# DYNAMICS OF A 2-DOF ORIENTING GEAR MECHANISM

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Recursive matrix relations for the kinematics and the dynamics analysis of a 2-DOF orienting gear train are established in this paper. The mechanism is a parallel system with five moving links and three bevel gear pairs controlled by two electric motors. Knowing the rotation motion of the end-effector, the inverse dynamics problem is solved using the fundamental principle of virtual work. Finally, some graphs for the input angles of rotation and the powers of the actuators are obtained.

LIST OF SYMBOLS

$q_{k,k-1}$ – orthogonal relative transformation matrix

$R$ – rotation matrix of the end-effector

$\vec{u}_1,\ \vec{u}_2,\ \vec{u}_3$ – three orthogonal unit vectors

$\alpha_1,\ \alpha_2$ – orientation angles giving the orientation of the end-effector

$\varphi_{k,k-1}$ – relative rotation angle of $T_k$ rigid body

$\vec{\omega}_{k,k-1}$ – relative angular velocity of $T_k$

$\vec{\omega}_{k0}$ – absolute angular velocity of $T_k$

$\tilde{\omega}_{k,k-1}$ – skew symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$

$\vec{\varepsilon}_{k,k-1}$ – relative angular acceleration of $T_k$

$\vec{\varepsilon}_{k0}$ – absolute angular acceleration of $T_k$

$\tilde{\varepsilon}_{k,k-1}$ – skew symmetric matrix associated to the angular acceleration $\vec{\varepsilon}_{k,k-1}$

$\vec{r}_k^{\,C}$ – position vector of the mass centre of $T_k$ rigid body

$\vec{r}_{k,k-1}^{\,A}$ – relative position vector of the centre $A_k$ of joint

$\vec{v}_{k,k-1}^{\,A}$ – relative velocity of the centre $A_k$

$\vec{\gamma}_{k,k-1}^{\,A}$ – relative acceleration of the centre $A_k$

$m_k$ – mass of $T_k$ rigid body

$\hat{J}_k$ – symmetric matrix of tensor of inertia of $T_k$ about the link-frame $x_k y_k z_k$

$m_{10}^A, m_{10}^B, p_{10}^A, p_{10}^B$ – torques and powers of two actuators.

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## 1. INTRODUCTION

A robot manipulator needs at least six degrees of freedom to manipulate an object freely in space. The first three moving links are used primarily for manipulating the position, while the second mechanism is used for controlling the orientation of the end-effectors. The subassembly associated with the last moving links is called the wrist, and their joint axes are designed to intersect at a common point called the wrist centre.

The orienting mechanisms are incorporated in the structure of industrial robots and have two or three output rotations. Generally, these mechanical systems have conical and cylindrical toothed elements in their structure, while the input axes are parallel and the output axes are orthogonal. The three rotary orientation movements are usually performed around the axes of a Cartesian orthogonal frame, having its axes linked to the last arm of the robot's positioning mechanism.

The industrial robots with orienting gear trains can perform several operations such as welding, flame cutting, spray painting, milling or assembling. Being comparatively simple and compact in size, the bevel-gear wrist mechanisms can be sealed in a metallic box that keeps the device of contamination. Furthermore, using bevel gear trains for power transmission, the actuators can be mounted remotely on the forearm, thereby reducing the weight and inertia of a robot manipulator.

Planetary gear trains with three degrees of freedom are adopted as the design concept for robotic wrist (Hsieh and Sheu [1]; Paul and Stevenson [2]; Willis [3]; Ma and Gupta [4]; White [5].

## 2. INVERSE KINEMATICS MODEL

Recursive matrix relations for kinematics and dynamics of a 2-DOF orienting gear train, which has a non-symmetrical kinematical scheme, are developed in the paper. The robot wrist must rotate around two orthogonal axes and the mechanism has two degrees of freedom. A matrix methodology for the kinematics analysis based on the concept of fundamental circuit of an open-loop chain is presented. This method involves the identification of all open-loop chains and the derivation of the geometric relationships between the orientation of the end-effector and the joint angles of the chains, including the input actuator displacements ([6–8]).

