DYNAMICS OF THE HEXAPOD PARALLEL ROBOT

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Recursive matrix relations in kinematics and dynamics of the Hexapod parallel robot are established in this paper. Controlled by six forces, the parallel manipulator prototype is a space six-degrees-of-freedom mechanical system with six fixed-length legs connecting to the moving platform. Knowing the position and the general motion of the platform, we develop first the inverse kinematics problem and determine the position, velocity and acceleration of each manipulator's body. Further, the inverse dynamic problem is solved using an approach based on the principle of virtual work. Finally, compact matrix relations and graphs of simulation for the input forces and powers are obtained.

1. INTRODUCTION

Parallel manipulators are closed-loop mechanisms presenting very good potential in terms of accuracy, rigidity and ability to manipulate large loads. In general, these manipulators consist of two main bodies coupled via numerous legs acting in parallel. One body is arbitrarily designated as fixed and is called *base*, while the other is regarded as movable and hence is called *moving platform* of the manipulator. Several mobile legs or limbs, made up as serial robots, connect the movable platform to the fixed frame. The bodies of the robot are connected one to the other by spherical joints, universal joints, revolute joints or prismatic joints. Typically, the number of actuators is equal to the number of degrees of freedom such that every link is controlled at or near the fixed base [1].

Parallel mechanisms can be found in practical applications, in which it is desired to orient a rigid body in space of high speed, such as aircraft simulators [2], positional tracker and telescopes [3, 4]. The first design for industrial purposes can be dated back to 1962, when Gough implemented a six-linear jack system for use as a Universal tire-testing machine. In fact, it was a huge force sensor, capable of measuring forces and torques on a wheel in all directions. Some years later, Stewart published a design of a platform robot for use as a flight simulator.

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Compared with serial mechanisms, parallel manipulator is a complex mechanical structure, behaving some special characteristics such as: greater rigidity, potentially higher kinematical precision, stabile capacity and suitable position of arrangement of actuators. However, they suffer the problems of relatively small useful workspace and design difficulties.

Considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2]; Merlet [5]; Parenti Castelli and Di Gregorio [6]). They are used in flight simulators and more recently for Parallel Kinematics Machines. The prototype of Delta parallel robot (Clavel [7]; Staicu [8]; Tsai and Stamper [9]) developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland as well as the Star parallel manipulator (Hervé and Sparacino [10]) are equipped with three motors and which train on the mobile platform in a three-degree-of-freedom general translation motion. Angeles [11], Gosselin and Gagné [12], Wang and Gosselin [13] analysed the direct kinematics, dynamics and singularity loci of the Agile Wrist spherical parallel robot with three concurrent actuators.

The analysis of parallel manipulators is usually implemented trough analytical methods in classical mechanics, in which projection and resolution of vector equations on the reference axes are written in a considerable number of cumbersome, scalar relations and the solutions are rendered by large scale computations together with time consuming computer codes [14, 15].

The Hexapod manipulator represents a new development in machine tools by utilising parallel kinematical structures. At the beginning, hexapods were developed based on the Stewart-Gough platform. More recently, commercial hexapods have been used by many companies in the development of high precision machine tools [16, 17], such as Variax from Giddings & Lewis, Tornado from Hexel Corp. and Geodetic from Geodetic Technology Ltd. Sliding-leg Hexapods with constant lengths have been envisaged, for example, HexaM from Toyoda [18].

Dynamic modelling and analysis of parallel mechanisms is an important part of hexapod design and control. A great deal of work has been done in this area. For example, Fitcher [19] used the Plücker line coordinates for dynamic analysis of parallel manipulators. Sugimoto [20] applied the motor algebra to address the same problem. Merlet [21] discussed the force transformation between the joint space and the task space, while Do and Yang [22] studied the inverse dynamics based on the Newton-Euler formulation. Geng et al. [23] studied the same problem using Lagrange's formalism. Hashimoto and Kimura [24] applied the parallel computational schemes to the inverse dynamics formulated based on the Newton-Euler method. Fijany and Bejczy [25] utilized a hierarchical graph-based mapping approach to improve the parallel computational schemes.

It should be noted that the previous studies on the dynamics analysis of parallel mechanisms were based on the assumption that leg inertia is negligible. This may be true for Stewart platform-based fight simulator, because the moving platform is much heavier. For machine tools, especially for high-speed machining, the moving platform is lighter and leg of inertia may not be negligible. Ji [26] first addressed the problem of the effect of leg inertia on the Stewart platform dynamics and provided a dynamic modelling based on the Newton-Euler approach. Recently, Codourey [27] developed a dynamic model including leg inertia for a revolute type of parallel mechanism called Delta. So far, however, no quantitative studies have been made on the effect of leg inertia. This problem is important, because the leg inertia of the Hexapod is compatible to that of the moving platform and dynamics becomes significant when operating at high speeds.

The natural orthogonal complement method has been applied to studying the serial or parallel manipulators and the flexible mechanisms (Angeles and Lee [28], Zanganeh et al. [29], Xi and Sinatra [30]). In this development, the Newton-Euler formulation is used to model the dynamics equations of each individual body, including the moving platform and the legs.

In the present paper, a new recursive matrix method is introduced. It has been proved to reduce the number of equations and computation operations significantly by using a set of matrices for kinematics and dynamics models.

2. INVERSE KINEMATICS

A spatial 6-DOF parallel manipulator, which can be existent in several applications including machine tools, is proposed in this paper. Since the pneumatic joints can easily achieve high accuracy and heavy loads, the majority of the 3-DOF or of the 6-DOF parallel mechanisms use the actuated prismatic joints.

The Hexapod under consideration is a six-degree-of-freedom parallel mechanism with constant length sliding legs. As shown in Fig. 1, it is a composed of six guide-ways, six legs, a base and a moving platform. Each leg is connected on one end by a universal joint to the guide-way along which the leg slides and on the other end by a spherical joint to the moving platform to which a tool is attached. The lengths of the legs and their guide-ways are constant.

Symbolically represented by six pairs of spherical joints A_4, B_4, C_4, D_4, E_4, F_4 a polygonal moving platform is driven by six sliding legs. Other two polygonal parallel platforms, which are connected by six guide-ways attached at the points A_0, B_0, C_0, D_0, E_0, F_0, constitute the fixed base of the manipulator. In what follows we consider that the moving platform is initially located at a *central configuration*, where the moving platform is not rotated with respect to the fixed base and the mass centre G is at an elevation $OG = h$ above the centre of the fixed base (Fig. 1).

For the purpose of analysis, we attach a Cartesian coordinate system $Ox_0y_0z_0(T_0)$ to the fixed base with its origin located at the centre O of fixed platform, the Oz_0 axis perpendicular to the base and the Ox_0 axis pointing to the midpoint line linking the points F_0, A_0. Another coordinate central frame $Gx_Gy_Gz_G$ could be linked just at the centre G of the moving platform.

