

EVALUATION OF VIBRATING COMPACTION PROCESS FOR FRESH CONCRETE

POLIDOR BRATU

This paper is focused on the research performed upon the dynamic model for vibrating compaction of the placed fresh concrete. The influence of the internal viscosity factor changing upon the internal energy dissipated during the concrete structure hardening is put into evidence.

1. INTRODUCTION

The vibrating compaction process for fresh concrete poured in moulds represents a basic procedure in the stage precursory to hardening having as final target to fulfil the following requirements:

- reducing the interior spaces volume containing air in order to obtain an appropriate porosity;
- assuring microstructure connections between cement, water, aggregates and additives in order to obtain an adequate structure physical homogeneity and consistency necessary for the workability provided by the technological procedures;
- to attain the rated breaking resistance on standard specimens.

The concrete vibrating process in case of prefabricated elements consists in two different stages, namely:

- linear-viscous behaviour of the fresh concrete when the internal viscous forces are proportional to the vibration speed. In this case, the damping

factor $n = \frac{c}{2m_1}$, where c represents the damping and proportionality

coefficient between the viscous force F_v and the concrete deformation speed \dot{x} , meaning that $F_v = c\dot{x}$ and m_1 is the global mass for the vibratory moving material system;

¹ Research Institute for Construction Equipment and Technology – ICECON SA, Bucharest, Romania, E-mail: icecon@icecon.ro.

- nonlinear-viscous behaviour for the fresh concrete where the viscous force modifies with the microstructure so that $n = n(t)$ and the viscous force is $F_v(t) = 2m_1n(t)\dot{x} = c(t)\dot{x}$.

2. ANALYSIS OF THE LINEAR-VISCOUS PARAMETERS FOR FRESH CONCRETE

The free vibration parameters for the mould-concrete system could characterise the linear/nonlinear behaviour as well as the system damping degree n , the critical damping fraction ζ and the damping coefficient c .

The linear-viscous behaviour is illustrated by the eigen motion vibro-record for the system marking an exponential decrease of the free vibration amplitude related to time.

The logarithmic decrement is experimentally determined as:

$$\Delta = \frac{1}{j-1} \ln \frac{A_1}{A_j}, \quad j = 2, 3, 4, \quad (1)$$

where A_1 is the first order amplitude measured on the diagram in the origin of time arbitrary selected; A_j – the j -order amplitude measured on the diagram at a time reference $\Delta t = jT^*$, with j the number of complete periods $j = 2, 3, 4$ and T^* the eigen period (pseudo-period).

The viscosity factor n is given by relation:

$$n = \frac{\Delta}{T^*}, \quad (2)$$

with T^* the pseudo-period of the form:

$$T^* = \frac{2\pi}{\sqrt{p^2 - n^2}}, \quad (3)$$

where p represents the natural eigen frequency for which the relation $m_1p^2 = k$ is fulfilled.

The critical damping fraction is determined as follows:

$$\zeta = \frac{c}{c_{cr}},$$

with $c_{cr} = 2m_1p$, so that we have:

$$\zeta = \frac{c}{2m_1p} = \frac{n}{p}. \quad (4)$$

The experiments carried out for two different concrete qualities C8/10 and C 20/25 for compaction intervals of less than 20 seconds, put into evidence a linear-viscous behaviour for the vibrated fresh concrete. Table 1 presents the linear-viscous parameters for the concrete while compacted by vibration.

The total mass is $m_1 = 60$ kg, the eigen pulsation $p = 3.14$ rad/s, the pseudo-period $T^* = 0.5$ s, the ratio water/cement denoted by w/c and the forced vibration interval Δt^* .

Table 1

Fresh concrete		Eigen vibration parameters				Linear viscosity parameters				Δt^* [s]
Quality	Ratio w/c	T^* [s]	J [cycle]	A_1 [mm]	A_2 [mm]	10^{-3} Δ	10^{-3} n [s]	10^{-3} ξ	C [Nsm ⁻¹]	
C 8/10	0.67	0.5	2	0.45	0.41	93	186	15	117	10
			3	0.42	0.32	136	272	21	171	60
			4	0.42	0.26	159	319	25	200	90
C 20/25	0.52	0.5	4	0.4	0.29	107	214	17	134	10
			5	0.33	0.20	126	252	20	159	60
			2	0.38	0.33	141	282	22	177	90

After the interval $\Delta t^* = 10$ s we found out the continuation of the vibration leads to a viscous nonlinear behaviour emphasised by the increase of the damping parameters n , ζ and c .