Let $Ox_0y_0z_0(T_0)$ be a fixed frame, about which the mechanism moves. The mechanism topology consists of five moving links, five turning pairs and three bevel gear pairs (Fig.1). Therefore, the wrist is a 2-DOF spherical mechanism, which has a limited rotation range about the vertical joint axis.

In the wrist mechanism, the gear $1a$, of radius $r_1^A$, mass $m_1^A$ and tensor of inertia $\hat{J}_1^A$ is adjoining to the link $2a$ of $l_1$ in length, which can serve as carrier for the $2b$ - $3b$ bevel gear pair. This element includes a bevel gear of radius $r_3^A$, mass $m_3^A$ and tensor of inertia $\hat{J}_3^A$. Four gears $1a, 2a, 1b, 2b$ are sun gears while $c = 3a$ is a bevel planet gear linked at the end-effector and adjacent to carrier $2a$.

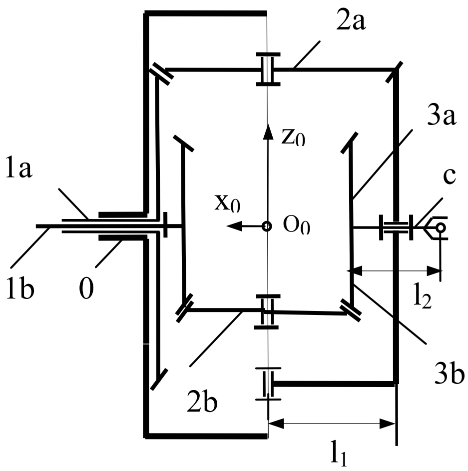


Fig. 1 – The 2-DOF orienting gear mechanism.

The gear $1b$ of radius $r_1^B$, mass $m_1^B$ and tensor of inertia $\hat{J}_1^B$ is connected to a second gear $2b$ of radius $r_3^B$, mass $m_3^B$ and tensor of inertia $\hat{J}_3^B$. Including the end-effector of length $l_2$ at the third gear $c = 3b$ of radius $r_5^B$, mass $m_5^B$, tensor of inertia $\hat{J}_5^B$, we obtain an assembly which is free to undergo arbitrary two concurrent rotations with respect to the centre $O_0$. From Fig. 1 we observe that the links $1a$ and $1b$ share one fixed common joint axis $x_0$ and that $2a$ and $2b$ share another common joint axis $z_0$.

In what follows, we introduce a matrix approach which utilizes the *theory of fundamental circuits* aroused by Tsai [6]. There exists a *real* or *fictitious* carrier for every gear pair in a planetary gear train and a fundamental matrix equation for each loop can be written as

$$q_{k+1,k-1} = q^{\varphi}_{k+1,k}\theta_y^{\delta_k} q_{k,k-1}, \quad \varphi_{k,k-1} = n_{k+1,k-1}\varphi_{k+1,k}, \quad \delta_k = \alpha_{k-1} + \alpha_{k+1}, \tag{1}$$

where $\varphi_{k,k-1}$ and $\varphi_{k+1,k}$ denote the relative angles of rotation of the carrier $T_k$ and the planet gear $T_{k+1}$, respectively, while $\alpha_{k-1}$, $\alpha_{k+1}$ are the angles of the gear $T_{k-1}$ and $T_{k+1}$. The gear ratios of a gear pair is defined as

$$n_{k+1,k-1} = r_{k+1} / r_{k-1} = z_{k+1} / z_{k-1}, \tag{2}$$

where $r_{k-1}$, $r_{k+1}$ and $z_{k-1}$, $z_{k+1}$ are the radius and the number of teeth of two gears, respectively (Fig. 2).

