-}A_j\frac{1}{2R_2}\left(r(\alpha_1)\cos\alpha_1\left(1-\ln\frac{r^2(\alpha_1)}{R_2^2}\right)-r(\alpha_2)\cos\alpha_2\left(1-\ln\frac{r^2(\alpha_2)}{R_2^2}\right)\right)+$$

$$+B_j\frac{1}{2R_2}\left(r(\alpha_1)\cos\alpha_1-r(\alpha_2)\cos\alpha_2\right)+C_j\frac{1}{4R_2}\left(r(\alpha_1)\cos\alpha_1-r(\alpha_2)\cos\alpha_2\right)- \tag{17}$$

$$-E_j\frac{1}{2R_2}\left(r(\alpha_1)\sin\alpha_1+r(\alpha_2)\sin\alpha_2\right)-\frac{r(\alpha_1)}{2R_2}\sin\alpha_1 I_{1j}+$$

$$+\frac{1}{2a_j}\frac{1}{2R_2}\left(r(\alpha_1)\cos\alpha_1\sqrt{F_j(\alpha_1)}-r(\alpha_2)\cos\alpha_2\sqrt{F_j(-\alpha_2)}\right)-$$

$$-\frac{1}{R_2}r(\alpha_1)\left(K_{1j}\sin\alpha_1-K_{2j}\cos\alpha_1\right)\Bigg],\ j=1,2.$$

We compute the stress resultants acting on surfaces $\theta=\alpha_1,\ r\in\left(r_{N_1}(\alpha_1),r_1(\alpha_1)\right)$ and we obtain

$$T_1^1 = \int_{r_{N_1}(\alpha_1)}^{r_1(\alpha_1)} t_{r\theta}^1 \Big|_{\theta=\alpha_1} \mathrm{d}r = R_2 v_2 \eta \Psi_{01}^{1\,''}(\alpha_1) \frac{R_1 - R_2}{2\delta_1 \sin\alpha_1} \left(\frac{1}{R_{N_1}} - \frac{1}{R_1} \right) +$$

$$+k \left[\frac{1}{2} \frac{\Psi_{01}^{0\,''}(\alpha_1)}{\sqrt{F_1(\alpha_1)}} + \varphi_1''(\alpha_1) \right] \frac{2\delta_1 \sin\alpha_1}{R_1 - R_2} \left(R_1 - R_{N_1} \right) \tag{18}$$

and

$$N_1^1 = \int_{r_{N_1}(\alpha_1)}^{r_1(\alpha_1)} t_{\theta\theta}^1 \Big|_{\theta=\alpha_1} \mathrm{d}r = -2R_2 v_2 \eta a_1 \frac{R_1 - R_2}{\delta_1 \sin\alpha_1} \left(\frac{1}{R_1} - \frac{1}{R_{N_1}} \right) -$$

$$-\frac{\eta v_2}{R_2} c_1 \frac{2\delta_1 \sin\alpha_1}{R_1 - R_2} \left(R_1 - R_{N_1} \right) - 2k \left\{ \left(\frac{\sqrt{F_1(\alpha_1)}}{2a_1} + \varphi_1'(\alpha_1) + \frac{C_1}{2} \right) \left(R_1 - R_{N_1} \right) + \right. \tag{19}$$

$$\left. +A_1 \left[R_1 \left(\ln \left(\frac{2\delta_1 R_1 \sin\alpha_1}{R_2 (R_1 - R_2)} \right)^2 - 1 \right) - R_{N_1} \left(\ln \left(\frac{2\delta_1 R_{N_1} \sin\alpha_1}{R_2 (R_1 - R_2)} \right)^2 - 1 \right) \right] \right\} \frac{2\delta_1 \sin\alpha_1}{R_1 - R_2} .$$

In the same way we compute T_1^2, T_1^3, T_2^1, T_2^2, T_2^3, N_1^2, N_1^3, N_2^1, N_2^2 and N_2^3.

3. THE DISCONTINUITY SURFACES

Considering surfaces S_1 and S_2 as singular surfaces for the velocity and stress field, the dynamical compatibility conditions are

$$\left[v_n \right] = 0 ,$$
$$\left[t_{kl} n_k \right] = 0 \tag{20}$$

and we shall have

$$\Psi_i(r,\theta) + v_j r_j \sin\theta = C_i ,\ i = 1,2 . \tag{21}$$

We impose that the discontinuity surface S_1 passes through the point P_1 , for $\theta = \alpha_1$ and we get the equation of surface S_1

$$R_2 v_2 \left[a_1(\theta - \alpha_1) + \frac{b_1}{2}\left(\sin 2\theta - \sin 2\alpha_1 \right) + \frac{e_1}{2}\left(\cos 2\theta - \cos 2\alpha_1 \right) \right] + \tag{22}$$

$$+\frac{k}{\eta} r_1^2 \varphi_1(\theta) + v_1\left(r_1(\theta)\sin\theta - r_1(\alpha_1)\sin\alpha_1\right) = 0 .$$

We impose that the discontinuity surface S_2 passes through the point P_2 , for $\theta = \alpha_2$ and we get the equation of surface S_2

$$R_2 v_2 \left[a_2(\theta - \alpha_1) + \frac{b_2}{2}(\sin 2\theta - \sin 2\alpha_1) + \frac{e_2}{2}(\cos 2\theta - \cos 2\alpha_1) \right] + \\ + \frac{k}{\eta} r_2^2 \varphi_2(\theta) + v_2\left(r_2(\theta)\sin\theta - r_2(\alpha_1)\sin\alpha_1\right) = 0 . \tag{23}$$

On surfaces S_{N_1} and S_{N_2} we have satisfied $[v_n] = 0$.

Imposing that S_{N_1} surface passes through $N_1\left(r_{N_1}(\alpha_1), \alpha_1\right)$, we will obtain the equation of surface S_{N_1} in the form

$$R_2 v_2 \left[\Psi_{01}^0(\theta) - \Psi_{01}^0(\alpha_1) - \left(\Psi_{03}^0(\theta) - \Psi_{03}^0(\alpha_1) \right) \right] + \\ + \frac{k}{\eta} r_{N_1}^2 \left[\varphi_1(\theta) - \varphi_3(\theta) \right] = 0 . \tag{24}$$

Similar, we get the equation of surface S_{N_2}

$$R_2 v_2 \left[\Psi_{02}^0(\theta) - \Psi_{02}^0(-\alpha_2) - \left(\Psi_{03}^0(\theta) - \Psi_{03}^0(-\alpha_2) \right) \right] + \\ + \frac{k}{\eta} r_{N_2}^2 \left[\varphi_2(\theta) - \varphi_3(\theta) \right] = 0 . \tag{25}$$

