

# MATHEMATICAL MODELLING OF THE ULTRASONIC NON-DESTRUCTIVE CONTROL FOR THE COMPOSITE MATERIALS

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This article presents the realization of a specialized software application for ultrasonic non-destructive testing of heterogeneous composite materials. Starting with the propagation of plane ultrasonic waves inside an infinite length layer and by using the Debye's series developing formalism it will determine the conception of a mathematical model adequate to the proposed purpose. The use of this software for an adapted programming language C++Builder5, it will facilitate the further manipulation of the less skilled computer users, the created interface being easy to handle. The results obtained in such way were compared to those obtained through experiments, having as consequence that the mathematical model corresponds to the physical reality and can be successfully used in the determination of mechanical properties or of the integrity state's of a heterogeneous composite material.

## 1. INTRODUCTION

The reflection and the transmission of the homogenous plane waves in an isotropic material layer with parallel faces is a subject of a lot of studies in ultrasonic field.

Primary, the calculation of the transmission and reflection coefficients were related to internal reflections and was elaborated by Schoch. Other authors extended the studies for a viscous layer (Conoir, 1987) and after that for an absorbent solid layer (Forito and others, 1985).

Experimental attempt to validate suggested mathematical models were performed by Billy and Quentin in 1984.

A lot of studies tried then to bring much more adaptability of the mathematical model to the real situation and the reflection and transmission coefficients will be rewrite in particular conditions of one of the ultrasonic characteristic, the resonance. Thus, Fiorito researches successively the case of a perfect fluid layer or elastic solid (1979), then viscous (1981), immersed in a perfect fluid and also the case of an absorbent layer (1985) in the same immersed

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conditions and with the reflection and refraction coefficients being calculated in resonance conditions.

On the other hand, the majority of the calculated models suggested the incident waves to have a pure homogenous nature, which obviously doesn't correspond to the physical reality, especially when the wired fluid is viscous. In this particular case, the plane heterogeneous wave will be present in both conduction mediums either in the wired fluid or in the solid layer. In the specialized literature we find works which treat the reflection and transmission of the heterogeneous waves through a solid simple interface (Deschamps and Hosten, 1989) and through a fluid layer (Fisk, 1979).

Much closer to the physical reality and to the proposed model of this article is the study of Deschamps and Chevee (1992) which deal with the case of an isotropic solid layer, viscoelastic, immersed in a viscous fluid and therefore submitted to the activity of some of the heterogeneous plane waves.

The structure of a composite material is mainly heterogeneous, because it is composed from a fiber texture more or less compact, included in a matrix, usually epoxidical resin. From the specialized literature and taking into consideration the working frequencies situated somewhere in the interval between 1-6 MHz, it may be assimilated approximately and with a good reflection in the reality, all this structure as being homogenous. Instead of it, the presence of the fibers makes the composite material to have a strong anisotropic character.

The exact problem to be discussed further it is to study the reflection and the transmission of the heterogeneous plane waves in an anisotropic and homogenous composite material layer.

## 2. THE METHOD

In physical sense, the aim pursued was to model an incident wave with known characteristics which is propagate under a special angle in a composite material layer immersed in a fluid medium.

**The isotropic case.** It will be treated further on the simplified case of the reflection and the refraction of a heterogeneous plane wave with unit amplitude at the origin, in an absorbent isotropic presumed layer, immersed in a fluid with knowing acoustic properties.

The field of the acoustic displacement in the interior of the layer, in stationary regime, will be given by the formula:

$$\underline{U}(M, \omega, t) = \Re \left\{ \sum_{p=1,2} \sum_{m=L,T} {}^*X_{pm} {}^*P_{pm} \exp i(\omega t - {}^*K_{pm} M) \right\}, \quad (1)$$

where:  ${}^*X_{pm}$  the wave amplitude, the sign “\*” denote the fact that the value may be complex;  ${}^*P_{pm}$  the wave polarization vector;  ${}^*K_{pm}$  wave vector;  $m = L, T$  the type of the propagated wave ( $L$  – longitudinal,  $T$  – transversal);  $p = 1, 2$  the interface where the waves diffraction take place (1 – superior, 2 – inferior).