3. DYNAMIC PARAMETERS FOR THE LINEAR-VISCOUS VIBRATING COMPACTION

Considering the dynamic model illustrated in Fig. 1, where the vibrating mass 1 (mass m_1) the mould 2 with fresh concrete having the mass m_2 , the whole assembly are supported by the elastic elements 3 having the elastic equivalent constant k . The concrete damping factor is c and the vibrator is unidirectional consisting on two eccentric masses M , each of them weighing $\frac{1}{2}m_0$ and located at distance r in respect to the rotation centres O_1 and O_2 . The masses M rotate synchronically with the angular speed $\dot{\varphi} = \omega$ and in respect to the initial position $\varphi = 0$ the eccentric masses $\frac{1}{2}m_0$ descend simultaneously with $\Delta y = r(1 - \cos \varphi)$ performing the total mechanical work $L_0 = m_0 gr(1 - \cos \varphi)$ according to Fig. 2.

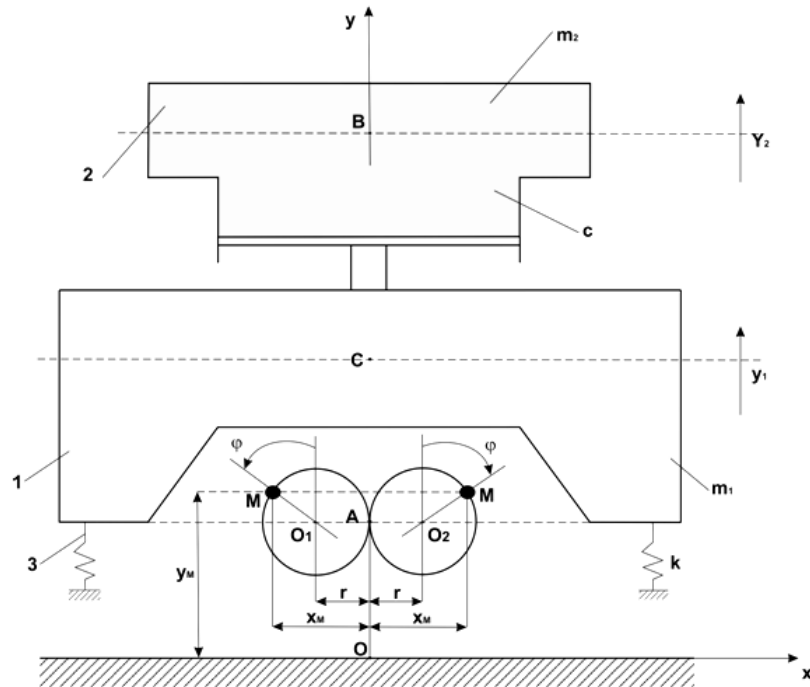


Fig. 1 – The dynamic model.

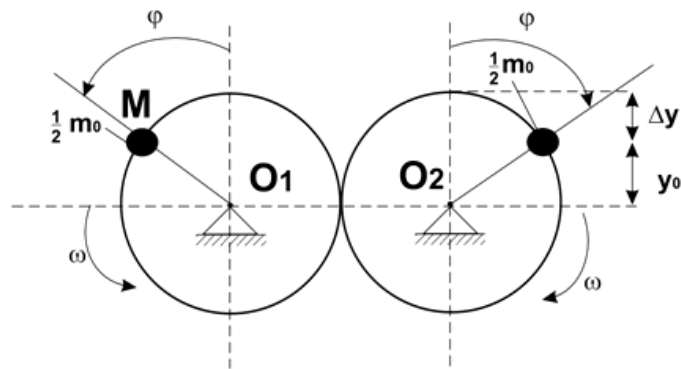


Fig. 2 – The rotating masses M and the eccentric masses $\frac{1}{2} m_0$.

The co-ordinates of the eccentric masses related to the fixed orthogonal system Oxy are as follows:

$$\begin{cases} x_M = r + r \sin \varphi \\ y_M = y_1 - AC + r \cos \varphi \end{cases} \quad (5)$$

and its speed is given by relations:

$$\begin{cases} \dot{x}_M = r\dot{\varphi} \cos \varphi \\ \dot{y}_M = \dot{y}_1 - r\dot{\varphi} \sin \varphi. \end{cases} \quad (6)$$

The quadratic functions for the material system are the kinetic energy E , the potential energy V and the dissipation function D .