4. BOUNDARY CONDITIONS

The following conditions will be used:

a) $v_\theta^i(r, \alpha_1) = 0$, $v_\theta^i(r, -\alpha_2) = 0$, $i = \overline{1,3}$;

b) the discontinuity surface S_1 passes through the point P_1' (also, S_2 passes through the point P_2');

c) we assume that $X^I = 0$, where

$$X^I = X \Big|_{\substack{r(\alpha_1) = r_1(\alpha_1) \\ r(\alpha_2) = r_1(\alpha_2)}} \quad \text{and} \quad X^{II} = -X \Big|_{\substack{r(\alpha_1) = r_{21}(\alpha_1) \\ r(\alpha_2) = r_2(\alpha_2)}}$$

d) the following friction conditions are imposed

$$t_{r\theta}^1 \Big|_{\theta=\alpha_1} = -m_1 \sqrt{\mathrm{II}_{\mathbf{t}'}^1} \Big|_{\theta=\alpha_1} , \; t_{r\theta}^3 \Big|_{\theta=\alpha_1} = m_1 \sqrt{\mathrm{II}_{\mathbf{t}'}^3} \Big|_{\theta=\alpha_1} ,$$

$t_{r\theta}^2\big|_{\theta=\alpha_1} = m_1\sqrt{\mathrm{II}_{\mathbf{t}'}^2}\big|_{\theta=\alpha_1}$, $t_{r\theta}^1\big|_{\theta=-\alpha_2} = -m_2\sqrt{\mathrm{II}_{\mathbf{t}'}^1}\big|_{\theta=-\alpha_2}$,

$t_{r\theta}^3\big|_{\theta=-\alpha_2} = -m_2\sqrt{\mathrm{II}_{\mathbf{t}'}^3}\big|_{\theta=-\alpha_2}$, $t_{r\theta}^2\big|_{\theta=-\alpha_2} = m_2\sqrt{\mathrm{II}_{\mathbf{t}'}^2}\big|_{\theta=-\alpha_2}$;

e) on surfaces S_{N_1} and S_{N_2} we have $[v_n]=0$, $X^1 = X_{N_1}^3$, $X^2 = X_{N_2}^3$;

f) $v_r^1\left(r_{N_1}(\alpha_1),\alpha_1\right) = -\omega_1\delta_1$, $v_r^1\left(r_{N_2}(-\alpha_2),-\alpha_2\right) = -\omega_2\delta_2$.

From condition a) we have $\varphi_i(\alpha_1)=0$, respectively $\varphi_i(-\alpha_2)=0$ and we obtain

$$\begin{gathered} A_i\alpha_1 + B_i\frac{1}{2}\sin 2\alpha_1 + D_i + E_i\frac{1}{2}\cos 2\alpha_1 = -\left[K_{1i}\cos 2\alpha_1 + K_{2i}\cos 2\alpha_1\right], \\ A_i\alpha_2 + B_i\frac{1}{2}\sin 2\alpha_2 - D_i - E_i\frac{1}{2}\cos 2\alpha_2 = 0,\ i=\overline{1,3}. \end{gathered} \tag{26}$$

Imposing that the discontinuity surface S_1 passes through the point P_1' $\left(r = r_1(-\alpha_2),\theta=-\alpha_2\right)$, we get

$$\begin{gathered} R_2v_2\left[-a_1(\alpha_1+\alpha_2) - \frac{b_1}{2}(\sin 2\alpha_1 + \sin 2\alpha_2) - \frac{e_1}{2}(\cos 2\alpha_1 - \cos 2\alpha_2)\right] - \\ -v_1\left[r_1(\alpha_2)\sin\alpha_2 + r_1(\alpha_1)\sin\alpha_1\right] = 0. \end{gathered} \tag{27}$$

Using the continuity equation $R_2v_2 = R_1v_1$ and

$$r_1(\alpha_2)\sin\alpha_2 + r_1(\alpha_1)\sin\alpha_1 = R_1, \tag{28}$$

we will have

$$a_1(\alpha_1+\alpha_2) + \frac{b_1}{2}(\sin 2\alpha_1 - \sin 2\alpha_2) + \frac{e_1}{2}(\cos 2\alpha_1 - \cos 2\alpha_2) = -1. \tag{29}$$

In a similar way, imposing that the discontinuity surface S_2 passes through the point P_2', we get

$$a_2(\alpha_1+\alpha_2) + \frac{b_2}{2}(\sin 2\alpha_1 + \sin 2\alpha_2) + \frac{e_2}{2}(\cos 2\alpha_1 - \cos 2\alpha_2) = -1. \tag{30}$$

From the friction condition $t_{r\theta}^1\big|_{\theta=\alpha_1} = -m_1\sqrt{\mathrm{II}_{\mathbf{t}'}^1}\big|_{\theta=\alpha_1}$, we obtain

$$b_1\sin 2\alpha_1 + e_1\cos 2\alpha_1 = m_1\sqrt{F_1(\alpha_1)}, \tag{31}$$

where

$$\sqrt{a_1^2 + b_1^2 + 2a_1b_1\cos 2\alpha_1 - 2a_1e_1\sin 2\alpha_1} = \sqrt{F_1(\alpha_1)}, \tag{32}$$

so

$$\frac{m_1^2}{1-m_1^2}=\frac{\left(b_1\sin 2\alpha_1+e_1\cos 2\alpha_1\right)^2}{\left(a_1+b_1\cos 2\alpha_1-e_1\sin 2\alpha_1\right)^2} \tag{33}$$

and with the notation

$$\frac{m_1^2}{1-m_1^2}=\gamma_1^2, \tag{34}$$

we get

$$b_1\sin 2\alpha_1+e_1\cos 2\alpha_1=-\gamma_1\left(a_1+b_1\cos 2\alpha_1-e_1\sin 2\alpha_1\right). \tag{35}$$

We also obtain that

$$\left(1-m_1^2\right)\varphi_1''\left(\alpha_1\right)-2m_1\frac{\Psi_{01}^{0\,'}\left(\alpha_1\right)}{\sqrt{F_1\left(\alpha_1\right)}}\varphi_1'\left(\alpha_1\right)=0, \tag{36}$$

with

$$\sqrt{1-m_1^2}=-\frac{\Psi_{01}^{0\,'}\left(\alpha_1\right)}{\sqrt{F_1\left(\alpha_1\right)}}. \tag{37}$$

Replacing (37) in (36), we have

$$\varphi_1''\left(\alpha_1\right)=-2\gamma_1\varphi_1'\left(\alpha_1\right). \tag{38}$$

Using the expressions of $\varphi_1'\left(\alpha_1\right)$ and $\varphi_1''\left(\alpha_1\right)$, we get

$$\begin{aligned}&2A_1\gamma_1+2B_1\left(\gamma_1\cos 2\alpha_1-\sin 2\alpha_1\right)+2E_1\left(-\gamma_1\sin 2\alpha_1-\cos 2\alpha_1\right)=\\&=-f_1\left(\alpha_1\right)+4\left[K_{11}\left(\gamma_1\sin 2\alpha_1+\cos 2\alpha_1\right)+K_{21}\left(\sin 2\alpha_1-\gamma_1\cos 2\alpha_1\right)\right].\end{aligned} \tag{39}$$