In the immersion fluid the field of the analogical acoustic displacement will be given by the relations:

$$\underline{U}_1(M, \omega, t) = \Re \left\{ {}^*P_1 \exp i(\omega t - {}^*K_1 M) + {}^*X_{1R} {}^*P_{1R} \exp i(\omega t + {}^*K_{1R} M) \right\}, \quad (2)$$

$$\underline{U}_2(M, \omega, t) = \Re \left\{ {}^*X_{2R} {}^*P_{2R} \exp i(\omega t - {}^*K_{2R} M) \right\} [0, 1]. \quad (3)$$

For estimating the global reflection/refraction coefficients of the layer, *i.e.* for finding the amplitude expression of all the excited waves in a solid and fluid layer, it will be considered that each interface of the layer is an acoustic diopter between two semi-infinite mediums, situated on the distance of  $\pm d/2$  on the median plan of it (Deschamps and Chengwei, 1991). The problem will be reduced then to calculate three amplitudes corresponding to five elementary possible cases, considered separately.

Each of this case it is a classical problem for four waves’ propagation and the solution is discussed in specialized literature (Brekhoskikh, 1960). The obtained solutions should take into consideration the fact that the interface on which the waves’ reflection/refraction occurred, it is situated at the distance  $\pm d/2$  of the centered mark in the median plan of the layer.

Thus, if  ${}^*X_{ps}^0$  ( $p, s$  – conversion coefficient of the incidental type  $p$  wave in the type  $s$  transmitted/reflected wave) is the amplitude of the evaluated wave, placing the origin of the cartesian mark on the interface, the expression of this amplitude on the distance  $z$  will be given with the relation:

$${}^*X_{ps} = {}^*X_{ps}^0 \exp(-i {}^*\phi_m), \quad (4)$$

where:  ${}^*\phi_m = {}^*K_m z \cos {}^*\theta_m$ .

The exponential  $\exp(-i {}^*\phi_m)$  it is called phase’s factor, and  $\phi_m$  it is called phase difference.

Further on, will be define three vectors composed from reflection and refraction coefficients of the layer, as follows:

$${}^*\underline{X}_0 = \left\{ {}^*R_R, {}^*R_L, {}^*R_T \right\}^T, \quad (5)$$

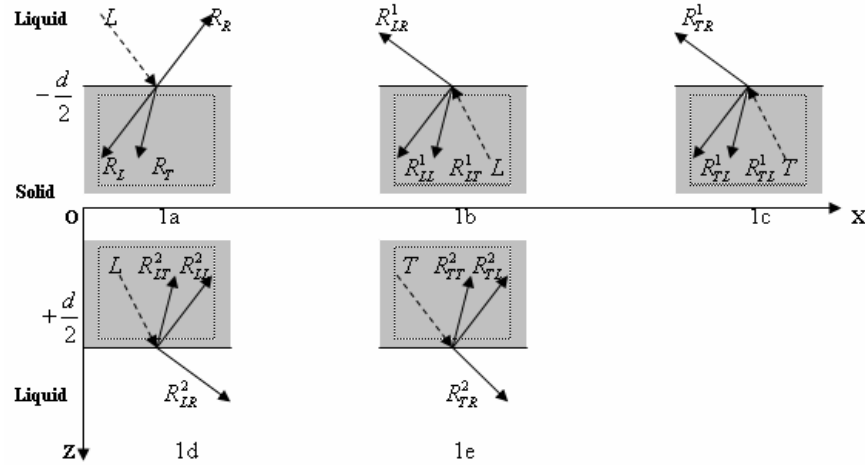


Fig. 1 – Propagation in a semi-infinite medium (5 possible cases). There are represented the incipient wave with a pointed line, and the reflected and/or transmitted wave with a continuous line.

$${}^* \underline{X}_p = \left\{ {}^* X_{pR}, {}^* X_{pL}, {}^* X_{pT} \right\}^T, \quad (6)$$

where the components  ${}^* R_m$  ( $m = R, L, T$ ), of the  $\underline{X}_0$  vector from the 5-th relation, are respectively,  $R$  reflection coefficient,  $L$  longitudinal transmission and  $T$  transversal transmission on the separation (dipoter) interface, fluid-solid, for an incidental wave in fluid.

In stead of it, vector's components  ${}^* \underline{X}_p$ , especially the quantities  ${}^* X_{pm}$ , with ( $p = 1, 2$ ) and ( $m = L, T$ ) are relative to excited waves in solid, at the interface of the separation  $p$ .

For the case when is considered an incidental (pointed) wave which propagate in fluid in the sense of the  $OZ$  axe, the components of the vector  $X_0$  will be obtained (Fig. 1a).