The system kinetic energy has the form:

$$E = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{1}{2} m_0 (\dot{x}_M^2 + \dot{y}_M^2) + \frac{1}{2} J \dot{\varphi}^2$$

or replacing \dot{x}_M and \dot{y}_M in (6) we obtain:

$$E = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{1}{2} m_0 (r^2 \dot{\varphi}^2 \cos^2 \varphi + \dot{y}_1^2 + r^2 \dot{\varphi}^2 \sin^2 \varphi - 2r\dot{y}_1\dot{\varphi} \sin \varphi) + \frac{1}{2} J \dot{\varphi}^2$$

and finally it results in:

$$E = \frac{1}{2} (m_0 + m_1) \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 - m_0 r \dot{y}_1 \dot{\varphi} \sin \varphi + \frac{1}{2} (m_0 r^2 + J) \dot{\varphi}^2. \quad (7)$$

The system potential energy is expressed by relation:

$$V = \frac{1}{2} k y_1^2 + m_0 r g (1 - \cos \varphi) \quad (8)$$

in respect to the steady static equilibrium position.

The dissipation function for the fresh concrete is given by:

$$D = \frac{1}{2} c (\dot{y}_1 - \dot{y}_2)^2. \quad (9)$$

For the generalised co-ordinates y_1 , y_2 , φ the second order Lagrange equation could be written under the form:

$$\begin{cases} \frac{d}{dt} \left(\frac{\partial E}{\partial \dot{y}_1} \right) - \frac{\partial E}{\partial y_1} = - \frac{\partial V}{\partial y_1} - \frac{\partial D}{\partial \dot{y}_1} \\ \frac{d}{dt} \left(\frac{\partial E}{\partial \dot{y}_2} \right) - \frac{\partial E}{\partial y_2} = - \frac{\partial V}{\partial y_2} - \frac{\partial D}{\partial \dot{y}_2} \\ \frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\varphi}} \right) - \frac{\partial E}{\partial \varphi} = - \frac{\partial V}{\partial \varphi} - \frac{\partial D}{\partial \dot{\varphi}} + Q_\varphi, \end{cases} \quad (10)$$

where Q_φ represents the generalised force corresponding to the vibrator actuating moment.

By derivation relations (10) become:

- for co-ordinate y_1

$$\frac{\partial E}{\partial \dot{y}_1} = (m_0 + m_1)\dot{y}_1 - m_0 r \dot{\varphi} \sin \varphi,$$

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{y}_1} \right) = (m_0 + m_1)\ddot{y}_1 - m_0 r \ddot{\varphi} \sin \varphi - m_0 r \dot{\varphi}^2 \cos \varphi,$$

$$\frac{\partial E}{\partial y_1} = 0; \quad \frac{\partial V}{\partial y_1} = ky_1; \quad \frac{\partial D}{\partial \dot{y}_1} = c(\dot{y}_1 - \dot{y}_2);$$

- for co-ordinate y_2

$$\frac{\partial E}{\partial \dot{y}_2} = m_2 \dot{y}_2; \quad \frac{d}{dt} \left(\frac{\partial E}{\partial \dot{y}_2} \right) = m_2 \ddot{y}_2; \quad \frac{\partial V}{\partial y_2} = 0; \quad \frac{\partial D}{\partial \dot{y}_2} = -c(\dot{y}_1 - \dot{y}_2);$$

- for co-ordinate φ

$$\frac{\partial E}{\partial \dot{\varphi}} = -m_0 r \dot{y}_1 \sin \varphi + (m_0 r^2 + J)\dot{\varphi},$$

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\varphi}} \right) = -m_0 r \ddot{y}_1 \sin \varphi - m_0 r \dot{y}_1 \dot{\varphi} \cos \varphi + (m_0 r^2 + J)\ddot{\varphi},$$

$$\frac{\partial E}{\partial \varphi} = -m_0 r \dot{y}_1 \dot{\varphi} \cos \varphi; \quad \frac{\partial V}{\partial \varphi} = m_0 r g \sin \varphi; \quad \frac{\partial D}{\partial \varphi} = 0.$$