From friction condition $t_{r\theta}^3\Big|_{\theta=\alpha_1}=m_1\sqrt{\mathrm{II}_{\mathbf{t}'}^3}\Big|_{\theta=\alpha_1}$, we have

$$\begin{aligned}&2A_1\gamma_1+2B_1\left(\gamma_1\cos 2\alpha_1-\sin 2\alpha_1\right)+2E_1\left(-\gamma_1\sin 2\alpha_1-\cos 2\alpha_1\right)=\\&=b_3\sin 2\alpha_1+e_3\cos 2\alpha_1=\gamma_1\left(a_3+b_3\cos 2\alpha_1-e_3\sin 2\alpha_1\right)\end{aligned} \tag{40}$$

and

$$\varphi_3''\left(\alpha_1\right)=2\gamma_1\varphi_3'\left(\alpha_1\right). \tag{41}$$

From the expressions of $\varphi_3'\left(\alpha_1\right)$ and $\varphi_3''\left(\alpha_1\right)$ we get

$$\begin{aligned}&2A_3\gamma_1+2B_3\left(\gamma_1\cos 2\alpha_1+\sin 2\alpha_1\right)+2E_3\left(-\gamma_1\sin 2\alpha_1+\cos 2\alpha_1\right)=\\&=f_3\left(\alpha_1\right)-4\left[K_{13}\left(\cos 2\alpha_1-\gamma_1\sin 2\alpha_1\right)+K_{23}\left(\sin 2\alpha_1+\gamma_1\cos 2\alpha_1\right)\right].\end{aligned} \tag{42}$$

In a similar way, using that $t_{r\theta}^2\Big|_{\theta=\alpha_1}=m_1\sqrt{\mathrm{II}_{\mathbf{t}'}^2}\Big|_{\theta=\alpha_1}$, we obtain

$$b_2 \sin 2\alpha_1 + e_2 \cos 2\alpha_1 = \gamma_1 \left(a_2 + b_2 \cos 2\alpha_1 - e_2 \sin 2\alpha_1 \right). \tag{43}$$

So, we have

$$\begin{aligned} & b_i \sin 2\alpha_1 + e_i \cos 2\alpha_1 = \begin{pmatrix} - \\ + \\ + \end{pmatrix} \gamma_1 \left(a_i + b_i \cos 2\alpha_1 - e_i \sin 2\alpha_1 \right),\ i = \overline{1,3}, \\ & \varphi_i''(\alpha_1) = \begin{pmatrix} - \\ + \\ + \end{pmatrix} 2\gamma_1 \varphi_i'(\alpha_1),\ i = \overline{1,3} \end{aligned} \tag{44}$$

and also

$$\begin{aligned} & b_i \sin 2\alpha_2 - e_i \cos 2\alpha_2 = \begin{pmatrix} - \\ + \\ - \end{pmatrix} \gamma_2 \left(a_i + b_i \cos 2\alpha_2 + e_i \sin 2\alpha_2 \right),\ i = \overline{1,3} \\ & \varphi_i''(-\alpha_2) = \begin{pmatrix} + \\ - \\ + \end{pmatrix} 2\gamma_2 \varphi_i'(-\alpha_2),\ i = \overline{1,3}. \end{aligned} \tag{45}$$

From the condition $v_r^1\left(r_{N_1}(\alpha_1),\alpha_1\right) = -\omega_1\delta_1$, we get the equation for r_{N_1}

$$\frac{k}{\eta}\varphi_1'(\alpha_1) r_{N_1}^2(\alpha_1) + R_2 \frac{R_2 v_2}{r_{N_1}(\alpha_1)} \Psi_{01}^{0\,'}(\alpha_1) = -\omega_1\delta_1, \tag{46}$$

which means

$$\mathrm{Bg}\varphi_1'(\alpha_1) r_{N_1}^2(\alpha_1) + \frac{\omega_1\delta_1}{v_2} R_2 r_{N_1}(\alpha_1) + R_2^2 \Psi_{01}^{0\,'}(\alpha_1) = 0, \tag{47}$$

from where

$$r_{N_1}(\alpha_1) = \frac{R_2\left[-\frac{\omega_1\delta_1}{v_2} + \sqrt{\left(\frac{\omega_1\delta_1}{v_2}\right)^2 - 4\mathrm{Bg}\varphi_1'(\alpha_1)\Psi_{01}^{0\,'}(\alpha_1)} \right]}{2\mathrm{Bg}\varphi_1'(\alpha_1)}. \tag{48}$$

Similar, from condition $v_r^1\left(r_{N_2}(-\alpha_2),-\alpha_2\right) = -\omega_2\delta_2$, we get the equation for r_{N_2}

$$\mathrm{Bg}\varphi_3'(-\alpha_2) r_{N_2}^2(\alpha_2) + \frac{\omega_2\delta_2}{v_2} R_2 r_{N_2}(\alpha_2) + R_2^2 \Psi_{03}^{0\,'}(-\alpha_2) = 0, \tag{49}$$

and from (49)

$$r_{N_2}(\alpha_2)=\frac{R_2\left[-\frac{\omega_2\delta_2}{v_2}+\sqrt{\left(\frac{\omega_2\delta_2}{v_2}\right)^2-4\mathrm{Bg}\varphi_3'(\alpha_1)\Psi_{03}^{0\,'}(-\alpha_2)}\right]}{2\mathrm{Bg}\varphi_3'(-\alpha_2)}. \tag{50}$$

Using that c) $X^I=0$, we have

$$\begin{gathered}a_1\frac{\delta_1+\delta_2}{2\delta_1\delta_2}+b_1\left(\frac{\cos^2\alpha_1}{\delta_1}+\frac{\cos^2\alpha_2}{\delta_2}\right)+\\+\frac{e_1}{2}\left(\frac{\mathrm{ctg}2\alpha_1\cos 2\alpha_1}{\delta_1}-\frac{\mathrm{ctg}2\alpha_2\cos 2\alpha_2}{\delta_2}\right)+c_1\frac{R_1^2}{2R_2^2(R_1-R_2)}=0\end{gathered} \tag{51}$$

and

$$\begin{gathered}A_1\left[\frac{\delta_1}{R_2}\sin^2\alpha_1\left(1-2\ln\frac{2\delta_1R_1\sin\alpha_1}{R_2(R_1-R_2)}\right)+\frac{\delta_2}{R_2}\sin^2\alpha_2\left(1-2\ln\frac{2\delta_2R_1\sin\alpha_2}{R_2(R_1-R_2)}\right)\right]+\\+B_1\frac{R_1-R_2}{2R_2}-C_1\frac{R_1-R_2}{4R_2}+E_1\frac{1}{2}\left(\frac{\delta_1}{R_2}\sin 2\alpha_1-\frac{\delta_2}{R_2}\sin 2\alpha_2\right)-\\-\frac{1}{2}\frac{\delta_1}{R_2}\sin 2\alpha_1\mathrm{I}_{11}-\frac{1}{2a_1}\left(\frac{\delta_1}{R_2}\sin^2\alpha_1\sqrt{F_1(\alpha_1)}+\frac{\delta_2}{R_2}\sin^2\alpha_2\sqrt{F_1(-\alpha_2)}\right)+\\+2\left[\frac{\delta_1}{R_2}\sin\alpha_1\left(K_{11}\cos\alpha_1+K_{21}\sin\alpha_1\right)\right]=0.\end{gathered} \tag{52}$$