To simplify the graphical image of the kinematical scheme of the mechanism, in what follows we will represent the intermediate reference systems by only two axes, so as is used in most of robotics papers [1, 5, 11]. The z_k axis is represented, of course, for each component element T_k. It is noted that the relative rotation with angle $\varphi_{k,k-1}$ or the relative translation of the body T_k with the displacement $\lambda_{k,k-1}$ must always be pointed along the direction of the z_k axis.

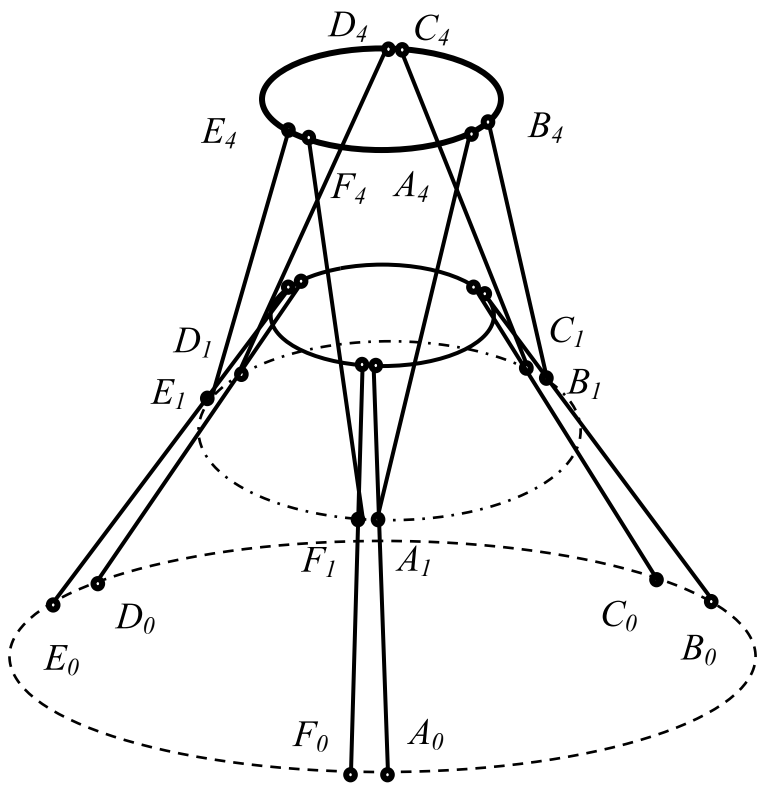

Fig. 1 – General scheme of the Hexapod parallel robot.

One of these identical active legs (for example leg $A \equiv A_1 A_2 A_3 A_4$) consists of a slider of mass m_1, which effect a translation with the velocity $v_{10}^A = \dot{\lambda}_{10}^A$ and the acceleration $\gamma_{10}^A = \ddot{\lambda}_{10}^A$, a moving Hooke joint characterized by the mass m_2, the angular velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and the angular acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$, and a leg of constant length l_3, mass m_3 and tensor of inertia $\hat{J}_3$, which is connected to the universal joint at the bottom end and a passive spherical joint at the other. This leg has a relative rotation about $A_3 z_3^A$ axis with the angle φ_{32}^A, so that $\omega_{32}^A = \dot{\varphi}_{32}^A$, $\varepsilon_{32}^A = \ddot{\varphi}_{32}^A$. Finally, a ball-joint or a spherical joint is attached to the moving platform, which can by schematised as a polygon of mass m_p and inertia tensor $\hat{J}_p$. Following notations are used: l_0 radius of the circle associated to the moving platform, $L_p = 2l_0 \sin\left(\frac{\pi}{3} - \alpha_0\right)$ long side, $l_p = 2l_0 \sin\alpha_0$ short side, L_0 radius of the circle associated to the fixed base, $L_b = 2L_0 \sin\left(\frac{\pi}{3} - \alpha_0\right)$ long side, $l_b = 2L_0 \sin\alpha_0$ short side, $s = 2l_1 = 2(L_b - L_p)/\sqrt{2}$ guide-way length, $\sin\beta_0 = \frac{L_0 - l_0}{l_1}$ inclination of the guide-way, γ guide-way angle, $\sin\beta = 2\left(\frac{l_0}{l_3}\right)\sin\left(\frac{\pi}{6} - \alpha_0\right)$ initial inclination of the leg and $h = l_1 \cos\beta_0 + l_3 \cos\beta$ as initial position of the center G of moving platform (Fig. 2).

At the central configuration, we also consider that the angles of orientation giving the positions of sliders, legs, universal joints and spherical joints are given by

$$\alpha_1^A = \alpha_0,\ \alpha_1^B = \frac{2\pi}{3} - \alpha_0,\ \alpha_1^C = \frac{2\pi}{3} + \alpha_0,\ \alpha_1^D = -\frac{2\pi}{3} - \alpha_0$$
$$\alpha_1^E = -\frac{2\pi}{3} + \alpha_0,\ \alpha_1^F = -\alpha_0.$$

$$\alpha_2^A = \alpha_2^C = \alpha_2^E = -\frac{\pi}{3} - \alpha_0,\ \alpha_2^B = \alpha_2^D = \alpha_2^F = \frac{\pi}{3} + \alpha_0, \tag{1}$$

$$\alpha_3^A = \alpha_3^C = \alpha_3^E = \frac{\pi}{3} - 2\alpha_0, \alpha_3^B = \alpha_3^D = \alpha_3^F = -\frac{\pi}{3} + 2\alpha_0.$$