For the rest of the cases, when the situation of some longitudinal and transversal waves incident on the 1 interface it is analyzed in turn, coming from the solid (the cases 1b and 1c), then of the incidence of the 2 surface of the same types of waves coming from the solid in the increasing sense of the  $O_z$  axe, it will result the components of four reflection/refraction vectors on each interface. For each of the evaluated cases the amplitude of the incidental wave's it is considered known and equal with the unity.

In their turn, these one admit the definition of two matrixes of reflection/refraction on each interface, considered in the following manner:

$$[R_i] = \begin{bmatrix} 0 & R_{LR}^i & R_{TR}^i \\ 0 & R_{LL}^i & R_{TL}^i \\ 0 & R_{LT}^i & R_{TT}^i \end{bmatrix}, \quad i = 1, 2. \quad (7)$$

Without relate in details of the calculus, the totality of the reflection/refraction coefficients presented in the above expression, is determined on writing the continuity equation in a point, on the considered interface (Brekhovkikh, 1960).

It will be noted with  $[T]$  and it is called double reflection matrix in the interior of the solid layer, the matrix:

$$[T] = [R_1][R_2]. \quad (8)$$

This one will have the form:

$$[T] = \begin{bmatrix} 0 & T_{LR} & T_{TR} \\ 0 & T_{LL} & T_{TL} \\ 0 & T_{LT} & T_{TT} \end{bmatrix}, \quad (9)$$

where the 6 non-zero coefficients of the double reflection matrix  $[T]$ , are given by the formula:

$$T_{ps} = R_{pL}^1 R_{Ls}^2 + R_{pT}^1 R_{Ts}^2. \quad (10)$$

This double reflection matrix is the geometrics series ratio, named Debye's series.

$$\underline{X}_1^{(n)} = ([1] + [T] + [T]^2 + [T]^3 + \dots + [T]^n) \underline{X}_0. \quad (11)$$

The coefficients' vector of the second interface it is obtained immediately with the relation:

$$\underline{X}_2^{(n)} = [R_2] \underline{X}_1^{(n)}. \quad (12)$$

The vector which gives reflection and transmission coefficients will be obtained taking into consideration an infinity of the successively reflections in the interior of the considered solid layer, *i.e.* it is the limit of the anterior series when  $(n) -$  is the number of the successively reflections, augments to the infinity.

We will have:

$$\underline{X}_1 = ([1] - [T])^{-1} \underline{X}_0, \quad (13)$$

$$\underline{X}_2 = [R_2] \underline{X}_1. \quad (14)$$

It is important to emphasize that all of these calculated coefficients are related to propagation of the waves through a single acoustic diopter which separate the two mediums considered half-infinites, fluid-solid, though they define the propagation of the acoustic waves in the interior of the layer with two separated interfaces fluid-solid.

### 3. THE RESULTS

For justification the use of the Debye's series, it will be evaluate in a transitory regime the response of an aluminum layer to the propagation of a homogenous plane wave.

In a classical mode, the evaluation of this response it is done by resolving a time convolution equation. The acoustical field resulted after scanning the solid layer it will be obtained with the Fourier integral in the frequency:

$$A(x, z, t, \theta) = \int_{-\infty}^{+\infty} E(f)H(f, \theta, z) \exp(i2\pi ft) df, \quad (15)$$

where  $E(f)$  is the Fourier's transformed data of the references sign and  $H(f, \theta, z)$  is the function of transfer of the solid layer immersed in fluid (in this particular case the coefficient of transmission of the layer, *i.e.*  $X_{2R}$ ,  $X_{2L}$  and  $X_{2T}$ ).

For simulating experimental conditions, the reference sign emitted by the palpate will be modulate being the multiplication between a function having a frequency  $f_0$  and the Gauss's bell with the opening  $2\sigma$  and the amplitude  $A_0$ .

The spectrum of such a signal will be (Fig. 2)

$$E(f) = A_0 \exp(-\pi\tau^2[f - f_0]^2) \exp(i2\pi ft). \quad (16)$$

For the integral from the (15) relation, being unable to be analytically resolved, it was used a numeric method for its calculation, the continuous sum being replaced with a finite terms sum (the calculus being performed with a program written in C++Builder5). In the following two figures are represented the transmitted sign by the aluminum layer for a incidental wave, calculated in two situations: when  $H = X_{2R}$  taking into consideration all the internal reflections and when  $H = X_{2R}^{(1)}$  is paid attention on a single doubled reflection in the interior of the layer.