From condition e), $X^1=X^3_{N_1}$, we get

$$\begin{gathered}a_1\frac{\delta_1+\delta_2}{2\delta_1\delta_2}+b_1\left(\frac{\cos^2\alpha_1}{\delta_1}+\frac{\cos^2\alpha_2}{\delta_2}\right)+c_1\left(\frac{R_{N_1}}{R_2}\right)^2\frac{1}{2(R_1-R_2)}+\\+\frac{e_1}{2}\left(\frac{\mathrm{ctg}\alpha_1\cos 2\alpha_1}{\delta_1}-\frac{\mathrm{ctg}\alpha_2\cos 2\alpha_2}{\delta_2}\right)=\\=a_3\frac{\delta_1+\delta_2}{2\delta_1\delta_2}+b_3\left(\frac{\cos^2\alpha_1}{\delta_1}+\frac{\cos^2\alpha_2}{\delta_2}\right)+c_3\left(\frac{R_{N_1}}{R_2}\right)^2\frac{1}{2(R_1-R_2)}+\\+\frac{e_3}{2}\left(\frac{\mathrm{ctg}\alpha_1\cos 2\alpha_1}{\delta_1}-\frac{\mathrm{ctg}\alpha_2\cos 2\alpha_2}{\delta_2}\right)\end{gathered} \tag{53}$$

and

$$\begin{gathered}A_1\left[\frac{\delta_1}{R_2}\sin^2\alpha_1\left(1-2\ln\frac{2\delta_1R_{N_1}\sin\alpha_1}{R_2(R_1-R_2)}\right)+\frac{\delta_2}{R_2}\sin^2\alpha_2\left(1-2\ln\frac{2\delta_2R_{N_1}\sin\alpha_2}{R_2(R_1-R_2)}\right)\right]+\\+B_1\frac{R_1-R_2}{2R_2}-C_1\frac{R_1-R_2}{4R_2}+E_1\frac{1}{2}\left(\frac{\delta_1}{R_2}\sin 2\alpha_1+\frac{\delta_2}{R_2}\sin 2\alpha_2\right)-\end{gathered} \tag{54}$$

$$-\frac{1}{2}\frac{\delta_1}{R_2}\sin 2\alpha_1 I_{11}-\frac{1}{2a_1}\left(\frac{\delta_1}{R_2}\sin^2\alpha_1\sqrt{F_1(\alpha_1)}+\frac{\delta_2}{R_2}\sin^2\alpha_2\sqrt{F_1(-\alpha_2)}\right)+$$
$$+2\left[\frac{\delta_1}{R_2}\sin\alpha_1\left(K_{11}\cos\alpha_1+K_{21}\sin\alpha_1\right)\right]=$$
$$=A_3\left[\frac{\delta_1}{R_2}\sin^2\alpha_1\left(1-2\ln\frac{2\delta_1 R_{N_1}\sin\alpha_1}{R_2(R_1-R_2)}\right)+\frac{\delta_2}{R_2}\sin^2\alpha_2\left(1-2\ln\frac{2\delta_2 R_{N_1}\sin\alpha_1}{R_2(R_1-R_2)}\right)\right]+$$
$$+B_3\frac{R_1-R_2}{2R_2}-C_3\frac{R_1-R_2}{4R_2}+E_3\frac{1}{2}\left(\frac{\delta_1}{R_2}\sin 2\alpha_1-\frac{\delta_2}{R_2}\sin 2\alpha_2\right)-$$
$$-\frac{1}{2}\frac{\delta_1}{R_2}\sin 2\alpha_1 I_{13}-\frac{1}{2a_3}\left(\frac{\delta_1}{R_2}\sin^2\alpha_1\sqrt{F_3(\alpha_1)}+\frac{\delta_2}{R_2}\sin^2\alpha_2\sqrt{F_3(-\alpha_2)}\right)+$$
$$+2\left[\frac{\delta_1}{R_2}\sin\alpha_1\left(K_{13}\cos\alpha_1+K_{23}\sin\alpha_1\right)\right].$$

Using that $X^2=X^3_{N_2}$, we obtain

$$a_1\frac{\delta_1+\delta_2}{2\delta_1\delta_2}+b_2\left(\frac{\cos^2\alpha_1}{\delta_1}+\frac{\cos^2\alpha_2}{\delta_2}\right)+c_2\left(\frac{R_{N_2}}{R_2}\right)^2\frac{1}{2(R_1-R_2)}+$$
$$+\frac{e_2}{2}\left(\frac{\text{ctg}\alpha_1\cos 2\alpha_1}{\delta_1}-\frac{\text{ctg}\alpha_2\cos 2\alpha_2}{\delta_2}\right)=$$
$$=a_3\frac{\delta_1+\delta_2}{2\delta_1\delta_2}+b_3\left(\frac{\cos^2\alpha_1}{\delta_1}+\frac{\cos^2\alpha_2}{\delta_2}\right)+c_3\left(\frac{R_{N_2}}{R_2}\right)^2\frac{1}{2(R_1-R_2)}+ \tag{56}$$
$$+\frac{e_3}{2}\left(\frac{\text{ctg}\alpha_1\cos 2\alpha_1}{\delta_1}-\frac{\text{ctg}\alpha_2\cos 2\alpha_2}{\delta_2}\right)$$

and

$$A_2\left[\frac{\delta_1}{R_2}\sin^2\alpha_1\left(1-2\ln\frac{2\delta_1 R_{N_2}\sin\alpha_1}{R_2(R_1-R_2)}\right)+\frac{\delta_2}{R_2}\sin^2\alpha_2\left(1-2\ln\frac{2\delta_2 R_{N_2}\sin\alpha_2}{R_2(R_1-R_2)}\right)\right]+$$
$$+B_2\frac{R_1-R_2}{2R_2}-C_2\frac{R_1-R_2}{4R_2}+E_1\frac{1}{2}\left(\frac{\delta_1}{R_2}\sin 2\alpha_1-\frac{\delta_2}{R_2}\sin 2\alpha_2\right)-$$
$$-\frac{1}{2}\frac{\delta_1}{R_2}\sin 2\alpha_1 I_{12}-\frac{1}{2a_2}\left(\frac{\delta_1}{R_2}\sin^2\alpha_1\sqrt{F_2(\alpha_1)}+\frac{\delta_2}{R_2}\sin^2\alpha_2\sqrt{F_2(-\alpha_2)}\right)+ \tag{57}$$
$$+2\left[\frac{\delta_1}{R_2}\sin\alpha_1\left(K_{12}\cos\alpha_1+K_{22}\sin\alpha_1\right)\right]=$$
$$=A_3\left[\frac{\delta_1}{R_2}\sin^2\alpha_1\left(1-2\ln\frac{2\delta_1 R_{N_1}\sin\alpha_1}{R_2(R_1-R_2)}\right)+\frac{\delta_2}{R_2}\sin^2\alpha_2\left(1-2\ln\frac{2\delta_2 R_{N_1}\sin\alpha_1}{R_2(R_1-R_2)}\right)\right]+$$

$$+B_3\frac{R_1-R_2}{2R_2}-C_3\frac{R_1-R_2}{4R_2}+E_3\frac{1}{2}\left(\frac{\delta_1}{R_2}\sin 2\alpha_1-\frac{\delta_2}{R_2}\sin 2\alpha_2\right)-$$

$$-\frac{1}{2}\frac{\delta_1}{R_2}\sin 2\alpha_1 \mathrm{I}_{13}-\frac{1}{2a_3}\left(\frac{\delta_1}{R_2}\sin^2\alpha_1\sqrt{F_3(\alpha_1)}+\frac{\delta_2}{R_2}\sin^2\alpha_2\sqrt{F_3(-\alpha_2)}\right)+$$

$$+2\left[\frac{\delta_1}{R_2}\sin\alpha_1\left(K_{13}\cos\alpha_1+K_{23}\sin\alpha_1\right)\right].$$