We consider the rotation angles $\varphi_{10}^A, \varphi_{10}^B$ of the actuators $A_1, B_1$ as variables giving the instantaneous position of the mechanism (Fig. 3). Pursuing two input-output circuits $A$ and $B$ , we obtain following successive matrices of transformation ([9, 10]):

$$a_{10} = a_{10}^{\varphi}\theta_1,\ a_{21} = a_{21}^{\varphi}\theta_3,\ a_{32} = a_{32}^{\varphi}\theta_2,\ a_{43} = a_{43}^{\varphi}\theta_1,$$

$$b_{10} = b_{10}^{\varphi}\theta_1,\ b_{21} = b_{21}^{\varphi}\theta_3, b_{32} = b_{32}^{\varphi}\theta_2\ ,\ \ b_{43} = b_{43}^{\varphi}\theta_3,\ b_{54} = b_{54}^{\varphi}\theta_2\ , \tag{3}$$

where one denoted

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},\ \theta_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix},\ \theta_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$q_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^{i} & \sin\varphi_{k,k-1}^{i} & 0 \\ -\sin\varphi_{k,k-1}^{i} & \cos\varphi_{k,k-1}^{i} & 0 \\ 0 & 0 & 1 \end{bmatrix}\quad (k = 1, 2, \dots , 5)\,, \tag{4}$$

$$q_{k0} = \prod_{s=1}^{k} q_{k-s+1,k-j}\ ,\ (q = a,\ b;\ i = A,\ B).$$

Let us suppose that the absolute motion of the platform attached at the planet gear $c = 3a = 3b$ is a rotation around the centre $O_0$. In the inverse geometric problem however, the orientation of the end-effector is known by intermediate of the two Euler angles $\alpha_1,\ \alpha_2$, which can be expressed by two analytical functions

$$\alpha_l = \alpha_l^{*}[1-\cos(\frac{\pi}{6}t)]\ \ (l = 1,\ 2)\ . \tag{5}$$

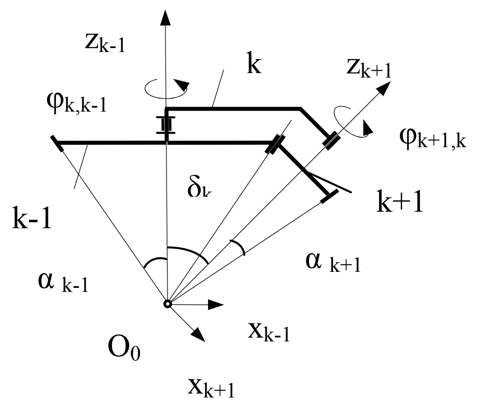


Fig. 2 – Gear fundamental circuit.

Representing the orientation of the platform in the fixed frame, the product of successive known matrices of rotation

$$R_1 = a_1\theta_3,\ R_2 = a_2\theta_1 \tag{6}$$

leads to the general rotation matrix

$$R = R_2 R_1 , \tag{7}$$

where

$$a_l = \mathrm{rot}(z, \alpha_l) = \begin{bmatrix} \cos\alpha_l & \sin\alpha_l & 0 \\ -\sin\alpha_l & \cos\alpha_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (l = 1, 2) . \tag{8}$$

The constraint geometric conditions for the rotation of the end-effector are given by the identities

$$a_{40} = b_{50} = R . \tag{9}$$

From these equations, we obtain the real-time evolution of all characteristic joint angles, as follows:

$$\begin{gathered} \varphi_{10}^A = \varphi_{21}^A = \frac{\alpha_1}{n_{31}^A} , \ \varphi_{32}^A = \alpha_1 , \ \varphi_{43}^A = \alpha_2 , \\ \varphi_{10}^B = \varphi_{21}^B = \frac{1}{n_{31}^B}(-\alpha_1 + \frac{\alpha_2}{n_{53}^B}) , \ \varphi_{32}^B = n_{31}^B \varphi_{10}^B , \ \varphi_{43}^B = \frac{\alpha_2}{n_{53}^B} , \ \varphi_{54}^B = \alpha_2 , \\ n_{31}^A = \frac{r_1^A}{r_3^A} , \ n_{31}^B = \frac{r_1^B}{r_3^b} , \ n_{53}^A = \frac{r_3^B}{r_5^B} . \end{gathered} \tag{10}$$

In the design of power transmission mechanisms, it is often necessary to analyze the velocity ratios between their input and output members and angular velocities or angular accelerations of the intermediate members.

