ON THE RESONANCE WAVE INTERACTION PHENOMENON

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Resonant wave interaction is a nonlinear process in which energy is transferred between different natural modes of a system by resonance. For a nonlinear system, the motion is not a summation of the linear modes, but consists of the linear harmonics plus their nonlinear coupling. This paper is studying the dynamic interaction between an attachment of cubic stiffness nonlinearity and the beam. The resonant interactions of the attachment with incident traveling wave propagating in the beam are studied by using the cnoidal method.

1. INTRODUCTION

It is of interest to explore the dynamic interaction of linear media as a beam with local nonlinear attachments, and to determine optimal configurations in terms of dispersive characteristics of the linear medium and system parameters of the attachment that result in maximum absorption of energy from the medium to the attachment. Under resonance conditions, the nonlinear coupling between different modes may lead to excitation of neutral modes. An interesting situation occurs in systems coupling a main structure with a nonlinear attachment, where isolated resonance captures are resulting as a consequence of the energy pumping [1–5]. The energy pumping is an irreversible transfer of vibration energy from the main structure to its attachment. It is interesting to note that this transient resonant interaction results in broadband passive absorption of energy by the attachment, in contrast to the linear vibration absorber whose effect is narrowband [6]. The lack of a linear part in the stiffness nonlinearity of the attachment makes it possible for it to engage in instantaneous resonance with incident waves as well as modes of the beam. This result is in agreement with similar findings of previous works where it was shown that essentially nonlinear passive attachments are capable of engaging in 1:1 resonance capture with (and extracting energy from) a series of linear modes of linear periodic chains to which they are weakly connected [6–8]. The interaction of incident travelling waves with the attachment can lead to phenomena such as speed up or slow down of the travelling wave, scattering of the wave to multiple

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independent wave packets, or trapping of the wave in the form of a localized wave [97]. The use of a nonlinear attachment applied on a beam is studied in this paper, for analyzing the energy exchange for an external sine excitation applied on the assembly. The result consists in a significant attenuation of the structural waves over a broad frequency range for arbitrarily located excitation. A specific application of such a system to a railway track is discussed in [10, 11].

2. THE THEORY

Consider an elastic beam of length L connected to a grounded local attachment of unit mass, viscous damping and stiffness nonlinearity. The connection between the rod and the nonlinear end attachment is made on the point $x = x_{\mathrm{A}}$ by means of a weak linear stiffness. Let us assume that $v(t)$ is the displacements of the attachment, the beam are initially at rest and that an external force $f(0,t) = A\sin\omega t$ is applied at the origin O of the coordinate system, at $t = 0$. The displacement $y(x_{\mathrm{A}},t)$ of the rod at the point of attachment A, in the direction of $v(t)$, can be written as [6]

$$y(x_{\mathrm{A}},t) = \int_{-\infty}^{t} f(0,\tau) g_{\mathrm{AO}}(t-\tau)\mathrm{d}\tau - \int_{-\infty}^{t} \varepsilon\big(y(x_{\mathrm{A}},\tau) - v(\tau)\big) f(0,\tau) g_{\mathrm{AA}}(t-\tau)\mathrm{d}\tau, \tag{1}$$

where the Green's function g_{AA} is the displacement at point A of the beam in the direction of $v(t)$, due to a unit impulse applied at the same point and the same direction, and the Green's function g_{AO} is the displacement at point A of the beam in the direction of $v(t)$, due to a unit impulse applied at origin O in the direction of the external force. The motion equation of the attachment is given by

$$\ddot{v}(t) + \lambda\dot{v}(t) + \sum_{j=1}^{p} \alpha_j v^j(t) = \varepsilon[y(x_{\mathrm{A}},t) - v(t)], \quad v(0) = \dot{v}(0) = 0, \tag{2}$$

where $0 \le \varepsilon \le 1$ scales the weak coupling, λ denotes the viscous damping coefficient, and α_j, $j = 1,...,p$, the coefficients of the stiffness nonlinearity. The lack of a linear part in the stiffness nonlinearity of the attachment makes possible for it to engage in instantaneous resonance with incident waves of the beam. This result is in agreement with similar findings of previous works where it was shown that essentially nonlinear passive attachments are capable of engaging in 1:1 resonance capture with (and extracting energy from) a series of linear modes of linear periodic chains to which they are weakly connected.