5. THE ROLLING TORQUES, ROLLING STRESS AND ROLLING PRESSURE

We will determine the rolling torque

$$M_1=\delta_1\left(T_1^1-T_3^1-T_2^1\right), \tag{58}$$

where

$$\frac{T_1^1}{\eta v_2}=\Psi_{01}^{0\,''}(\alpha_1)\frac{R_1-R_2}{2\delta_1\sin\alpha_1}\left(\frac{R_2}{R_{N_1}}-\frac{R_2}{R_1}\right)+$$
$$+\mathrm{Bg}\left[\frac{1}{2}\frac{\Psi_{01}^{0\,''}(\alpha_1)}{\sqrt{F_1(\alpha_1)}}+\varphi_1''(\alpha_1)\right]\frac{2\delta_1\sin\alpha_1}{R_1-R_2}\left(\frac{R_1}{R_2}-\frac{R_{N_1}}{R_2}\right),$$
$$\frac{T_3^1}{\eta v_2}=\Psi_{03}^{0\,''}(\alpha_1)\frac{R_1-R_2}{2\delta_1\sin\alpha_1}\left(\frac{R_2}{R_{N_1}}-\frac{R_2}{R_1}\right)+$$
$$+\mathrm{Bg}\left[\frac{1}{2}\frac{\Psi_{03}^{0\,''}(\alpha_1)}{\sqrt{F_3(\alpha_1)}}+\varphi_3''(\alpha_1)\right]\frac{2\delta_1\sin\alpha_1}{R_1-R_2}\left(\frac{R_{N_1}}{R_2}-\frac{R_{N_2}}{R_2}\right), \tag{59}$$
$$\frac{T_2^1}{\eta v_2}=\Psi_{02}^{0\,''}(\alpha_1)\frac{R_1-R_2}{2\delta_1\sin\alpha_1}\left(1-\frac{R_2}{R_{N_2}}\right)+$$
$$+\mathrm{Bg}\left[\frac{1}{2}\frac{\Psi_{02}^{0\,''}(\alpha_1)}{\sqrt{F_2(\alpha_1)}}+\varphi_2''(\alpha_1)\right]\frac{2\delta_1\sin\alpha_1}{R_1-R_2}\left(\frac{R_{N_2}}{R_2}-1\right),$$

so

$$\frac{M_1}{\delta_1\eta v_2}=\frac{T_1^1-T_3^1-T_2^1}{\eta v_2}\,. \tag{60}$$

Also, for the rolling torque of roll 2 we have

$$M_2=\delta_2\left(T_1^2+T_3^2-T_2^2\right). \tag{61}$$

Total rolling torque will have the expression

$$M=M_1+M_2\,. \tag{62}$$

Introducing the stress

$$\sigma_{x2} = \frac{X^{II}}{R_2} \tag{63}$$

and $\sigma_y = k\sqrt{3}$, is the mean yield stress of the material we obtain the relative rolling stress the relation

$$\begin{aligned}
\frac{\sigma_{x2}}{\frac{2}{\sqrt{3}}\sigma_y} = \frac{1}{\mathrm{Bg}}\Bigg[& a_2 \frac{(\delta_1+\delta_2)(R_1-R_2)}{2\delta_1\delta_2} + b_2(R_1-R_2)\left(\frac{\cos^2\alpha_1}{\delta_1}+\frac{\cos^2\alpha_2}{\delta_2}\right)+ \\
& +\frac{e_2}{2}(R_1-R_2)\left(\frac{\operatorname{ctg}\alpha_1\cos 2\alpha_1}{\delta_1}-\frac{\operatorname{ctg}\alpha_2\cos 2\alpha_2}{\delta_2}\right)+\frac{c_2}{2}\Bigg]- \\
& -\Bigg\{A_2\frac{2}{R_1-R_2}\left[\delta_1\sin^2\alpha_1\left(1-2\ln\frac{2\delta_1\sin\alpha_1}{R_1-R_2}\right)+\right. \\
& \left.+\delta_2\sin^2\alpha_2\left(1-2\ln\frac{2\delta_2\sin\alpha_2}{R_1-R_2}\right)\right]+B_2-\frac{C_2}{2}+ \\
& +E_2\frac{1}{R_1-R_2}(\delta_1\sin 2\alpha_1-\delta_2\sin 2\alpha_2)-\frac{\delta_1}{R_1-R_2}\sin 2\alpha_1 I_{12}- \\
& -\frac{1}{a_2}\frac{1}{R_1-R_2}\left(\delta_1\sin^2\alpha_1\sqrt{F_2(\alpha_1)}+\delta_2\sin^2\alpha_2\sqrt{F_2(-\alpha_2)}\right)+ \\
& \frac{4}{R_1-R_2}\left[\delta_1\sin\alpha_1\left(K_{12}\cos\alpha_1+K_{22}\sin\alpha_1\right)\right]\Bigg\}
\end{aligned} \tag{64}$$

and the rolling pressure

$$\frac{t_{\theta\theta}^{i}}{\frac{\eta v_2}{R_2}} = \frac{2R_2^2}{r^2}a_i - c_i - \mathrm{Bg}\left(\frac{\sqrt{F_i(\theta)}}{a_i} + 4A_i\ln\frac{r}{R_2} + 2\varphi_i'(\theta) + C_i\right). \tag{65}$$

6. NUMERICAL RESULTS AND CONCLUSIONS

Were considered different cases:

A. Rolls with different diameters (geometrical asymmetry);

B. Rolls with same diameters, asymmetry given by different roll speed

Were studied different characteristics of the asymmetrical rolling process.