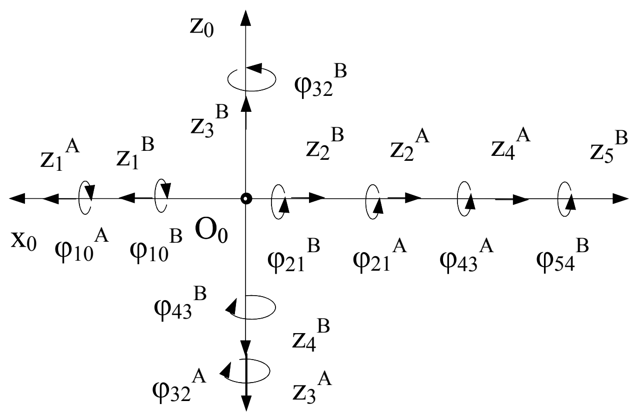


Fig. 3 – Kinematical scheme of the mechanism.

The analysis of the kinematics of bevel-gear wrist mechanisms of gyroscopic structure is very complex, due to the fact that the carriers and planet gears may possess simultaneous angular velocities about nonparallel axes. The conventional tabular or analytical method, which concentrates on planar epicyclical gear trains, is no longer applicable. To overcome this difficulty, Freudenstein, Longman and Chen [11] applied the dual relative velocity and dual matrix of transformation for the analysis of epicyclical bevel-gear trains. Tsai, Chen and Lin [12], Chang and Tsai [13] and Hedman [14] showed that the kinematical analysis of geared robotic mechanisms can be accomplished by applying the theory of fundamental circuits.

Since a kinematical chain is an assemblage of links and joints, these can be symbolized in a more abstract form known as equivalent graph representation (Fig. 4). For the reason that will be clear later we use the *associated graph* to represent the topology of the mechanism. In the kinematical graph representation we denote the links by vertices and the joints by edges (Yan and Hsieh [15, 16]). Two small concentric circles label the vertex denoting the fixed link $0$ .

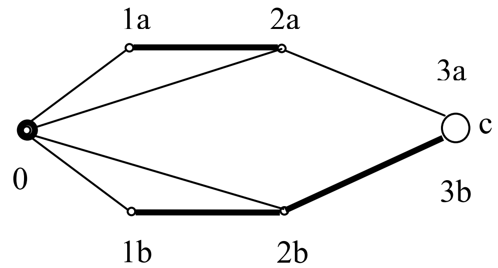


Fig. 4 – Associated graph of the mechanism.

To distinguish the difference between the pairs connections, the gear pairs $1a$ - $2a$ , $1b$ - $2b$ , $2b$ - $3b$ are designed by thick edges and revolute joints $0$ - $1a$ , $0$ - $1b$ , $2a$ - $3a$ , $0$ - $1b$ , $0$ - $2b$ by thin edges. Four edged paths, which start from the base link $0$ and end at the end-effector link $3a$ or $3b$ consist of vertices $1a, 2a, 3a$ and $1b, 2b, 3b$. There are three independent loops, three fundamental circuits and we identify three *fictitious* carriers.