Substituting (2) in (1), the following equation for the oscillation of the attachment is obtained

$$\ddot{v}(t)+\lambda\dot{v}(t)+\sum_{j=1}^{p}\alpha_j v^j(t)=\sum_{n=1}^{N}(-1)^{n-1}\varepsilon^n\Delta_n\ ,$$
$$\Delta_n=f(0,t)*g_{\mathrm{AO}}(t)*g_{\mathrm{AA}}^{n-1}(t)+(-1)^n v(t)*g_{\mathrm{AA}}^{n-1}(t)\,. \tag{3}$$

The Green functions are expanded by a set of cnoidal functions [10]

$$g(t)=\sum_{n=0}^{N}\frac{1}{a_n}\mathrm{cn}^2(\omega_n t;m_n)\,, \tag{4}$$

where the modulus m of the Jacobean elliptic function is $m=\dfrac{e_2-e_3}{e_1-e_3}$ with $e_1>e_2>e_3$ the real roots of the equation $4y^3-g_1y-g_2=0$, $g_1,g_2\in\mathbb{R}$ and satisfying the condition $g_1^3-27g_2^2>0$. The motion equation of the beam is

$$y_{tt}(x,t)+\omega_0^2y(x,t)-y_{xx}(x,t)=f(0,t)\,,\ \ y_x(x_{\mathrm{A}},t)+\varepsilon[v(t)-y(x_{\mathrm{A}},t)]=0\,,$$
$$y(x_{\mathrm{A}}-L,t)=0\,,\ \ y(x,0)=y_t(x,0)=v(0)=\dot{v}(0)=0\,, \tag{5}$$

where $x_A=L+e$ ($x=0$ and $x=L+e$ are the ends of the rod) and ω_0^2 is the normalized stiffness of the elastic foundation.

3. RESULTS

The calculations are carried out for $L=200$, $\omega_0=\sqrt{1.2}$, $A=10$, $\lambda=0.4$, $N=5$, $e=1$, $\varepsilon=0.14$ and the cubic stiffness nonlinearity $p=3$ and $\alpha_1=3.3$, $\alpha_2=3.5$, $\alpha_1=4$ [10]. The response $v(t)$ of the attachment at $x=L+e$ is displayed in Fig. 1. Instantaneous frequency of the nonlinear attachment is depicted in Fig. 2. These figures put into evidence the presence of four regimes of transient responses. The first regime (0–90s) describes the interaction of the nonlinear attachment with incoming travelling waves with frequency $\omega>\omega_0$. After a short transition, the attachment passes to periodic oscillation of the second regime (140–260s) with frequency nearly below ω_0, and after another short transition to a weakly oscillation of the third regime (340–460s) with frequency nearly above ω_0. The periodic motion of the second and third regimes are the consequence of energy pumping where the attachment engages in 1–1 resonance capture with a linear structural mode [6, 12].

The last regime (550–800s) consists in weakly modulated periodic motions in the neighbourhood of ω_0. The transition between the third and fourth regimes

(480–540s) describes the case when the attachment can no longer sustain resonance capture, and escape from resonance capture occurs. The energy is radiated back to the rod and the instantaneous frequency decreases until it reaches a frequency ≈ 0 [12]. By comparing our results with those obtained in [6] for impulse excitation and step initial displacement distribution, we observe that in the case of a sine external force, four regimes are depicted, and not three as in [6]. This can be explained by an oscillatory irreversible transfer of vibration energy from the rod to its nonlinear attachment. Two steps of energy pumping for 1–1 resonance capture with the linear structural mode are depicted, for two weakly modulated periodic motions with nearly equal frequency (below and above ω_0). We can term this phenomenon as an oscillatory energy pumping.

The response of the attachment in the second regime (140–260s) with frequency nearly below ω_0 is shown in Fig. 3. The response of the attachment in the third regime (340–460s) with frequency nearly above ω_0, and respectively, in the transition zone between the third and the fourth regime (480–540s) are shown in Fig. 3, respectively, Fig. 4.