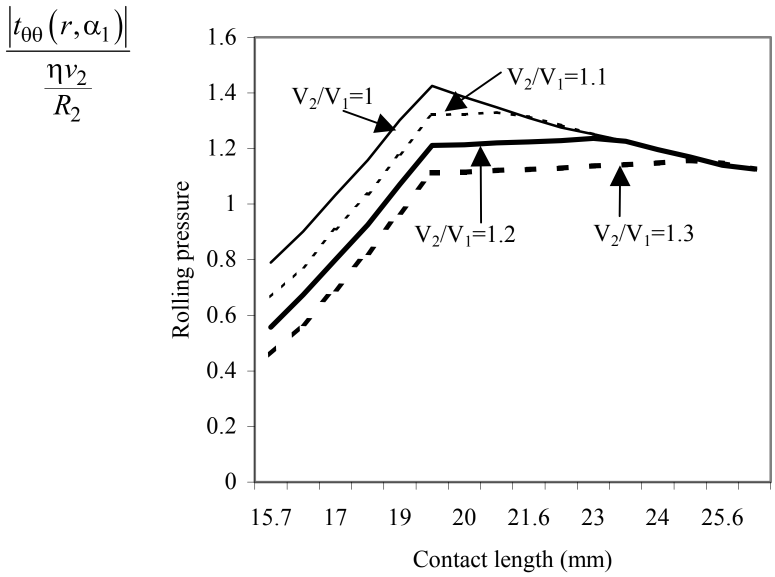

Fig. 2 – Rolling pressure for various role speed ratios when $R_1 = 2.5$ mm, $R_2 = 1.5$ mm, $\alpha_1 = 3^0$, $\alpha_2 = 2.477^0$, $\delta_1 = 100$ mm, $\delta_2 = 121$ mm, Bg = 0.5, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$.

We observe that the rolling pressure (Fig. 2 in case A and Fig. 3 in case B) increases, with the decreasing of the roll speed ratio. When the roll speed ratio is 1 the neutral points become a single one. The process is taking place in better conditions when roll speed ratio increases.

When the friction factor ratio increases, the rolling pressure increases (Fig. 4).

The rolling pressure decreases with increasing strip speed (Fig. 5).

The rolling pressure decreases with decreasing of the reduction (Fig. 6).

The back force increases with the increasing of the friction factor; for $\gamma = 0.6027$, the back force is zero, which allows us to conclude that the process is taking place only by means of the friction forces, without aplling a back force (Fig. 7).

The relative rolling stress increases with the increasing of the friction factor and decreases with the increasing of the roll speed.

The back force decreases with the increasing of the roll speed ratio (Fig. 8).

The back force vertical resultant is small and increases with the increasing of the friction factor (Fig. 9).

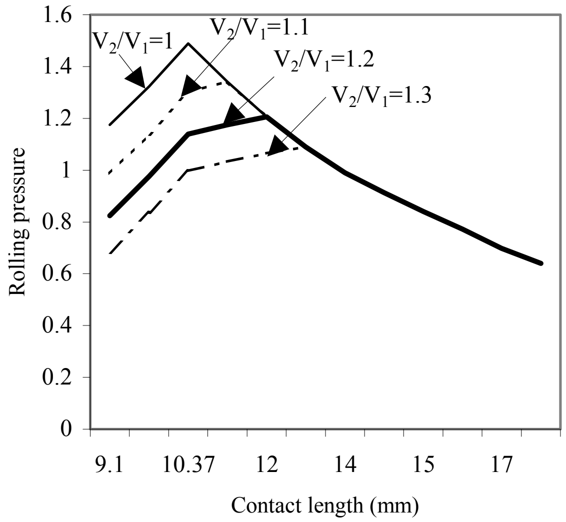

Fig. 3 – Rolling pressure for various role speed ratios when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.3, $\gamma_1 = \gamma_2 = 0.2$, $V_2 = 0.9v_2$.

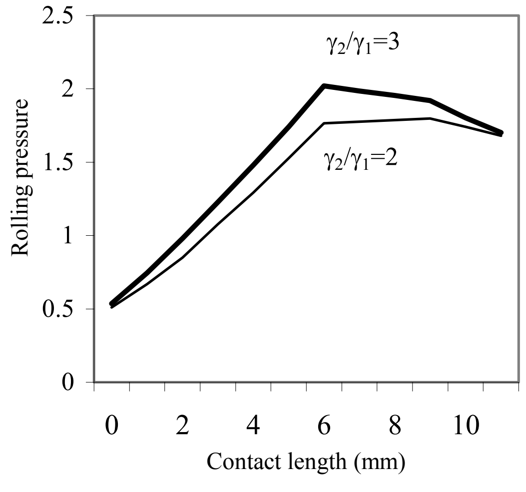

Fig. 4 – Rolling pressure for various friction factors ratios when $R_1 = 2.3$ mm, $R_2 = 1.5$ mm, $\alpha_1 = 3^0$, $\alpha_2 = 1.38^0$, $\delta_1 = 100$ mm $\delta_2 = 217$ mm, Bg = 0.8, $V_1 = 0.65v_2$, $V_2 = 0.8v_2$, $\gamma_1 = 0.1$.

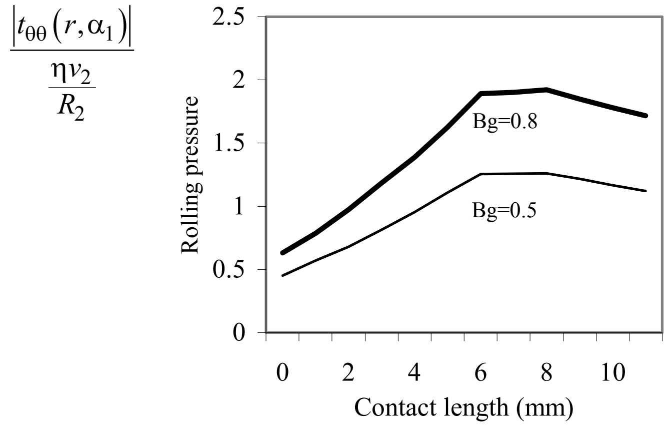

Fig. 5 – Rolling pressure for the strip speed when R_1= 2.3 mm, R_2= 1.5 mm, $\alpha_1 = 3^0$, $\alpha_2 = 1.38^0$, δ_1 = 100 mm, δ_2 = 217 mm, $V_1 = 0.7v_2$, $V_2 = 0.8v_2$, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$.

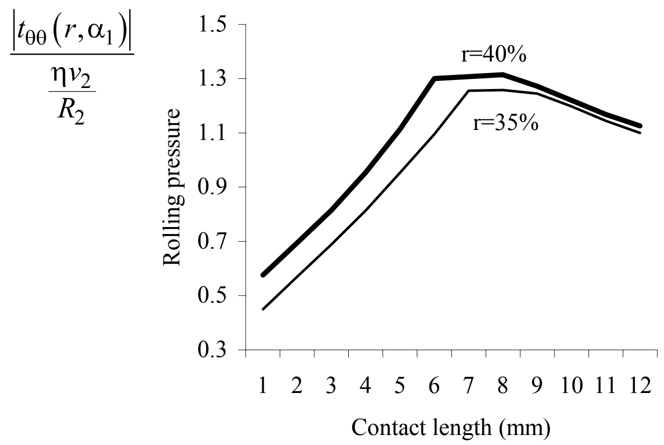

Fig. 6 – Rolling pressure for various reductions when R_2 = 1.5 mm, $\alpha_1 = 3^0$, δ_1 = 100 mm, Bg = 0.5, $V_1 = 0.7v_2$, $V_2 = 0.8v_2$, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$.