The kinematics of an element for each circuit is characterized by skew-symmetric matrices given by the recursive relations [17]:

$$\tilde{\omega}^i_{k0} = q_{k,k-1}\tilde{\omega}^i_{k-1,0}q^{\mathrm{T}}_{k,k-1} + \tilde{\omega}^i_{k,k-1} \,,\ \tilde{\omega}^i_{k,k-1} = \dot{\varphi}^i_{k,k-1}\tilde{u}_3 \,, \tag{11}$$

where $\tilde{u}_3$ is a skew-symmetric matrix associated with the unit vector $\vec{u}_3$. These matrices are *associated* to the angular velocities

$$\vec{\omega}^i_{k0} = q_{k,k-1}\vec{\omega}^i_{k-1,0} + \vec{\omega}^i_{k,k-1},\ \omega^i_{k,k-1} = \dot{\varphi}^i_{k,k-1}\vec{u}_3 \,. \tag{12}$$

Knowing the rotation motion of the platform by the relations (5), one develops the inverse kinematical problem and determines the velocities $\vec{v}^i_{k0}$, $\vec{\omega}^i_{k0}$ and accelerations $\vec{\gamma}^i_{k0}, \vec{\varepsilon}^i_{k0}$ of each of the moving links.

Based on the important remark

$$\omega_{k,k-1} = n_{k+1,k-1}\omega_{k+1,k} \,, \tag{13}$$

the derivatives with respect to time of the relations (10) lead to the relative angular velocities of all links as function of the angular velocities $\dot{\alpha}_1$, $\dot{\alpha}_2$ of the end-effector:

$$\omega_{10}^A = \omega_{21}^A = \frac{\dot{\alpha}_1}{n_{31}^A},\ \omega_{32}^A = \dot{\alpha}_1,\ \omega_{43}^A = \dot{\alpha}_2,$$

$$\omega_{10}^B = \omega_{21}^B = \frac{1}{n_{31}^B}(-\dot{\alpha}_1 + \frac{\dot{\alpha}_2}{n_{53}^B}),\ \omega_{32}^B = n_{31}^B \omega_{10}^B,\ \omega_{43}^B = \frac{\dot{\alpha}_2}{n_{53}^B},\ \omega_{54}^B = \dot{\alpha}_2. \tag{14}$$

Starting from these results, a complete expression of the Jacobian of the mechanism is easily written in an invariant form. This square invertible matrix is an essential element for the analysis of singularity loci into robot workspace.

Let us assume now that the mechanism has successively two independent virtual motions. Characteristic virtual velocities expressed as function of robot's position are given by the relations (14). First, we consider following input angular velocities $\omega_{10a}^{Av} = 1$, $\omega_{10a}^{Bv} = 0$ and we obtain the set of virtual velocities:

$$\begin{gathered} \omega_{21a}^{Av} = 1,\ \omega_{32a}^{Av} = n_{31}^A,\ \omega_{43a}^{Av} = n_{31}^A n_{53}^B, \\ \omega_{21a}^{Bv} = 0,\ \omega_{32a}^{Bv} = 0,\ \omega_{43a}^{Bv} = n_{31}^A,\ \omega_{54a}^{Bv} = n_{31}^A n_{53}^B, \\ \omega_{1a}^{Cv} = n_{31}^A,\ \omega_{2a}^{Cv} = n_{31}^A n_{53}^B. \end{gathered} \tag{15}$$

A second virtual motion is defined by the input velocities $\omega_{10b}^{Bv} = 1$, $\omega_{10b}^{Av} = 0$ and the following results:

$$\begin{gathered} \omega_{21b}^{Bv} = 1,\ \omega_{32b}^{Bv} = n_{31}^B,\ \omega_{43b}^{Bv} = n_{31}^B, \omega_{54b}^{Bv} = n_{31}^B n_{53}^B, \\ \omega_{21b}^{Av} = 0,\ \omega_{32a}^{Av} = 0,\ \omega_{43b}^{Av} = n_{31}^B n_{53}^B, \\ \omega_{1b}^{Cv} = 0,\ \omega_{2b}^{Cv} = n_{31}^B n_{53}^B. \end{gathered} \tag{16}$$

Concerning the relative angular accelerations of the compounding elements of the mechanism, these are immediately given by deriving the relations on the velocities (14): $\varepsilon_{k,k-1}^i = \dot{\omega}_{k,k-1}^i$.