In this case, the second order Lagrange equations (10) become:

$$\begin{cases} (m_0 + m_1)\ddot{y}_1 - m_0 r \ddot{\varphi} \sin \varphi - m_0 r \dot{\varphi}^2 \cos \varphi = -k_1 y_1 - c(\dot{y}_1 - \dot{y}_2) \\ m_2 \ddot{y}_2 = -c(\dot{y}_1 - \dot{y}_2) \\ (m_0 r + J)\ddot{\varphi} - m_0 r \ddot{y}_1 \sin \varphi - m_0 r \dot{y}_1 \dot{\varphi} \cos \varphi - (-m_0 r \dot{y}_1 \dot{\varphi} \cos \varphi) = -m_0 r g \sin \varphi + Q_\varphi. \end{cases}$$

Aiming the dynamic analysis under steady regime where $\varphi = \omega t$ and $\omega = \text{constant}$ we have $\dot{\varphi} = \omega$ and $\ddot{\varphi} = 0$ so that the differential equation system can be written of the form:

$$\begin{cases} (m_0 + m_1)\ddot{y}_1 + ky_1 + c(\dot{y}_1 - \dot{y}_2) = m_0 r \omega^2 \cos \omega t \\ m_2 \ddot{y}_2 - c(\dot{y}_1 - \dot{y}_2) = 0 \\ -m_0 r \ddot{y}_1 \sin \omega t + m_0 g r \sin \omega t = Q_\phi. \end{cases} \quad (11)$$

The first two equations are coupled by the viscous damping and the third equation depends upon the mass m_1 motion only so that it results in the actuating moment Q_ϕ . In this case, we have the matrix formulation of the form:

$$\begin{bmatrix} m_0 + m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \cos \omega t \\ 0 \end{Bmatrix} \quad (12)$$

or compressed

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{f}, \quad (13)$$

where \mathbf{M} is the mass matrix; \mathbf{C} – the damping matrix; \mathbf{K} – the elastic matrix; \mathbf{y} – the displacement vector; \mathbf{f} – the external forces vector having the form $\begin{Bmatrix} F_0 e^{i\omega t} \\ 0 \end{Bmatrix}$.

The solutions for the steady forced vibrations are following:

$$\begin{aligned} y_1 &= A_1 \cos(\omega t - \theta_1) \\ y_2 &= A_2 \cos(\omega t - \theta_2), \end{aligned}$$

where A_1 and A_2 are the amplitudes for displacements x_1 and x_2 ; θ_1 and θ_2 are the difference in phase between the displacements x_1 and x_2 and the perturbing force $F = F_0 \cos \omega t$.

In order to find the solutions we use the complex numbers method of the form:

$$\tilde{y} = A e^{i(\omega t + \theta)} = A \cos(\omega t - \theta) + iA \sin(\omega t - \theta),$$

where $i = \sqrt{-1}$ is the imaginary unit. Thus, for both solutions we have:

$$\begin{aligned} \tilde{y}_1 &= A_1 e^{i(\omega t - \theta_1)} = A_1 \cos(\omega t - \theta_1) + iA_1 \sin(\omega t - \theta_1), \\ \tilde{y}_2 &= A_2 e^{i(\omega t + \theta_2)} = A_2 \cos(\omega t + \theta_2) + iA_2 \sin(\omega t + \theta_2). \end{aligned}$$

The solutions derivatives in the complex form are expressed by relations:

$$\begin{aligned} \dot{\tilde{y}}_1 &= i\omega A_1 e^{i(\omega t - \theta_1)} = i\omega \tilde{y}_1, \\ \ddot{\tilde{y}}_1 &= i\omega \dot{\tilde{y}}_1 = i\omega(i\omega \tilde{y}_1) = -\omega^2 \tilde{y}_1, \end{aligned}$$

$$\dot{\tilde{y}}_2 = i\omega\tilde{y}_2; \quad \ddot{\tilde{y}}_2 = -\omega^2\tilde{y}_2.$$

The matrix equation (10) becomes:

$$-\omega^2\mathbf{M}\tilde{y} + i\mathbf{C}\omega\tilde{y} + \mathbf{K}\tilde{y} = \tilde{\mathbf{f}},$$

with $\tilde{\mathbf{f}} = [\tilde{F} \quad 0]^T$ so that we have:

$$[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}]\tilde{y} = \tilde{\mathbf{f}}, \quad (14)$$

where matrix $\tilde{\mathbf{S}}$ is noted as

$$\tilde{\mathbf{S}} = [-\omega^2\mathbf{M} + \mathbf{K} + i\omega\mathbf{C}] \quad (15)$$

or

$$\tilde{\mathbf{S}} = \begin{bmatrix} -(m_0 + m_1)\omega^2 & -ic\omega \\ -ic\omega & -m_2\omega^2 + ic\omega \end{bmatrix} \quad (16)$$