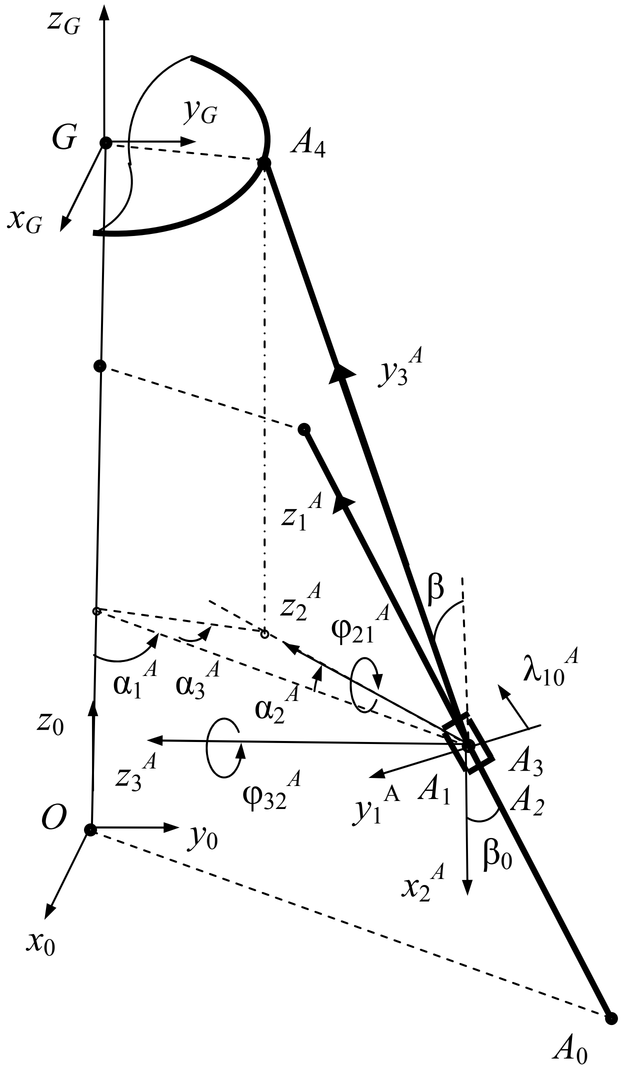

Fig. 2 – Kinematical scheme of first leg A of the mechanism.

Assuming that the each leg is connected to the fixed base by the slider and the universal joint such that it cannot rotate about the longitudinal axis, the orientation of the leg A with respect to the fixed base can be described by two Euler angles, namely a rotation angle φ_{21}^{A} about the $A_2 z_2^A$ axis, followed by another rotation of angle φ_{32}^{A} about the rotated $A_3 z_3^A$ axis.

Pursuing the first leg A in the $OA_1A_2A_3A_4$ way, we obtain the following matrices of transformation [31]:

$$a_{10} = a_{\beta}^{\circ}\theta_3 a_{1\alpha}^{A},\ a_{21} = a_{21}^{\varphi}\theta_1 a_{2\alpha}^{AT} a_{\beta}^{\circ T},\ a_{32} = a_{32}^{\varphi} a_{\beta}\theta_1\theta_2, \tag{2}$$

where

$$a_{i\alpha}^{A} = \begin{bmatrix} \cos\alpha_i^A & \sin\alpha_i^A & 0 \\ -\sin\alpha_i^A & \cos\alpha_i^A & 0 \\ 0 & 0 & 1 \end{bmatrix}\ (i = 1,\ 2,\ 3),$$

$$a_{\beta}^{\circ} = \begin{bmatrix} \cos\beta_0 & 0 & -\sin\beta_0 \\ 0 & 1 & 0 \\ \sin\beta_0 & 0 & \cos\beta_0 \end{bmatrix},\ a_{\beta} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},\ \theta_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \theta_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

$$a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^A & \sin\varphi_{k,k-1}^A & 0 \\ -\sin\varphi_{k,k-1}^A & \cos\varphi_{k,k-1}^A & 0 \\ 0 & 0 & 1 \end{bmatrix},\ a_{k0} = \prod_{s=1}^{k} a_{k-s+1,k-s},\quad (k = 1, 2, 3).$$

Analogous relations can be written for other five legs of the mechanism.

Six independent displacements λ_{10}^A, λ_{10}^B, λ_{10}^C, λ_{10}^D, λ_{10}^E, λ_{10}^F of the active links are the input variables that can give the instantaneous position of the mechanism. But, in the inverse geometric problem, it can be considered that three coordinates x_0^G, y_0^G, z_0^G of mass centre G of the moving platform and others three Euler angles α_1, α_2, α_3 of successive rotations about the Gx_G, Gy_G, Gz_G axes gives the position of the mechanism. Since all rotations take place successively about the moving coordinate axes, the resulting rotation matrix is obtained by multiplying three basic rotation matrices:

$$R_1 = R(x,\ \alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

$$R_2 = R(y, \alpha_2) = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix} \tag{4}$$

$$R_3 = R(z,\alpha_3) = \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Then, the general rotation matrix R of the platform from $Ox_0y_0z_0(T_0)$ to $Gx_Gy_Gz_G$ reference system is given by

$$R = R_3R_2R_1\,. \tag{5}$$

We suppose that the coordinates of the platform's centre G and the angles α_1, α_2, α_3, which are expressed by following analytical functions

$$x_0^G = x_0^{G*}\left(1-\cos\frac{\pi}{3}t\right),\ y_0^G = y_0^{G*}\left(1-\cos\frac{\pi}{3}t\right),\ z_0^G = h - z_0^{G*}\left(1-\cos\frac{\pi}{3}t\right)$$

$$\alpha_l = \alpha_l^*\left(1-\cos\frac{\pi}{3}t\right)\ \ (l=1,\ 2,\ 3), \tag{6}$$

can describe the general absolute motion of the moving platform.

The set of 18 variables $\lambda_{10}^A,\ \varphi_{21}^A,\ \varphi_{32}^A,\ \dots,\ \lambda_{10}^F,\ \varphi_{21}^F,\ \varphi_{32}^F$ will be determined by several vector-loop equations established along the branches of the leg-guide-way system, as follows

$$\vec{r}_{10}^{\,A} + \sum_{k=1}^{3} a_{k0}^T\vec{r}_{k+1,k}^{\,A} - R^T\vec{r}_G^{\,A_4} = \vec{r}_{10}^{\,B} + \sum_{k=1}^{3} b_{k0}^T\vec{r}_{k+1,k}^{\,B} - R^T\vec{r}_G^{\,B_4} = \dots\dots$$

$$\dots\dots = \vec{r}_{10}^{\,F} + \sum_{k=1}^{3} f_{k0}^T\vec{r}_{k+1,k}^{\,F} - R^T\vec{r}_G^{\,F_4} = \vec{r}_0^{\,G}\,, \tag{7}$$

where

$$\vec{r}_{10}^{\,A} = L_0a_{1\alpha}^{A\,T}\vec{u}_1 + (l_1+\lambda_{10}^A)a_{10}^T\vec{u}_3),\ \vec{r}_{21}^{\,A} = \vec{0},\ \vec{r}_{32}^{\,A} = \vec{0},\ \vec{r}_{43}^{\,A} = l_3\vec{u}_2,\ \vec{r}_G^{\,A_4} = l_0a_{1\alpha}^{AT}a_{3\alpha}^{AT}\vec{u}_1\,,$$

$$\vec{u}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix},\ \vec{u}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix},\ \vec{u}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix},\ \vec{r}_0^{\,G} = \begin{bmatrix}x_0^G\\y_0^G\\z_0^G\end{bmatrix},$$

$$\tilde{u}_1 = \begin{bmatrix}0&0&0\\0&0&-1\\0&1&0\end{bmatrix},\ \tilde{u}_2 = \begin{bmatrix}0&0&1\\0&0&0\\-1&0&0\end{bmatrix},\ \tilde{u}_3 = \begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}. \tag{8}$$

Knowing the general motion of the platform by the relations (6), we develop the inverse kinematical problem and determine the absolute velocities $\vec{v}_{k0}^A,\ \vec{\omega}_{k0}^A$ and accelerations $\vec{\gamma}_{k0}^A,\ \vec{\varepsilon}_{k0}^A$ of each of the moving bodies.