The angular accelerations $\vec{\varepsilon}_{k0}^i$ and the useful square matrices $\tilde{\omega}_{k0}^i \tilde{\omega}_{k0}^i + \tilde{\varepsilon}_{k0}^i$ are calculated with the following recursive formulae [18]:

$$\begin{gathered} \vec{\varepsilon}_{k0}^i = q_{k,k-1} \vec{\varepsilon}_{k-1,0}^i + \varepsilon_{k,k-1}^i \vec{u}_3 + \omega_{k,k-1}^i q_{k,k-1} \tilde{\omega}_{k-1,0}^i q_{k,k-1}^T \vec{u}_3, \\ \tilde{\omega}_{k0}^i \tilde{\omega}_{k0}^i + \tilde{\varepsilon}_{k0}^i = q_{k,k-1} \left( \tilde{\omega}_{k-1,0}^i \tilde{\omega}_{k-1,0}^i + \tilde{\varepsilon}_{k-1,0}^i \right) q_{k,k-1}^T + \\ + \omega_{k,k-1}^i \omega_{k,k-1}^i \tilde{u}_3 \tilde{u}_3 + \varepsilon_{k,k-1}^i \tilde{u}_3 + 2\omega_{k,k-1}^i q_{k,k-1} \tilde{\omega}_{k-1,0}^i q_{k,k-1}^T \tilde{u}_3. \end{gathered} \tag{17}$$

The velocity $\vec{v}_k^{Ci}$ and the acceleration $\vec{\gamma}_k^{Ci}$ of mass centre of $T_k^i$ rigid body are calculated from two basic matrix relations

$$\vec{v}_k^{Ci} = \tilde{\omega}_{k0}^i \vec{r}_k^{Ci},\ \vec{\gamma}_k^{Ci} = \{\tilde{\omega}_{k0}^i \tilde{\omega}_{k0}^i + \tilde{\varepsilon}_{k0}^i\} \vec{r}_k^{Ci}. \tag{18}$$

For simulation purposes let us consider a mechanism which has the following characteristics

$$l_1 = 0.03\,\text{m},\ l_2 = 0.055\,\text{m},$$

$$r_1^A = 0.025\,\text{m},\ r_3^A = 0.04\,\text{m},\ r_1^B = 0.02\,\text{m},\ r_3^B = 0.035\,\text{m},\ r_5^B = 0.015\,\text{m},$$
$$m_1^A = 0.25\,\text{kg},\ m_3^A = 0.4\,\text{kg},\ m_4^A = 1\,\text{kg},\ m_1^B = 0.2\,\text{kg},\ m_3^B = 0.35\,\text{kg},\ m_5^B = 1\,\text{kg},$$
$$\alpha_1^* = \frac{\pi}{4},\ \alpha_2^* = \pi\,,\ m_r = -0.5\,\text{Nm},\ \Delta t = 6\,\text{s}. \tag{19}$$

A program which implements the suggested algorithm is developed in MATLAB to solve first the inverse kinematics of the orienting gear train. For illustration, it is assumed that for a period of six second the end-effector starts at rest from its initial position and is moving in a known rotation motion. A numerical study of the robot kinematics is carried out by computation of the input angles of rotation $\varphi_{10}^A$, $\varphi_{10}^B$, for example, of two revolute actuators (Fig. 5).