The determinant for matrix $\tilde{\mathbf{S}}$ is

$$\tilde{\Delta} = \det \tilde{\mathbf{S}} = D + iE, \quad (17)$$

where

$$\begin{cases} D = (m_0 + m_1)m_2\omega^4 - m_2k\omega^2 \\ E = ck\omega - c(m_0 + m_1 + m_2)^3. \end{cases} \quad (18)$$

The modulus of $\tilde{\Delta}$ is $\Delta^2 = \tilde{\Delta}^2 = D^2 + E^2$ leading to the condition to determine the eigen pulsations

$$\Delta^2 = D^2 + E^2 = 0. \quad (19)$$

Finally, expression $\Delta = \sqrt{D^2 + E^2}$ can be written

$$\Delta = \omega R, \quad (20)$$

with

$$R = [r_6\omega^6 + r_4\omega^4 + r_2\omega^2 + r_0]^{\frac{1}{2}}, \quad (21)$$

where

$$\begin{cases} r_6 = (m_0 + m_1)^2 m_2^2 \\ r_4 = c^2 (m_0 + m_1 + m_2)^2 - 2(m_0 + m_1)m_2^2 k \\ r_2 = k^2 m_2^2 - 2kc^2 (m_1 + m_2) \\ r_0 = k^2 c^2. \end{cases}$$

The amplitudes are given by relations:

$$A_1 = \frac{m_0 r \omega^2}{R} \sqrt{m_2^2 \omega^2 + c^2} = \frac{m_0 r \omega^2}{R} \sqrt{m_2 \omega^2 + 4m_2^2 n^2}, \quad (22)$$

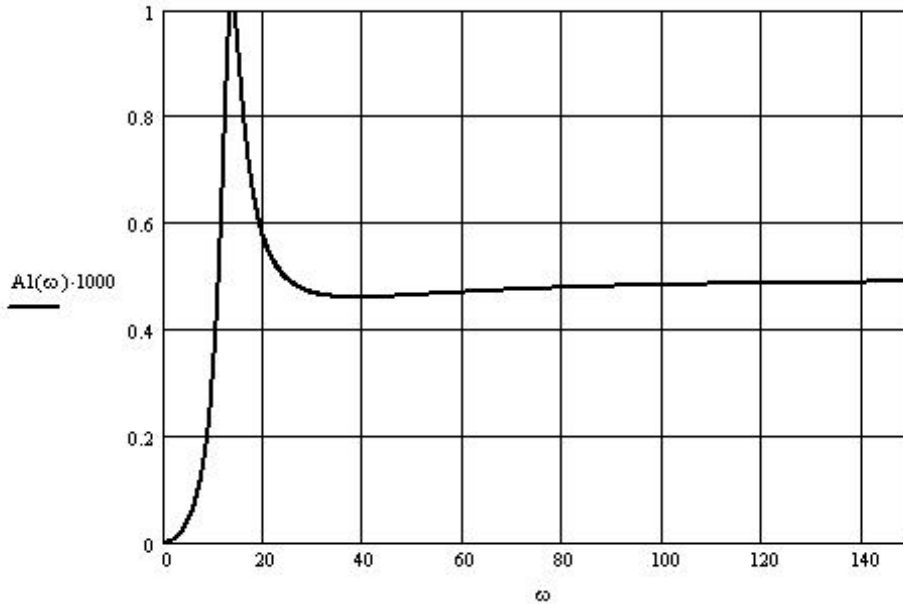
$$A_2 = \frac{m_0 r \omega^2}{R} c = \frac{2m_0 r \omega^2}{R} m_2 n, \quad (23)$$

with $c = 2m_2 n$.

The difference in phase between the perturbing force $F = m_0 r \omega^2 \cos \omega t$ and instantaneous displacements x_1 and x_2 are determined by

$$\begin{cases} \operatorname{tg} \theta_1 = \frac{cD + \omega m_2 E}{cE - \omega m_2 D} \\ \operatorname{tg} \theta_2 = \frac{D}{E}. \end{cases} \quad (24)$$

The variation curves for $A_1(\omega)$ and $A_2(\omega)$ are presented in Fig. 3, where A_1 and A_2 are expressed in mm. Figure 4 illustrates amplitudes A_1 and A_2 as a function of n , for $\omega = 3.14 \text{ s}^{-1}$.