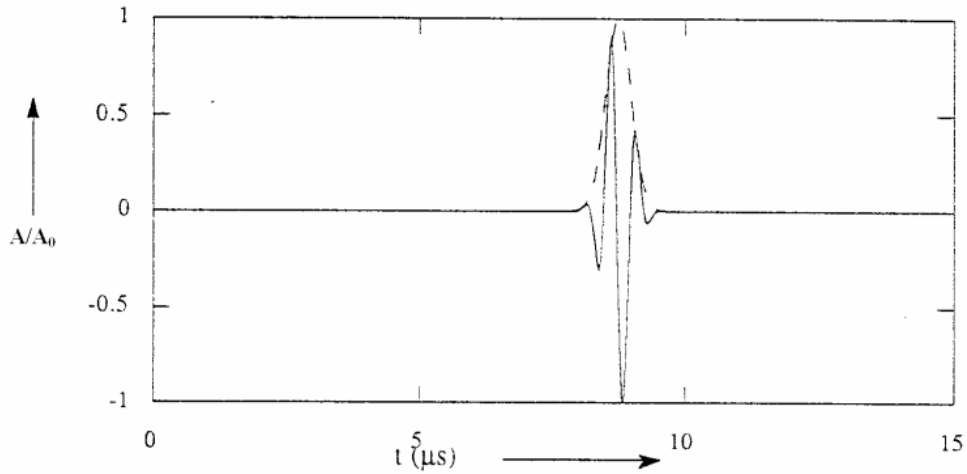
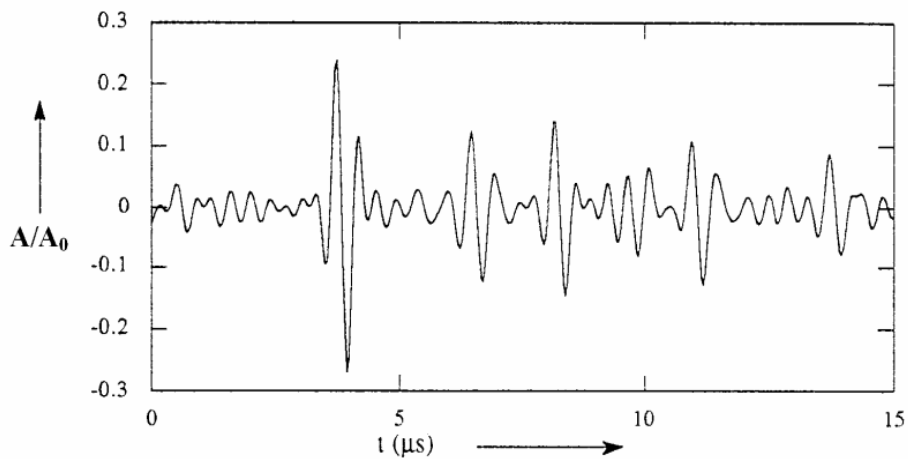


Fig. 2 – The reference signal simulated depending on time.

The other parameters that interfere are:

$$k_{01} = 8, k_{0L} = 2, k_{0T} = 4, \rho_1 = 1, \rho_{Al} = 2,78, \frac{\omega d}{2\pi} = 20, d = 10 \text{ mm} .$$

It may be revealed that in the second case, that one using the Debye's series, the echo's become much more visible and easy to read in comparison with the first case, where the totality of the reflections determine an accentuated superposition of them.



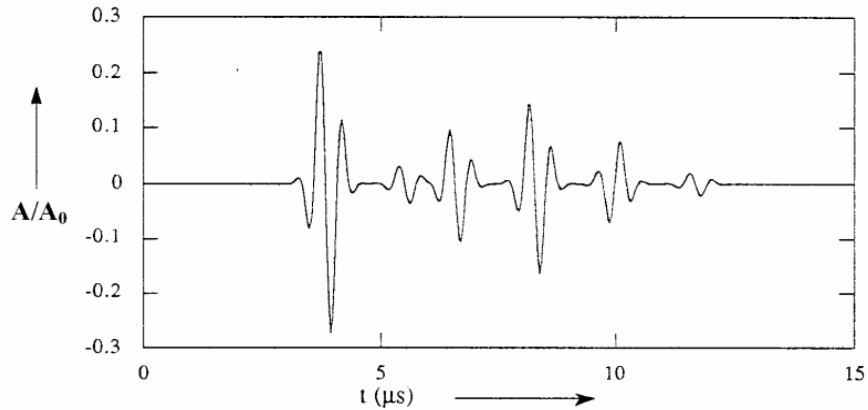


Fig. 3 – The response of the immersed layer in two cases: a) using a classical transmitted coefficient; b) using Debye's series.