First, we compute the angular velocities of six legs and the velocities of the joints in terms of the angular velocity of the moving platform and the velocity of its centre G

$$\vec{\omega}_{60}^{\circ G} = R^T \vec{\omega}_{60}^G = \dot{\alpha}_1 R_1^T \vec{u}_1 + \dot{\alpha}_2 R_1^T R_2^T \vec{u}_2 + \dot{\alpha}_3 R^T \vec{u}_3$$
$$\dot{\vec{r}}_0^G = R^T \vec{v}_{60}^G = [\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{z}_0^G]^T . \tag{9}$$

The motions of the compounding elements of each leg (for example the leg A) are characterized by following skew-symmetric matrices [32]:

$$\tilde{\omega}_{k0}^A = a_{k,k-1} \tilde{\omega}_{k-1,0}^A a_{k,k-1}^T + \omega_{k,k-1}^A \tilde{u}_3 \quad (k = 2,\ 3), \tag{10}$$

which are *associated* to the absolute angular velocities given by the recursive formula

$$\vec{\omega}_{k0}^A = a_{k,k-1} \vec{\omega}_{k-1,0}^A + \omega_{k,k-1}^A \vec{u}_3 . \tag{11}$$

Following relations give the velocities $\vec{v}_{k0}^A$ of joints A_k :

$$\vec{v}_{k0}^A = a_{k,k-1} \vec{v}_{k-1,0}^A + a_{k,k-1} \tilde{\omega}_{k-1,0}^A \vec{r}_{k,k-1}^A + v_{k,k-1}^A \vec{u}_3 ,\ v_{\sigma,\sigma-1}^A = 0 \quad (\sigma = 2,\ 3). \tag{12}$$

Equations of geometrical constraints (7) will be derivate with respect to time to obtain the following *matrix conditions of connectivity* established for the characteristic relative velocities of first leg A (for example):

$$v_{10}^A \ \vec{u}_j^T a_{10}^T \vec{u}_3 + \omega_{21}^A l_3 \ \vec{u}_j^T a_{20}^T \ \tilde{u}_3 a_{32}^T \vec{u}_2 - \omega_{32}^A l_3 \vec{u}_j^T a_{30}^T \vec{u}_1 = \vec{u}_j^T \dot{\vec{r}}_0^G + \vec{u}_j^T R^T \tilde{\omega}_{60}^G \vec{r}_G^{A_4} , \tag{13}$$
$$(j = 1,2,3)$$

where

$$\tilde{\omega}_{60}^{\circ G} = R^T \tilde{\omega}_{60}^G R = \dot{\alpha}_1 R_1^T \tilde{u}_1 R_1 + \dot{\alpha}_2 R_1^T R_2^T \tilde{u}_2 R_2 R_1 + \dot{\alpha}_3 R^T \tilde{u}_3 R \tag{14}$$

denotes the skew-symmetric matrix associated to the absolute angular velocity $\vec{\omega}_{60}^{\circ G}$ of the moving platform [33]. From these equations, we obtain the relative velocities v_{10}^A, ω_{21}^A, ω_{32}^A as functions of angular velocity of the platform and velocity of mass centre G. Derived from (13), the complete Jacobian matrix of the robot is a fundamental element for the analysis of singularity loci and workspace of the robot.

Let us assume now that the manipulator has a *virtual motion* determined by the following virtual velocities

$$v_{10a}^{Av} = 1 , v_{10a}^{Bv} = 0 , v_{10a}^{Cv} = 0 , v_{10a}^{Dv} = 0 , v_{10a}^{Ev} = 0 ,\ v_{10a}^{Fv} = 0 . \tag{15}$$

The relations of connectivity (13) express immediately the relative virtual velocities as function of the position of the manipulator. Other five sets of virtual velocities can be obtained if we consider successively that $v_{10b}^{Bv} = 1$, $v_{10c}^{Cv} = 1$, $v_{10d}^{Dv} = 1$, $v_{10e}^{Ev} = 1$, $v_{10f}^{Fv} = 1$.

As for the relative accelerations γ_{10}^{A}, ε_{21}^{A}, ε_{32}^{A} of the elements of first leg A of the mechanism, following other *conditions of connectivity* are imposed

$$\gamma_{10}^{A}\,\vec{u}_j^{\,T} a_{10}^{T}\vec{u}_3 + \varepsilon_{21}^{A} l_3\,\vec{u}_j^{\,T} a_{20}^{T}\,\tilde{u}_3 a_{32}^{T}\vec{u}_2 - \varepsilon_{32}^{A} l_3 \vec{u}_j^{\,T} a_{30}^{T}\vec{u}_1 = \vec{u}_j^{\,T}\ddot{\vec{r}}_0^{\,G} + \vec{u}_j^{\,T} R^{T}\{\tilde{\omega}_{60}^{G}\tilde{\omega}_{60}^{G} + \tilde{\varepsilon}_{60}^{G}\}\vec{r}_G^{\,A_4}$$

$$-l_3\omega_{21}^{A}\omega_{21}^{A}\vec{u}_j^{\,T} a_{20}^{T}\tilde{u}_3\tilde{u}_3 a_{32}^{T}\vec{u}_2 + l_3\omega_{32}^{A}\omega_{32}^{A}\vec{u}_j^{\,T} a_{30}^{T}\vec{u}_2 + 2l_3\omega_{21}^{A}\omega_{32}^{A}\vec{u}_j^{\,T} a_{20}^{T}\tilde{u}_3 a_{32}^{T}\vec{u}_1\,,$$

$$(j = 1, 2, 3)\,, \tag{16}$$

where an useful square matrix is introduced

$$\begin{aligned}\tilde{\omega}_{60}^{\circ G}\tilde{\omega}_{60}^{\circ G} + \dot{\tilde{\omega}}_{60}^{\circ G} &= R^{T}(\tilde{\omega}_{60}^{G}\tilde{\omega}_{60}^{G} + \dot{\tilde{\omega}}_{60}^{G})R = \ddot{\alpha}_1 R_1^{T}\tilde{u}_1 R_1 + \ddot{\alpha}_2 R_1^{T} R_2^{T}\tilde{u}_2 R_2 R_1 + \ddot{\alpha}_3 R^{T}\tilde{u}_3 R + \\ &+ \dot{\alpha}_1^{2} R_1^{T}\tilde{u}_1\tilde{u}_1 R_1 + \dot{\alpha}_2^{2} R_1^{T} R_2^{T}\tilde{u}_2\tilde{u}_2 R_2 R_1 + \dot{\alpha}_3^{2} R^{T}\tilde{u}_3\tilde{u}_3 R + \\ &+ 2\dot{\alpha}_1\dot{\alpha}_2 R_1^{T}\tilde{u}_1 R_2^{T}\tilde{u}_2 R_2 R_1 + 2\dot{\alpha}_2\dot{\alpha}_3 R_1^{T} R_2^{T}\tilde{u}_2 R_3^{T}\tilde{u}_3 R + 2\dot{\alpha}_3\dot{\alpha}_1 R_1^{T}\tilde{u}_1 R_2^{T} R_3^{T}\tilde{u}_3 R\,.\end{aligned} \tag{17}$$