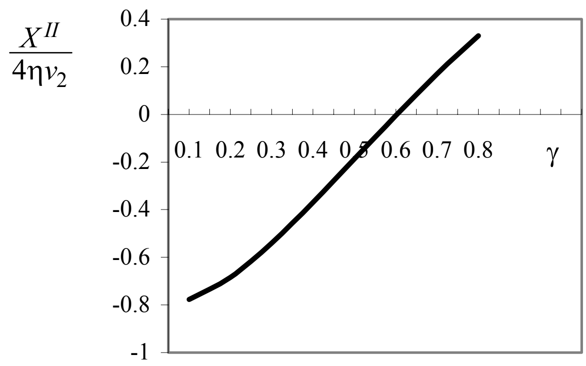

Fig. 7 – The back force for friction factor when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.3, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$.

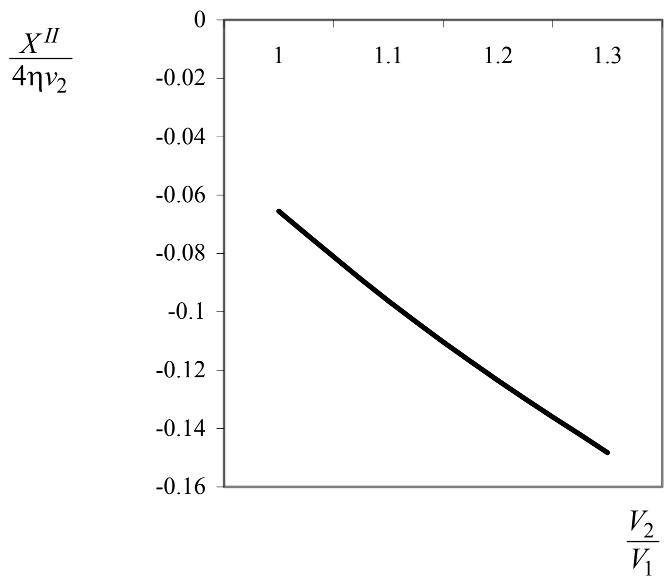

Fig. 8 – The back force for various role speed ratios when $R_1 = 2.5$ mm, $R_2 = 1.5$ mm, $\alpha_1 = 3^0$, $\alpha_2 = 2.477^0$, $\delta_1 = 100$ mm, $\delta_2 = 121$ mm, Bg = 0.5, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$.

The back force vertical resultant is small and remains constant with the increasing of the roll speed ratio (Fig. 10).

The total rolling torque M decreases with the increasing of the roll speed ratio (Fig. 11).

The total rolling torque M increases with the increasing of the friction factor (Fig. 12).

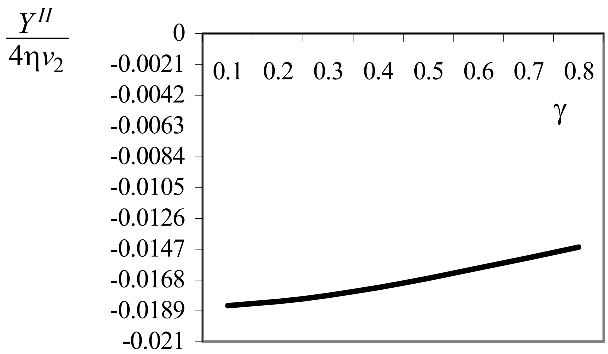

Fig. 9 – The back force vertical resultant for friction factor when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.3, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$.

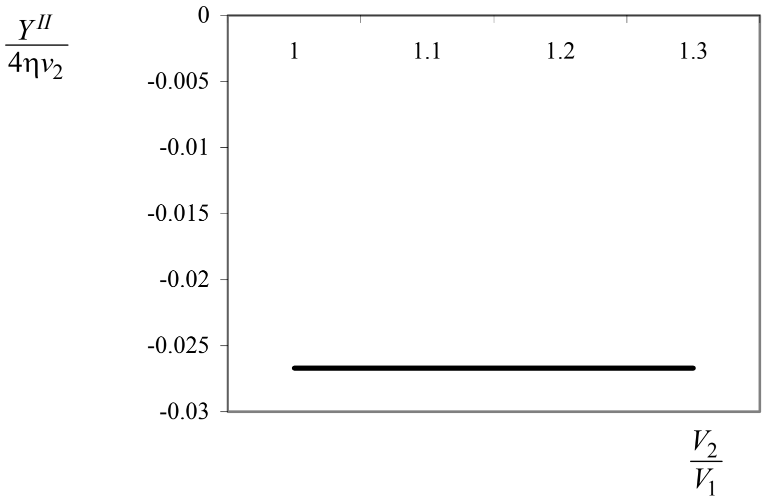

Fig. 10 – The back force vertical resultant for various roll speed ratios when $R_1 = 2.5$ mm, $R_2 = 1.5$ mm, $\alpha_1 = 3^0$, $\alpha_2 = 2.477^0$, $\delta_1 = 100$ mm, $\delta_2 = 121$ mm, Bg = 0.5, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$

The rolling pressure for the lower roll is smaller than the rolling pressure for upper roll in deformation zone *III*. In zones *I* and *II*, they are practically, equal (Fig. 13).

The neutral point for upper roll is moving towards the exit of the viscoplastic deformation zone, when the friction factor increases and is moving towards the entrance in the viscoplastic deformation zone, when the rolling speed increases (Figs. 14, 15).

The neutral point for lower roll has an opposite variation then the neutral point for upper roll.

$$M = \frac{M_1}{\delta_1} + \frac{M_2}{\delta_2}$$

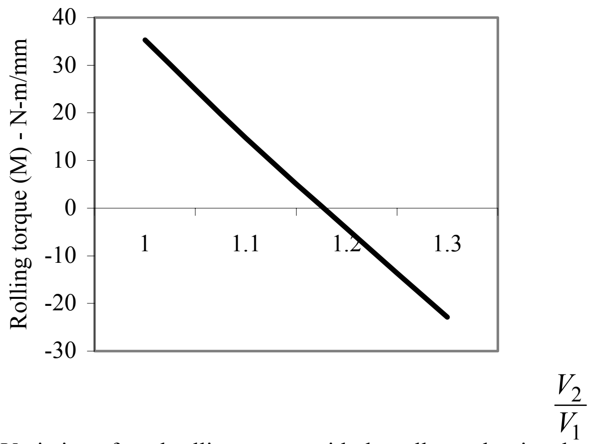

Fig. 11 – Variation of total rolling torque with the roll speed ratio when $R_1 = 2.5$ mm, $R_2 = 1.5$ mm, $\eta = 3$ Nmm^{-2}s, $k = 400$ Nmm^{-2}, $v_2 = 400$ mms^{-1}, $\alpha_1 = 3^0$, $\alpha_2 = 2.477^0$, $\delta_1 = 100$ mm, $\delta_2 = 121$ mm, Bg = 0.5, $\gamma_1 = 0.1$, $\gamma_2 = 0.2$.