The abundance of the visible signs in first case is caused by the discretisation of the spectrum, the calculated sign containing beside its useful part the superposition of all other trances with the same duration ( $20 \mu\text{s}$ ) which constitute its "queue".

Instead of it, the use of Debye's series allow truncate the sign duration, fixing the repeated number of echoes in the interior of the layer, so that in correlation with the reverse Fourier's discrete transformed data, not to originate in undesirable superposition.

At the final, a confrontation theory-experiment done on a layer of composite material immersed in water confirm the anterior results.

There are represented transmission and reflection modules depending on the angle of the incidence calculated theoretically with the (13) and (14) relations.

Several conclusions may be extracted: on the one hand the concordance of the theory with the experiment proves the existence of the heterogeneous plane waves' interaction in the interior of the layer immersed in the fluid. It is necessary to specify that all the anterior theory is not valid in case when the faces which form the diopter of the separation between the two mediums, fluid and solid, are not parallel.

The presented oscillations from Fig. 4 are not caused by the numeric data processing but by the waves' interference phenomenon, which is though very amortized, keep being visible.

The other parameters which interfere in the calculus are:



$$k_{01} = 14.6, k_{0L} = 9.16, k_{0L} = 0.083, k_{0T} = 1.7,$$

$$k_{0R} = 0.25, \rho_1 = 1, \rho_{PVC} = 1.46, \frac{d}{\lambda} = 35,$$

$$d = 10 \text{ mm}, f = 3.5 \text{ MHz}, C_{11} = 8.42, C_{44} = 1.82.$$

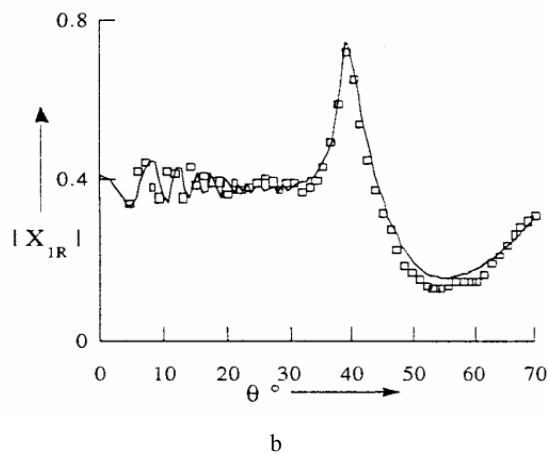
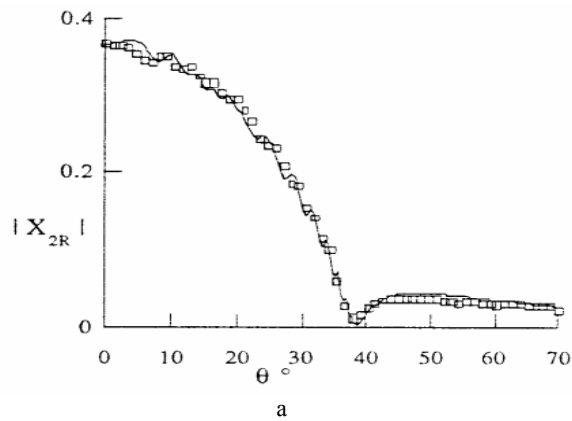


Fig. 4 – The global reflection coefficient module depending on the incidental angle: — theory;  $\square$  experiment.

#### 4. CONCLUSIONS

As a result of this study, it was demonstrated that the use of Debye's series in the calculus of reflection and transmission coefficients of a layer from a composite material, immersed in a fluid, it is justified from the physical reality point of view.

Definition of the mathematical machine, as well as its inclusion in a specialized software application on a platform C++Builder5, it will permit in the future the imagination and prosecution of a numerous trying for determination of the behavior of different materials while passing ultrasound waves.

The particular application for which was created the program and especially that to be use for the study of composite materials, may be easily extended and to other situations excepting the fluid immersion, because this software contains modules easy to be change and use for any other application from the domain.

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