The accelerations $\vec{\gamma}_{k0}^{A}$ of the joints A_k and the angular accelerations $\vec{\varepsilon}_{k0}^{A}$ are expressed with some recurrence relations, founded by derivatives of equations (10), (11) and (12)

$$\vec{\varepsilon}_{k0}^{A} = a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{A} + \vec{\varepsilon}_{k,k-1}^{A} + \omega_{k,k-1}^{A} a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\vec{u}_3$$

$$\begin{aligned}\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A} &= a_{k,k-1}(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A})a_{k,k-1}^{T} + \tilde{\omega}_{k,k-1}^{A}\tilde{\omega}_{k,k-1}^{A} + \tilde{\varepsilon}_{k,k-1}^{A} + \\ &+ 2a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}\,a_{k,k-1}^{T}\tilde{\omega}_{k,k-1}^{A}\end{aligned}$$

$$\vec{\gamma}_{k0}^{A} = a_{k,k-1}\vec{\gamma}_{k-1,0}^{A} + a_{k,k-1}(\tilde{\omega}_{k-1,0}^{A}\,\tilde{\omega}_{k-1.0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A})\vec{r}_{k,k-1}^{A} + 2v_{k,k-1}^{A} a_{k,k-1}\tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T}\vec{u}_3 + \gamma_{k,k-1}^{A}\vec{u}_3$$

$$\gamma_{\sigma,\sigma-1}^{A} = 0 \quad (\sigma = 2,\ 3)\,. \tag{18}$$

If other five kinematical chains of the manipulator are pursued, analogous relations can be easily obtained.

The relations (13) and (16) represent *the inverse kinematics model* of the Hexapod parallel robot.

3. EQUATIONS OF MOTION

The dynamics of parallel manipulators is complicated by existence of multiple closed-loop kinematical chains. Difficulties commonly encountered in dynamics modelling of parallel robots include problematic issues such as: complicated spatial kinematical structure with possess a large number of passive degrees of freedom, dominance of inertial forces over the frictional and gravitational components and the problem linked to the solution of the inverse dynamics.

In the recent years, many research works have been conducted on the dynamics of the parallel manipulators. There are three methods, which can provide the same results concerning the determination of the inputs, which must be exerted by the actuators in order to produce a given motion of the end-effector. The first one is using the Newton-Euler classic procedure [14, 15, 34], the second one applies the Lagrange equations and multipliers formalism [1, 9, 23] and the third one is based on the fundamental principle of virtual work [35, 36, 37].

A lot of works have focused on the dynamics of Stewart platform. Dasgupta and Mruthyunjaya [14] used the Newton-Euler approach to develop closed-form dynamic equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints. Tsai [1] presented an algorithm to solve the inverse dynamics for a Stewart platform-type using Newton-Euler equations, which can be reduced to six if a proper sequence is taken. This classical approach requires computation of all constraint forces and moments between the links. However, these computations are not necessary for simulation and control of a manipulator.

Geng [23] and Tsai [9] developed Lagrange equations of motion under some simplifying assumptions regarding the geometry and inertia distribution of the manipulator. The Lagrange formulation is well structured and can be expressed in closed form, but a large amount of symbolic computation is needed to find partial derivatives of the Lagrange's function, the analytical calculi involved are too long for each scheme of the manipulator and they have risk of making errors. Liu et al. [38] derived a set of differential equations for the forward dynamics of legs and moving platform, using the Huston form.

Knowing the position and kinematics state of each link as well as the external forces acting on the robot, in the present paper we apply the principle of virtual work for the inverse dynamic problem in order to establish some definitive recursive matrix relations. The six input forces required in a given motion of the moving platform will easily be computed using a matrix recursive procedure.

Six independent pneumatic or hydraulic systems A, B, C, D, E, F, that generate six input forces $\vec{f}_{10}^{A} = f_{10}^{A}\vec{u}_3$, $\vec{f}_{10}^{B} = f_{10}^{B}\vec{u}_3$, $\vec{f}_{10}^{C} = f_{10}^{C}\vec{u}_3$, $\vec{f}_{10}^{D} = f_{10}^{D}\vec{u}_3$, $\vec{f}_{10}^{E} = f_{10}^{E}\vec{u}_3$, $\vec{f}_{10}^{F} = f_{10}^{F}\vec{u}_3$, which are oriented along the axes $A_1 z_1^A$, $B_1 z_1^B$, $C_1 z_1^C$, $D_1 z_1^D$, $E_1 z_1^E$, $F_1 z_1^F$ control the motion of six moving pistons of the legs.

The parallel robot can artificially be transformed in a set of six open kinematical chains C_i $(i = A, B, C, D, E, F)$ subject to the constraints. This is possible by cutting each joint for moving platform, and takes its effect into account by introducing the corresponding constraint conditions. The first and more simplified open tree system could comprise the moving platform only.

The wrench of two vectors $\vec{F}_k^*$ and $\vec{M}_k^*$ evaluates the influence of the action of the weight $m_k\vec{g}$ and of other external and internal forces applied to the same element T_k of the mechanism

$$\vec{F}_k^* = 9.81 m_k a_{k0}\vec{u}_3\,,\;\; \vec{M}_k^* = 9.81 m_k \tilde{r}_k^C a_{k0}\vec{u}_3\,. \tag{19}$$

Now, we compute the force of inertia $\vec{F}_k^{in}$ and the resulting moment of inertia forces $\vec{M}_k^{in}$ of an arbitrary rigid body T_k of mass m_k with respect to the centre of its first joint:

$$\begin{aligned}\vec{F}_k^{in} &= -m_k\{\vec{\gamma}_{k0} + (\tilde{\omega}_{k0}^2 + \tilde{\varepsilon}_{k0})\vec{r}_k^C\}\\ \vec{M}_k^{in} &= -m_k\{\tilde{r}_k^C\vec{\gamma}_{k0} + \hat{J}_k\vec{\varepsilon}_{k0} + \tilde{\omega}_{k0}\hat{J}_k\vec{\omega}_{k0}\}.\end{aligned} \tag{20}$$

Introducing a 3×3 skew-symmetric matrix $\tilde{r}_k$ *associated* to the vector $\vec{r}_k$, we can express the symmetrical tensor of inertia $\hat{J}_k = \oint \tilde{r}_k \tilde{r}_k^T \mathrm{d}m_k$ of the body T_k.