$$M = \frac{M_1}{\delta_1} + \frac{M_2}{\delta_2}$$

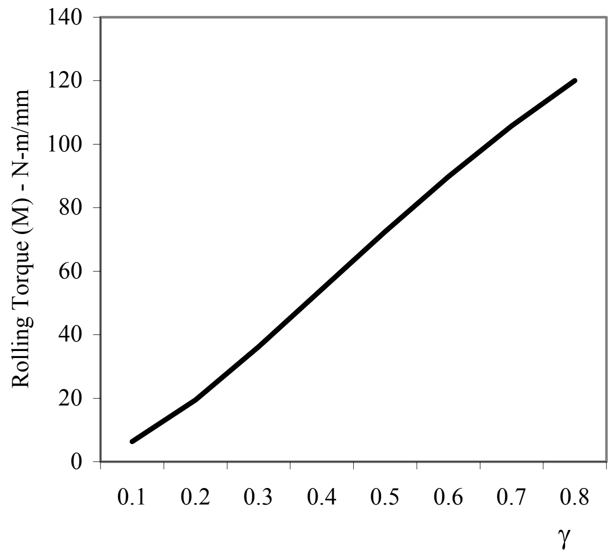

Fig. 12 – Variation of the relative rolling torque versus friction factor when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\eta = 3$ Nmm^{-2}s, $k = 400$ Nmm^{-2}, $v_2 = 250$ mms^{-1}, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.9, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$.

The results obtained with perturbation method are in good agreement with the results given in [4], where the problem was solved with slab method.

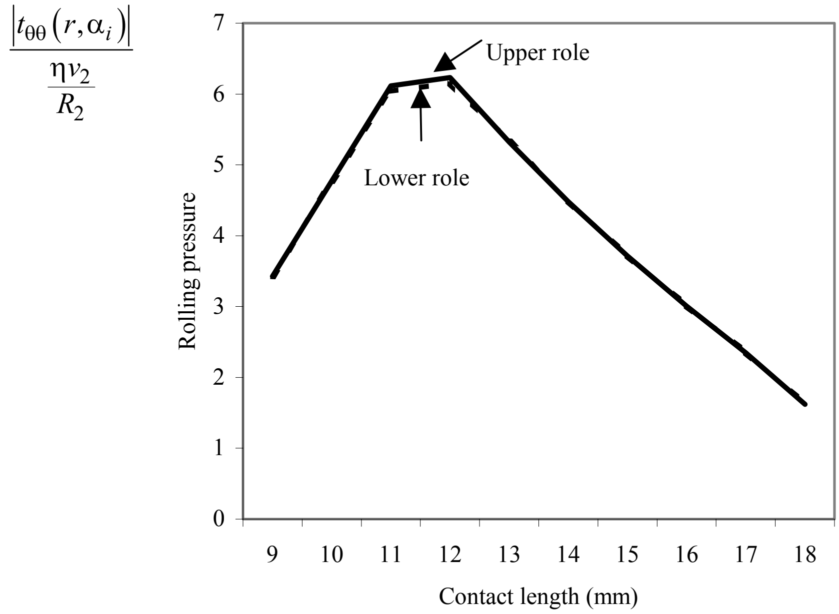

Fig. 13 – Variation of rolling pressure for upper and lower roll when $X^{II} = 0$, $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.9, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$, $\gamma_1 = \gamma_2 = 0.6027$.

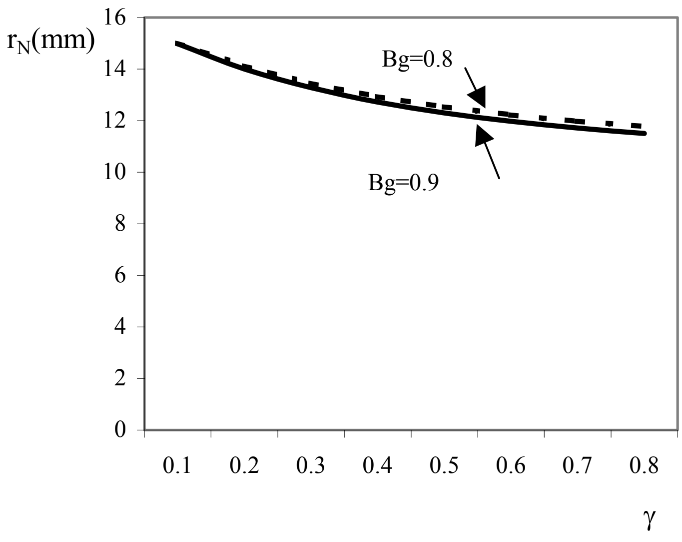

Fig. 14 – Variation of the position of the neutral point 1 with friction factor for different speed ratios when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.3, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$.

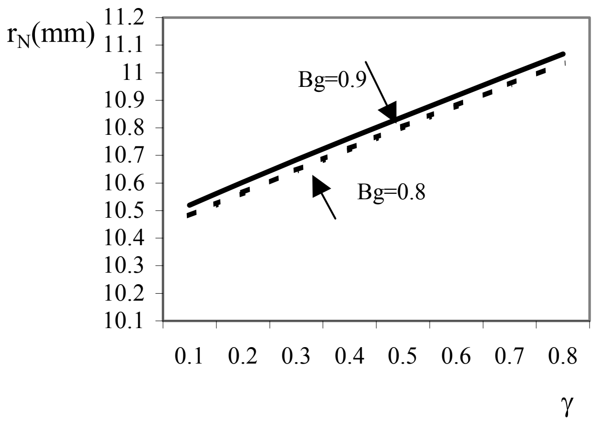

Fig. 15 – Variation of the position of the neutral point 2 with friction factor for different speed ratios when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.3, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$.

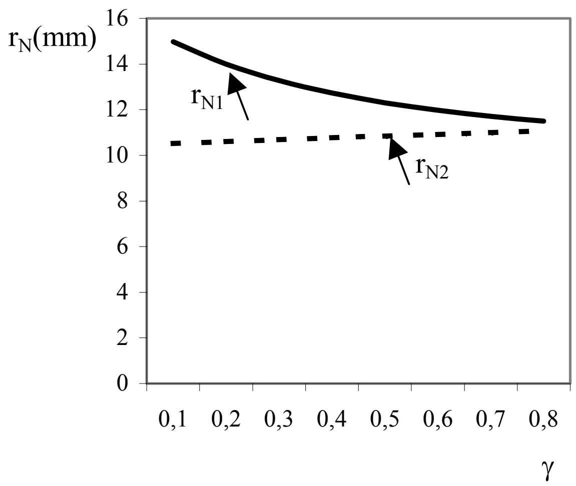

Fig. 16 – Variation of the position of the neutral points with friction factor when $R_1 = 1.5$ mm, $R_2 = 0.75$ mm, $\alpha_1 = \alpha_2 = 2.36645^0$, $\delta_1 = \delta_2 = 110$ mm, Bg = 0.3, $V_1 = 0.6v_2$, $V_2 = 0.9v_2$.

The influence of rolling speed on whole process was described using the Bingham number.

Until now, using other methods to solve the asymmetrical rolling problem, this influence cannot be described.

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