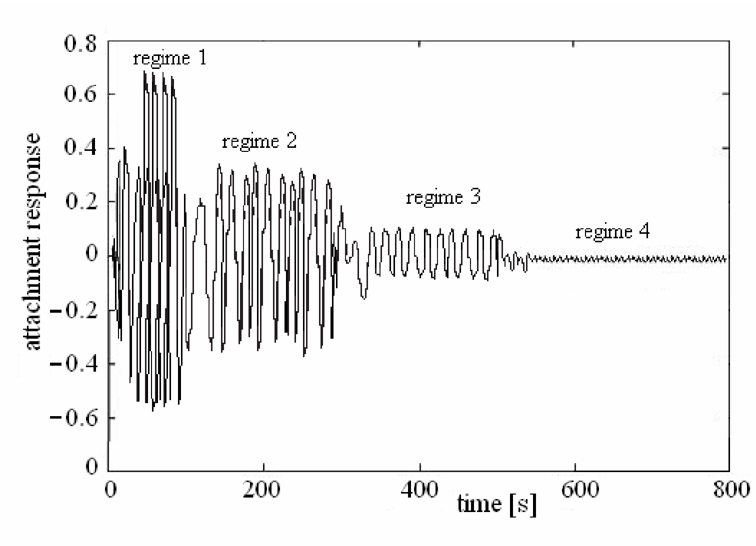

Fig. 1 – The response of the attachment.

From these figures we see that the nonlinear attachment behaved as a passive, broadband, adaptive boundary controller. The resonant interactions of the nonlinear attachment with traveling waves in the pass band of the beam, can be considered as analytical continuations of resonance capture cascades as the number of subsystems of the connected linear chain tends to infinity, and the dynamics approaches the continuum limit [6]. As the energy of the attachment decreases due to damping and energy radiation back to the beam in the form of traveling waves, its instantaneous frequency continuously decreases and approaches the bounding frequency ω_0. Then, the attachment engages in 1:1 resonance capture with the in-phase mode of the rod, in similarity to resonance captures studied in earlier works in alternative finite-chain attachment configurations.

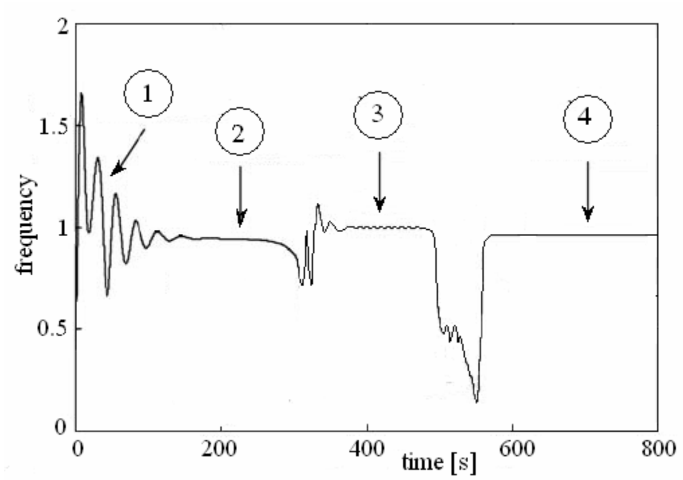

Fig. 2 – Instantaneous frequency of the nonlinear attachment.

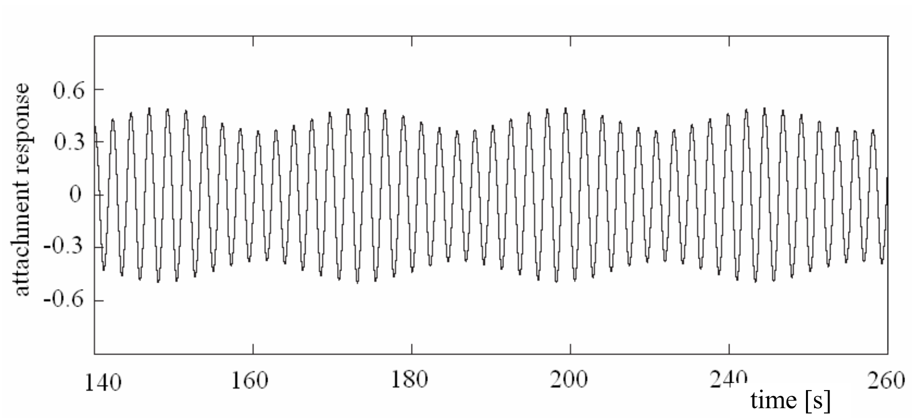

Fig. 3 – The response of the attachment in the second regime.