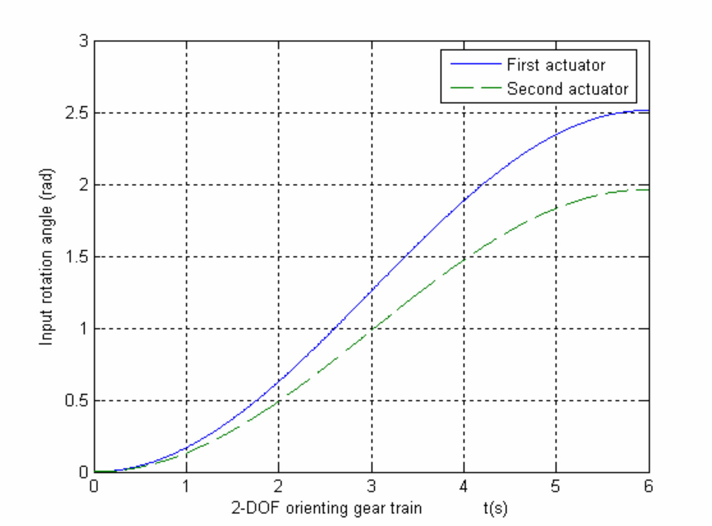


Fig. 5 – Input rotation angles $\varphi_{10}^A$, $\varphi_{10}^B$ of two actuators.

### 3. DYNAMICS SIMULATION

Two torques of moment $\vec{m}_{10}^A = m_{10}^A \vec{u}_3$, $\vec{m}_{10}^B = m_{10}^B \vec{u}_3$ control by intermediate of electric motors the motion of the orienting gear train. The derivation of a dynamic model has a very important effect in the determination of the actuator torques.

In the inverse dynamic problem, in the present paper one applies the principle of virtual work in order to establish some recursive matrix relations for the torques and the powers of the active systems.

The parallel mechanism can artificially be transformed in a set of three open serial chains $C_j$ $(j = A, B, C)$ subject to the constraints. This is possible by cutting successively the joints for the end-effector and taking their effects into account by introducing the corresponding constraint conditions.

Considering that the mobile platform motion is given, the position, angular velocity, angular acceleration as well as the velocity and acceleration of the centre of mass are known of each element. The force of inertia of an arbitrary rigid body $T_k^A$, for example

$$\vec{f}_{k0}^{\,inA} = -m_k^A \left[ \vec{\gamma}_{k0}^A + \left( \tilde{\omega}_{k0}^A \tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A \right) \vec{r}_k^{\,CA} \right] \tag{20}$$

and the resulting moment of the forces of inertia

$$\vec{m}_{k0}^{\,inA} = -[ m_k^A \tilde{r}_k^{CA} \vec{\gamma}_{k0}^A + \hat{J}_k^A \vec{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A \hat{J}_k^A \vec{\omega}_{k0}^A ] \tag{21}$$

are determined with respect to the common centre of rotation $O_0$. On the other hand, the wrench of two vectors $\vec{f}_k^{\,*A}$ and $\vec{m}_k^{\,*A}$ evaluates the influence of the action of the external and internal forces applied to the same element $T_k^A$ or of its weight $m_k^A \vec{g}$, for example:

$$\vec{f}_k^{\,*A} = 9.81 m_k^A a_{k0} \vec{u}_3 ,\ \vec{m}_k^{\,*A} = 9.81 m_k^A \tilde{r}_k^{CA} a_{k0} \vec{u}_3 \ \ (k = 1,\ 2,\ ...,\ 5)\,. \tag{22}$$

Finally, two recursive relations generate the vectors

$$\begin{aligned} \vec{F}_k^A &= \vec{F}_{k0}^A + a_{k+1,k}^T \vec{F}_{k+1}^A , \\ \vec{M}_k^A &= \vec{M}_{k0}^A + a_{k+1,k}^T \vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T \vec{F}_{k+1}^A , \end{aligned} \tag{23}$$

where one denoted

$$\vec{F}_{k0}^A = -\vec{f}_{k0}^{\,inA} - \vec{f}_k^{\,*A} ,\ \vec{M}_{k0}^A = -\vec{m}_{k0}^{\,inA} - \vec{m}_k^{\,*A} \,. \tag{24}$$

In the context of the real-time control, neglecting the frictional forces and considering the gravitational effect, the relevant objective of a dynamic model is to determine the input torques, which must be exerted by the actuators in order to produce a given trajectory of the end-effector.