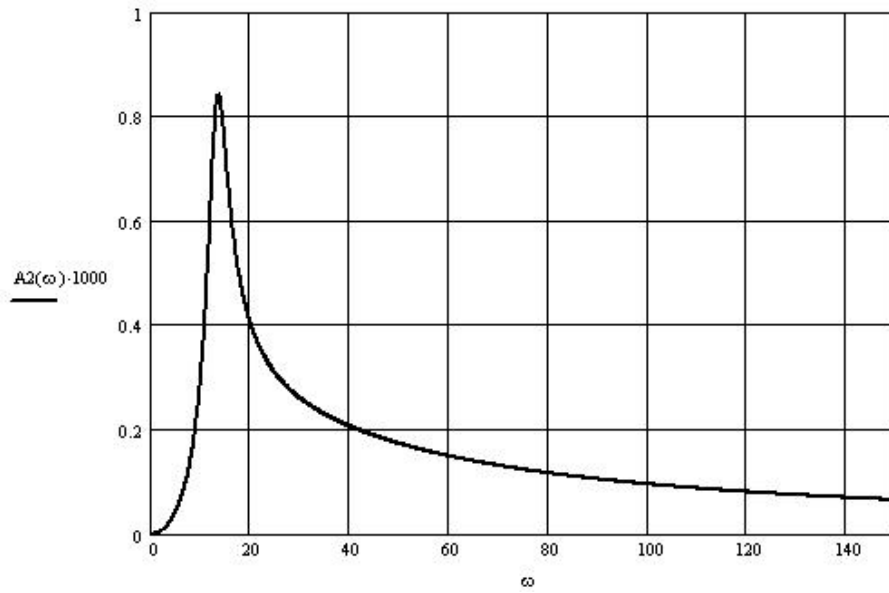
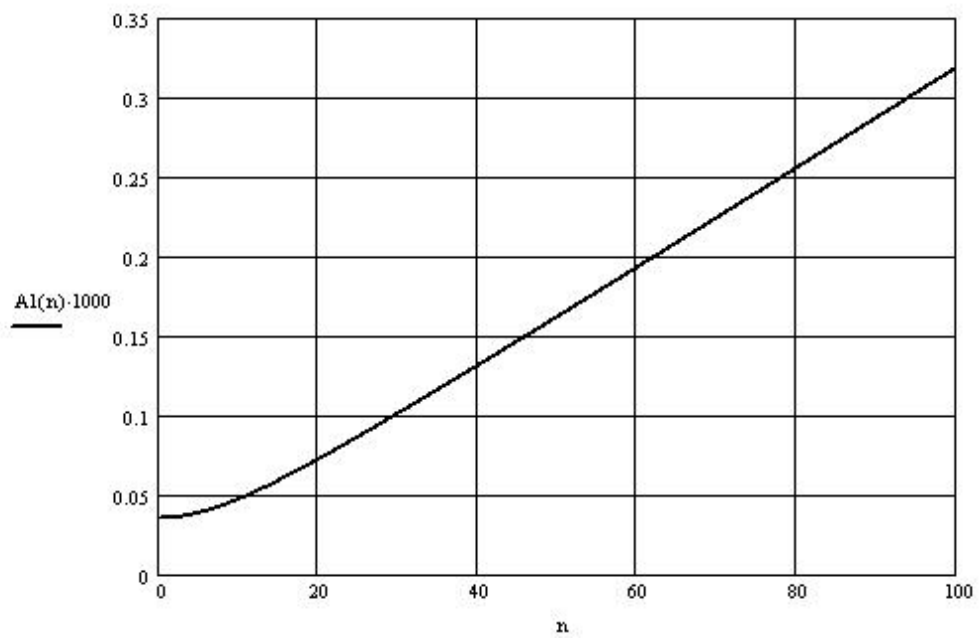


Fig. 3 – The variation curves for $A_1(\omega)$ and $A_2(\omega)$.