Considering six successive independent virtual motions of the robot, virtual displacements and velocities should be compatible with the virtual motions imposed by all kinematical constraints and joints at a given instant in time. By intermediate of the Jacobian matrix expressed by the conditions of connectivity (13), the absolute virtual velocities $\vec{v}_{k0}^v$, $\vec{\omega}_{k0}^v$ associated with all moving links are related to a set of independent *relative virtual velocities* $\vec{v}_{k,k-1}^v = v_{k,k-1}^v\vec{u}_3$, $\vec{\omega}_{k,k-1}^v = \omega_{k,k-1}^v\vec{u}_3$.

The fundamental principle of the virtual work [1, 5, 11] states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Assuming that frictional forces at the joints are negligible, the virtual work produced by the forces of constraint at the joints is zero.

Total virtual work contributed by the first active force $\vec{f}_{10}^A$, for example, inertia forces and moments of inertia forces $\vec{F}_k^{in}$, $\vec{M}_k^{in}$ and by the wrench of known external forces $\vec{F}_k^*$, $\vec{M}_k^*$ can be written in a compact form, based on the relative virtual angular velocities *only* $\omega_{k,k-1}^v$. Applying *the fundamental equations of the parallel robots dynamics* established by Stefan Staicu [39], the following matrix relation results for the input force of first prismatic actuator:

$$f_{10}^A = \vec{u}_3^T\{\vec{F}_1^A + \omega_{21a}^{Av}\vec{M}_2^A + \omega_{32a}^{Av}\vec{M}_3^A + \omega_{21a}^{Bv}M_2^B + \omega_{32a}^{Bv}M_3^B + \omega_{21a}^{Cv}\vec{M}_2^C + \omega_{32a}^{Cv}\vec{M}_3^C +$$

$$+\omega_{21a}^{Dv}\vec{M}_2^D+\omega_{32a}^{Dv}\vec{M}_3^D+\omega_{21a}^{Ev}\vec{M}_2^E+\omega_{32a}^{Ev}\vec{M}_3^E+\omega_{21a}^{Fv}\vec{M}_2^F+\omega_{32a}^{Fv}\vec{M}_3^F+$$
$$+v_{10a}^{Gv}\vec{F}_1^G+v_{21a}^{Gv}\vec{F}_2^G+v_{32a}^{Gv}F_3^G+\omega_{43a}^{Gv}\vec{M}_4^G+\omega_{54a}^{Gv}\vec{M}_5^G+\omega_{65a}^{Gv}\vec{M}_6^G\}. \quad (21)$$

Two recursive relations generate the vectors

$$\vec{F}_k^A=\vec{F}_{k0}^A+a_{k+1,k}^T\vec{F}_{k+1}^A,$$

$$\vec{M}_k^A=\vec{M}_{k0}^A+a_{k+1,k}^T\vec{M}_{k+1}^A+\tilde{r}_{k+1,k}^A a_{k+1,k}^T\vec{F}_{k+1}^A, \quad (22)$$

where one denoted

$$\vec{F}_{k0}^A=-\vec{F}_k^{inA}-\vec{F}_k^{*A},\ \ \vec{M}_{k0}^A=-\vec{M}_k^{inA}-\vec{M}_k^{*A}. \quad (23)$$

The relations (21), (22) and (23) represent *the inverse dynamics model* of the Hexapod parallel robot, which can be transformed in a model for automatic command.

However, in the forward dynamics, the method based on the principle of virtual work is not straightforward because of the complicated velocity transform between the joint-space and task-space.

Compared with Tsai's analytical method based on the principle of virtual work [1, 9, 35], the advantages of the present approach are the followings:

• Geometrical constraint relations, under matrix form, generate through successive recast the connectivity conditions that will supply all the relative velocities and relative accelerations, which characterize the independent kinematical chains.

• The accelerations of the mass centers, the angular accelerations and the twists of the inertia forces are expressed through matrix formulae, which contain the kinematical characteristics of the relative motion of the building elements of the manipulator.

• A single matrix relation supplies all the virtual velocities.

• The explicit dynamics equation represents a *definitive formula*, obtained by the transformation of the general expression of the virtual work where the relative virtual velocities only appear, generated by recursive relations.

• All intermediate analytical calculations were eliminated and the numerical computation is achieved through the numerical code, for each active force or torque applied by the driving system.

As applications let us consider a manipulator, which has the following characteristics

$$\alpha_1^*=0,\ \alpha_2^*=0,\ \alpha_3^*=\frac{\pi}{12},\ \alpha_0=\frac{\pi}{36},\ \gamma=\frac{\pi}{4},\ \Delta t=3\ \text{s}$$

$$x_0^{G*}=0.1\ \text{m},\ y_0^{G*}=0\ \text{m},\ z_0^{G*}=0.15\ \text{m}$$

$$m_1=0.1\ \text{kg},\ m_2=0.05\ \text{kg},\ m_3=0.398\ \text{kg},\ m_p=3.983\ \text{kg} \quad (24)$$

$$L_0 = OA_0 = 0.6 \text{ m}, \; l_0 = l_4 = GA_4 = 0.3 \text{ m}, \; l_1 = l_2 = s/2 \,, \; l_3 = 0.5 \text{ m}$$

$$\hat{J}_3 = \begin{bmatrix} 0.0722 & & \\ & 0 & \\ & & 0.0722 \end{bmatrix}, \; \hat{J}_p = \begin{bmatrix} 0.068 & & \\ & 0.068 & \\ & & 0.136 \end{bmatrix}.$$

4. DYNAMICS SIMULATIONS

Based on the algorithm derived from above equations, a computer program was developed to solve the inverse dynamics of the Hexapod manipulator, using the MATLAB software. To validate the dynamics modelling, it is assumed that the moving platform starts at rest from a central configuration and moves along or rotates about one of three orthogonal directions. Furthermore, at the starting time $t = 0$, the centre of moving platform is assumed to be located lower the fixed base at the initial position of coordinates $x_0^G = 0, \; y_0^G = 0, \; z_0^G = h$.