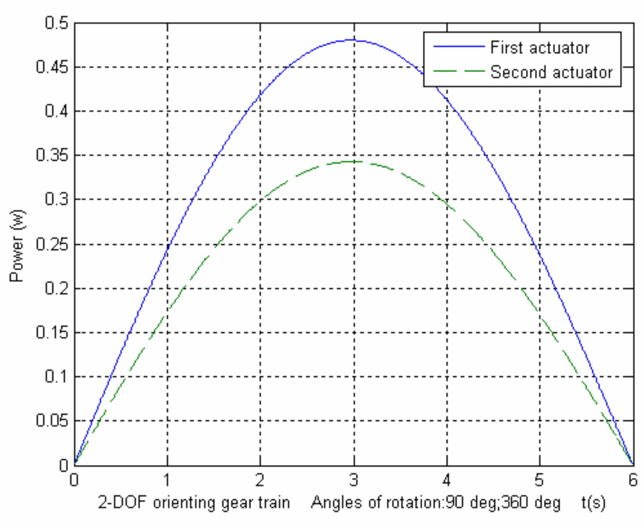


Fig. 6 – Input powers $p_{10}^A$, $p_{10}^B$ of two actuators.

The fundamental principle states that a mechanism is under dynamic equilibrium if and only if the virtual power developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Applying *the fundamental equations of parallel robots dynamics* obtained in a compact form by Stefan Staicu [19], the following matrix relations results

$$\begin{aligned} m_{10}^{A} = \vec{u}_3^{T} \Big[ & \vec{M}_1^{A} + \omega_{21a}^{Av} \vec{M}_2^{A} + \omega_{32a}^{Av} \vec{M}_3^{A} + \\ & + \omega_{21a}^{Bv} \vec{M}_2^{B} + \omega_{32a}^{Bv} \vec{M}_3^{B} + \omega_{1a}^{Cv} \vec{M}_1^{C} + \omega_{2a}^{Cv} \vec{M}_2^{C} \Big] \end{aligned} \tag{25}$$

for the torque of first actuator, and

$$\begin{aligned} m_{10}^{B} = \vec{u}_3^{T} \Big[ & \vec{M}_1^{B} + \omega_{21b}^{Bv} \vec{M}_2^{B} + \omega_{32b}^{Bv} \vec{M}_3^{B} + \\ & + \omega_{21b}^{Av} \vec{M}_2^{A} + \omega_{32b}^{Av} \vec{M}_3^{A} + \omega_{1b}^{Cv} \vec{M}_1^{C} + \omega_{2b}^{Cv} \vec{M}_2^{C} \Big] \end{aligned} \tag{26}$$

for the torque of second actuator. The relations (23), (25) and (26) represent the *inverse dynamic model* of the 2-DOF orienting gear train. The procedure leads to very good estimates of the actuators torques for given displacement of end-effector, provided that the inertial properties of the gears are known with sufficient accuracy and that friction is not significant. This new dynamic approach developed here can be extended to any gyroscopic bevel-gear train with revolute actuators.

Based on the algorithm derived from the above recursive relations, a computer program solve the inverse dynamics modelling of the robot, using the MATLAB software. Assuming that a resistant torque of constant moment $m_r = -0.5$ Nm applied at the end-effector and the weights $m_k^A \vec{g}$ of compounding rigid bodies constitute the external forces acting on the mechanism during its evolution, a numerical computation in the dynamics is developed, based on the determination of the two input powers $p_{10}^A = \omega_{10}^A m_{10}^A$ and $p_{10}^B = \omega_{10}^B m_{10}^B$ (Fig. 6). The time-history evolution of the powers required by two active systems are shown for a period of six second of motion.

## 4. CONCLUSIONS

Within the inverse kinematics analysis, some exact matrix relations giving the position, velocity and acceleration of each link for a 2-DOF orienting gear train have been established.

Based on the principle of virtual work, the new approach described above is very efficient and establishes a direct recursive determination of the variation in real-time of torques and powers of the actuators. The matrix relations, given by this dynamic simulation, can be transformed in a model for automatic command of the gear mechanism.

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