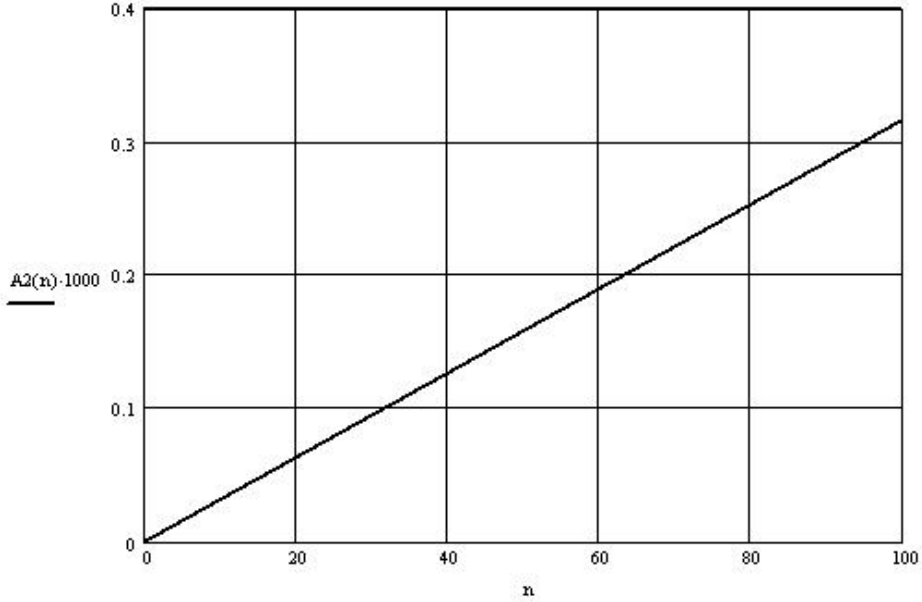


Fig. 4 – The amplitudes A_1 and A_2 as a function of n , for $\omega = 3.14 \text{ s}^{-1}$.

The actuation moment Q_φ for the driving shaft of the vibro-generator necessary to keep the harmonic motion under forced regime is obtained basing on the third relation in (8), using solution $y_1 = A_1 \cos(\omega t - \theta_1)$ with its derivative $\dot{y}_1 = -\omega^2 A_1 \cos(\omega t - \theta_1)$. Thus we have:

$$Q_\varphi = m_0 r A_1 \omega^2 \cos(\omega t - \theta_1) \sin \omega t + m_0 g r \sin \omega t$$

or transforming the trigonometrically product in sums we obtain:

$$Q_\varphi = \frac{1}{2} m_0 r A_1 \omega^2 [\sin \theta_1 + \sin(2\omega t - \theta_1)] + m_0 g r \sin \omega t . \quad (25)$$

2. ENERGY DISSIPATED DURING THE VIBRATING COMPACTION PROCESS

The elementary mechanical work performed by the generalised force Q_φ corresponding to the actuation moment of the vibro-generator for an elementary angle $d\varphi$ is defined by relation:

$$dL = Q_\varphi d\varphi,$$

where $\varphi = \omega t$ and $d\varphi = \omega dt$.

Aiming to perform a complete rotation, the necessary mechanical work is:

$$L = \int_0^{2\pi} Q_\varphi d\varphi$$

or

$$L = \int_0^{2\pi} \frac{1}{2} m_0 r A_1 \omega^2 \sin \theta_1 d\varphi + \int_0^{2\pi} \frac{1}{2} m_0 r A_1 \omega^2 \sin(2\omega t - \theta_1) d\varphi + \int_0^{2\pi} m_0 g r \sin \omega t d\varphi$$

from where finally it results in:

$$L = \pi m_0 r A_1 \omega^2 \sin \theta_1. \quad (26)$$

The average power N_m corresponds to one period interval, namely for $T = \frac{2\pi}{\omega}$, leading to the following expression:

$$N_m = \frac{L}{T} = \frac{L\omega}{2\pi}$$

or

$$N_m(\omega) = \frac{1}{2} m_0 r A_1 \omega^3 \sin \theta_1. \quad (27)$$

Basing on the technical data in case of an industrial vibrating table the main vibration parameters have been graphically illustrated for the following two situations:

- a) for constant internal damping, for a given n , the exciting pulsation ω has been monotonously increased to the technological values (Fig. 5).
- b) for constant exciting pulsation of the technological vibrations, for a given ω , the internal damping n has been modified as a result of the of the internal structure evolution of the fresh material according to the dynamic compaction process (Fig. 6).