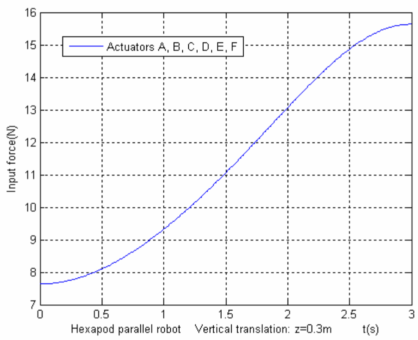

Fig. 3 – Input forces of six actuators: vertical translation.

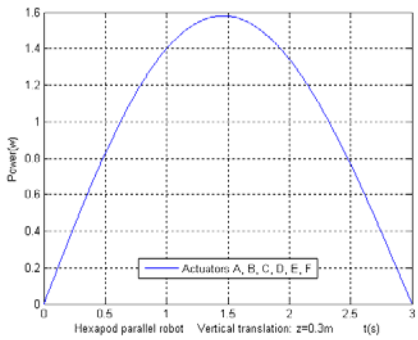

Fig. 4 – Powers of six actuators: vertical translation.

Assuming that there are not external forces and moments acting on the moving platform, the time-history evolution of the input forces and powers required by three pneumatic or hydraulic active systems are shown for a period of three second of platform's motion.

The following examples are solved to illustrate the algorithm. For the first example, the moving platform moves along the *vertical* z_0 direction with variable acceleration while all the other positional parameters are held equal to zero.

As can be seen from Fig. 3 and Fig. 4 it is proved to be true that all input forces and powers are permanently equal to one another. When the moving platform is going to the fixed base, the limbs become more horizontally oriented, therefore increasing the actuating forces.

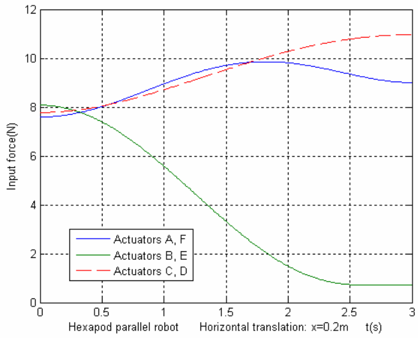

Fig. 5 – Input forces of six actuators: horizontal translation.

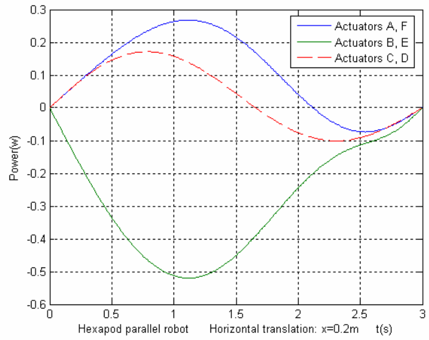

Fig. 6 – Powers of six actuators: horizontal translation.

If the centre G moves along a *rectilinear trajectory* along the horizontal x_0 direction without rotation of the platform, the six input forces (Fig. 5) and the powers (Fig. 6) required by the actuators are calculated by the program and plotted versus time.

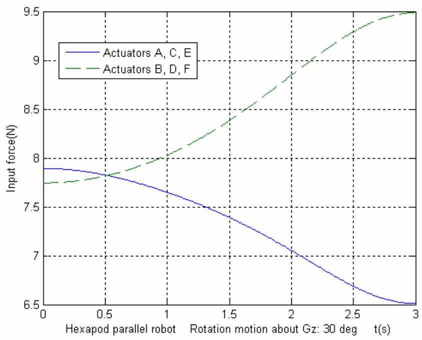

Fig. 7 – Input forces of six actuators: rotation motion.

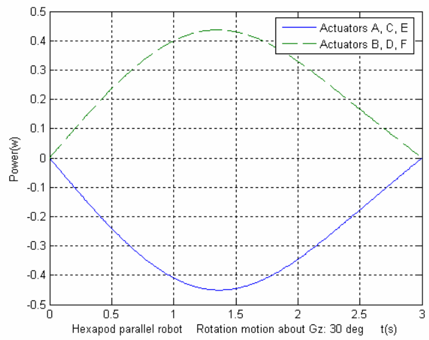

Fig. 8 – Powers of six actuators: rotation motion.

For the third example we consider the *rotation motion* of the moving platform about z_0 axis with a variable angular acceleration $\ddot{\alpha}_3$. From Fig. 7 and Fig. 8, it is remarked that, during the rotation motion of the platform, the actuators B, D, F only are working in an active regime with increasing forces and positive values of the absorbed powers.

5. CONCLUSIONS

Most of dynamical models based on the Lagrange formalism neglect the weight of intermediate bodies and take into consideration the active forces or moments only and the wrench of applied forces on the moving platform. The number of relations given by this approach is equal to the total number of the position variables and Lagrange multipliers inclusive. Also, the analytical calculations involved in these equations are very tedious, thus presenting an elevated risk of errors. The commonly known Newton-Euler method, which takes into account the free-body-diagrams of the mechanism, leads to a large number of equations with unknowns including also the connecting forces in the joints. Finally, the actuating forces could be obtained.

Within the inverse kinematics analysis, the conditions of connectivity (13), (16) that give in real-time the relative velocity and acceleration of each element of the parallel robot have been established in the present paper. The dynamics model takes into consideration the mass, the tensor of inertia and the action of weight and inertia force introduced by each element of the manipulator. Based on the principle of virtual work, the new approach is far more efficient, can eliminate all forces of internal joints and establishes a direct determination of the time-history evolution of active forces and powers required by the three actuators. Also, the method described above is quite available in forward and inverse mechanics of serial and parallel mechanisms, the platform of which behaves in translation, spherical evolution or more general six-degree-of-freedom motion. The recursive matrix relations (21), (22) represent a set of explicit equations of the dynamic simulation and, in a context of automatic command, can easily be transformed into a robust model for computerized control of the Hexapod parallel robot.

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REFERENCES

1. L-W. TSAI, *Robot analysis: the mechanics of serial and parallel manipulators*, John Wiley & Sons, Inc., 1999.
2. D. STEWART, *A Platform with Six Degrees of Freedom*, Proc. Inst. Mech. Eng., Part 1, **15**, *180*, pp. 371-378, 1965.
3. G.R. DUNLOP, T.P. JONES, *Position analysis of a two DOF parallel mechanism-Canterbury tracker*, Mechanism and Machine Theory, **34**, pp. 599-614, 1999.
4. J.A. CARRETERO, R.P. PODHORODESKI, M.N. NAHON, C.M. GOSSELIN, *Kinematic analysis and optimization of a new three-degree-of-freedom spatial parallel manipulator*, Journal of Mechanical Design, **122**, pp. 17-24, 2000.
5. J-P. MERLET, *Parallel robots*, Kluwer Academic Publishers, 2000.
6. V. PARENTI- CASTELLI, R. DI GREGORIO, *A new algorithm based on two extra-sensors for real-time computation of the actual configuration of generalized Stewart-Gough manipulator*, Journal of Mechanical Design, **122**, 2000.