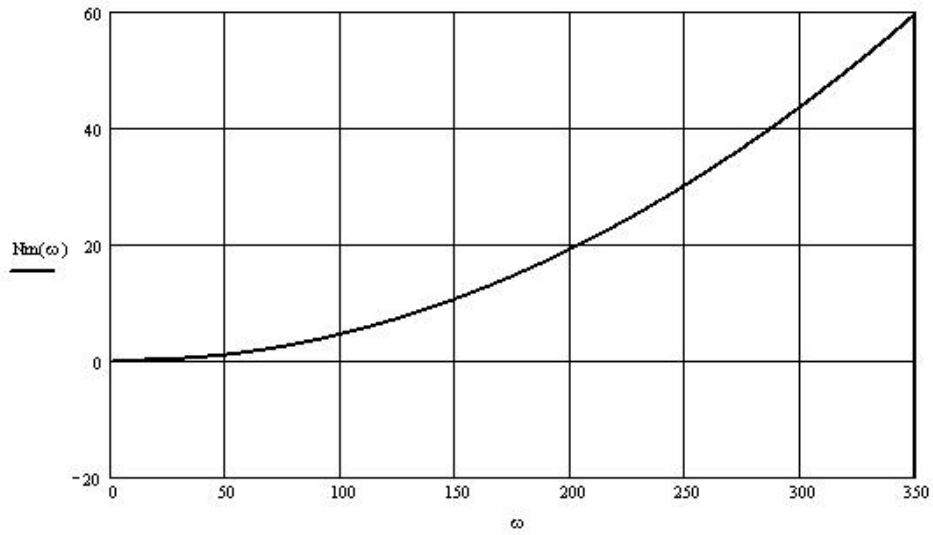


Fig. 5 – The exciting pulsation ω monotonously increased to the technological values.

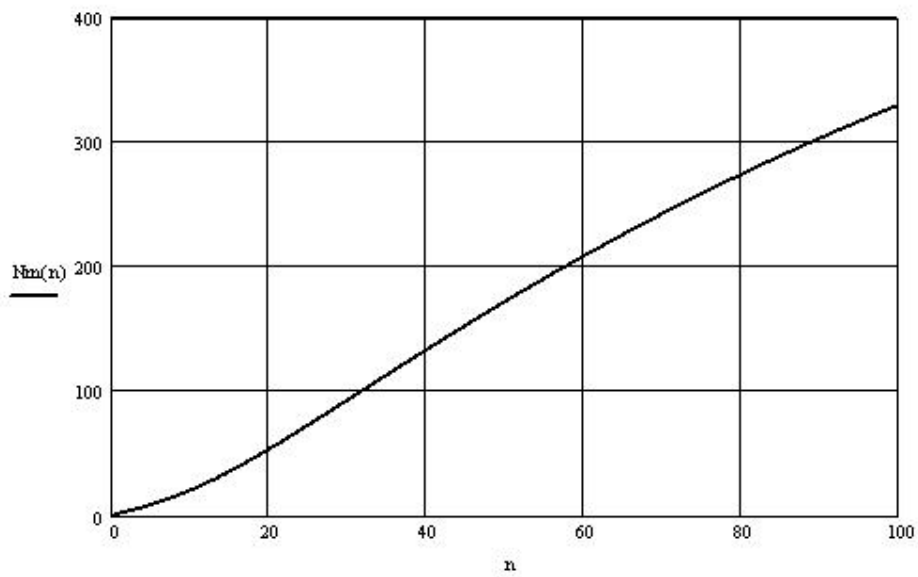


Fig. 6 – The internal damping n modified as a result of the of the internal structure evolution of the fresh material.

3. CONCLUSIONS

The vibrating compaction process for fresh concrete can be modelled by means of a two masses system having a single connection elastic element at its base and a single viscous element between the vibrating mould and concrete.

The curves are specific for the fresh concrete compaction process in case of a vibratory system consisting on: $m_2 = 200$ kg; $m_0 = 2$ kg; $m_{0r} = 0.1$ kg·m; $k = 64,000$ N/m; $\omega \in [0; 350]s^{-1}$; $n \in [0.02; 10] s^{-1}$.

Basing on the data included in this study new analysis concerning the structure micro-processes as a result of the dynamic compaction of the fresh concrete placed as prefabricates, pillars moulded in formworks, beams or slabs can be developed.

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REFERENCES

1. BRATU, P., *Elastic system vibrations*, Technical Publishing House, Bucharest, 2000.
2. BRATU, P., *Concrete as a viscous damper for vibratory tables having vertical oscillation*, Proceedings of the XXIIth National Conference of Solid Mechanics, **IV**, Braşov, 1998.
3. BRATU, P., MIHALCEA, A., *The rheological modelling and analysis of the fresh concrete compacting process subjected to harmonic vibrations*, Proceedings of the International Conference on Engineering Rheology, ICER'99, **4**, Zielona Gora, 1999.