7. R. CLAVEL, *Delta: a fast robot with parallel geometry*, Proceedings of 18th International Symposium on Industrial Robots, Lausanne, pp. 91-100, 1988.
8. S. STAICU, *Recursive modelling in dynamics of Delta parallel robot*, Robotica, Cambridge University Press, **27**, *2*, pp. 199-207, 2009.
9. L-W. TSAI, R. STAMPER, *A parallel manipulator with only translational degrees of freedom*, ASME Design Engineering Technical Conferences, Irvine, CA, 1996.
10. J-M. HERVÉ, F. SPARACINO, *Star. A New Concept in Robotics*, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, pp. 176-183, 1992.
11. J. ANGELES, *Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms*, Springer, 2002.
12. C. GOSSELIN, M. GAGNÉ, *Dynamic models for spherical parallel manipulators,* Proceedings of the IEEE International Conference on Robotics & Automation ICRA'95, Milan, Italy, 1995.
13. J. WANG, C. GOSSELIN, *A new approach for the dynamic analysis of parallel manipulators*, Multibody System Dynamics, **2**, *3*, 1998.
14. B. DASGUPTA, T.S. MRUTHYUNJAYA, *A Newton-Euler formulation for the inverse dynamics of the Stewart platform manipulator*, Mechanism and Machine Theory, Elsevier, **34**, 1998.
15. Y-W. LI, J. WANG, L-P. WANG, X-J. LIU, *Inverse dynamics and simulation of a 3-DOF spatial parallel manipulator*, Proceedings of the IEEE International Conference on Robotics & Automation ICRA'2003, Taipei, Taiwan, pp. 4092-4097, 2003.
16. T. HUANG, Z. LI, M. LI, D. CHETWYND, C.M. GOSSELIN, *Conceptual Design and Dimensional Synthesis of a Novel 2-DOF Translational Parallel Robot for Pick-and-Place Operations*, ASME Journal of Mechanical Design, **126**, pp. 449-455, 2004.
17. D. ZHANG, *Kinetostatic Analysis and Optimization of Parallel and Hybrid Architectures for Machine Tools*, Ph.D. Thesis, Laval University, Québec, Canada, 2000.
18. M. SUZUKI, K. WATANABE, T. SHIUKAWA, T. TOOYAMA, K. HATTORI, *Development of milling machine with parallel mechanism*, Toyota Technical Review, **47**, *1*, pp. 125-130, 1997.
19. E.F. FICHTER, *A Stewart platform-based manipulator: general theory and practical construction*, International Journal of Robotics Research, **5**, *2*, pp. 157-182, 1986.
20. K. SUGIMOTO, *Kinematic and dynamic analysis of parallel manipulators by means of motor algebra*, ASME Journal of Mechanisms, Transmissions and automation in Design, **109**, pp. 3-7, 1987.
21. J-P. MERLET, *Direct kinematics and assembly modes of parallel manipulators,* International Journal of Robotics Research, **11**, *2*, pp. 150-162, 1992.
22. W.Q.D. DO, D.C.H. YANG, *Inverse dynamics and simulation of a platform type of robot*, International Journal of Robotics Research, **5**, *3*, pp. 209-227, 1988.
23. Z. GENG, L.S. HAYNES,T.D. LEE, R.L. CAROLL, *On the dynamic and kinematic analysis of a class of Stewart platforms*, Robotics and Autonomous Systems, **9**, *4*, pp. 237-254, 1992.
24. K. HASHIMOTO, H. KIMURA, *A new parallel algorithm for the inverse dynamics*, International Journal of Robotics Research, **8**, *1*, pp. 63-76, 1989.
25. A. FIJANY, A.K. BEJEZY, *Parallel computation of manipulator inverse dynamics*, Journal of Robotic Systems, **8**, *5*, pp. 599-635, 1991.
26. Z. JI, *Study of the effect of leg inertia in Stewart platforms*, Proceedings of the IEEE International Conference on Robotics & Automation ICRA'93, Atlanta, GA, 1993, pp. 121-126.
27. A. CODOUREY, *Dynamic modelling of parallel robotics for computed-torque control implementation,* International Journal of Robotics Research, **17**, *12*, pp. 1325-1336, 1998.
28. J. ANGELES, S. LEE, *The formulation of dynamical equations of holonomic mechanical systems using a natural orthogonal complement*, ASME Journal of Applied Mechanics, **55**, pp. 243-254, 1988.
29. E. ZANGANEH, R. SINATRA, J. ANGELES, *Dynamics of a six-degree-of-freedom parallel manipulator with revolute legs*, Robotica, Cambridge University Press, **15**, *4*, 1997.
30. F-F. XI, R. SINATRA, *Effect of dynamic balancing on four-bar linkage vibrations*, Mechanism and Machine Theory, Elsevier, **32**, *6*, pp. 715-728, 1997.

31. S. STAICU, *Recursive modelling in dynamics of Agile Wrist spherical parallel robot*, Robotics and Computer-Integrated Manufacturing, Elsevier, **25**, *2*, pp. 409-416, 2009.
32. S. STAICU, *Power requirement comparison in the 3-RPR planar parallel robot dynamics*, Mechanism and Machine Theory, Elsevier, **44**, *5*, pp. 1045-1057, 2009.
33. S. STAICU, X-J. LIU, J. WANG, *Inverse dynamics of the HALF parallel manipulator with revolute actuators*, Nonlinear Dynamics, Springer, **50**, *1-2*, pp. 1-12, 2007.
34. G. LEBRET, K. LIU, F.L. LEWIS, *Dynamic analysis and control of a Stewart platform manipulator*, Journal of Robotic Systems, **10**, *5*, pp. 629-655, 1993.
35. L-W. TSAI, *Solving the inverse dynamics of Stewart-Gough manipulator by the principle of virtual work*, ASME Journal of Mechanical Design, **122**, 2000.
36. S. STAICU, D. ZHANG, *A novel dynamic modelling approach for parallel mechanisms analysis*, Robotics and Computer-Integrated Manufacturing, Elsevier, **24**, *1*, pp. 125-130, 2008.
37. C-D. ZHANG, S-M. SONG, *An Efficient Method for Inverse Dynamics of Manipulators Based on Virtual Work Principle*, Journal of Robotic Systems, **10**, *5*, 1993.
38. M-J. LIU, C-X. LI and C-N. LI, *Dynamic analysis of the Gough-Stewart platform manipulator*, IEEE Transactions on Robotics and Automation, **16**, *1*, pp. 94-98, 2000.
39. S. STAICU, *Relations matricielles de récurrence en dynamique des mécanismes*, Rev. Roum. Sci. Techn. – Méc. Appl., **50**, *1-3*, 2005.