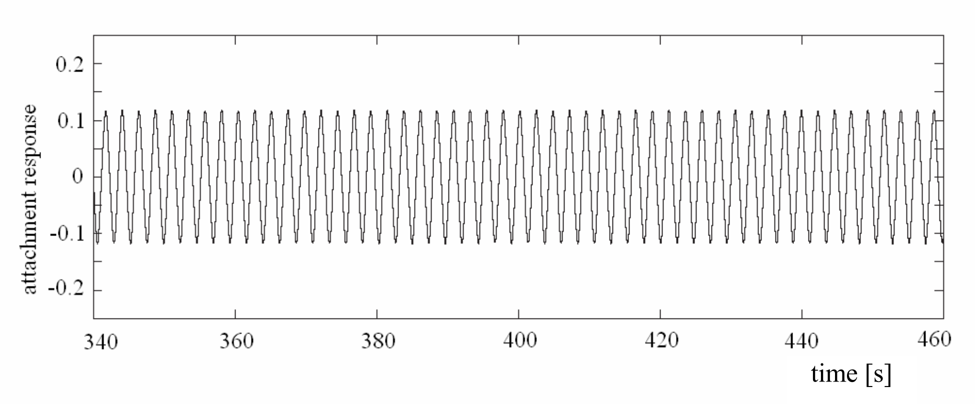

Fig. 4 – The response of the attachment in the third regime.

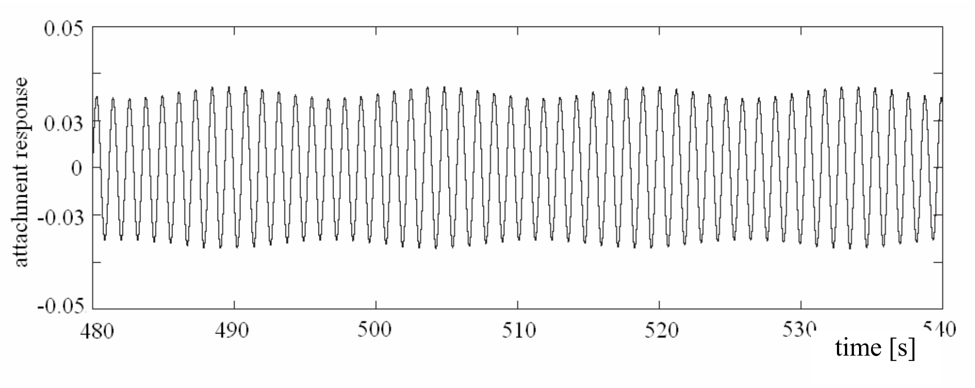

Fig. 5 – The response of the attachment in the transition zone between the third and the fourth regime.

From this analysis, it results that it is necessary to increase the effectiveness of the energy pumping and to use very small nonlinear attachments to be able to apply it to real structures. The nonlinear additional structure which is coupled to the initial beam by giving desired results must be carefully analyzed.

Now it is the time to explain the chosen values for the weak stiffness parameter ε and the cubic stiffness parameters $\alpha_i, i=1,2,3$, namely $\varepsilon = 0.14$ and $\alpha_1 = 3.3$, $\alpha_2 = 3.5$, $\alpha_1 = 4$. We have chosen these parameters so that the energy transfer from the beam to the attachment to be optimal. The energy transfer must be effective for a fixed and not large time interval. For a quantitative valuation of this transfer we estimate the following: if a loss of energy for the initially perturbed linear oscillator $\dfrac{T-T_1}{T}$ during this time interval is equal to 70% or more, the energy transfer is considered as optimal [13]. The optimal energy transfer in a plane of the system parameters (ε, α_i), $i = 1,2,3$ is represented in Fig. 6. The parameter ε varies in the interval (0.08, 0.2), and the parameters α_i, $i = 1,2,3$ in the interval (2.5, 5.5).

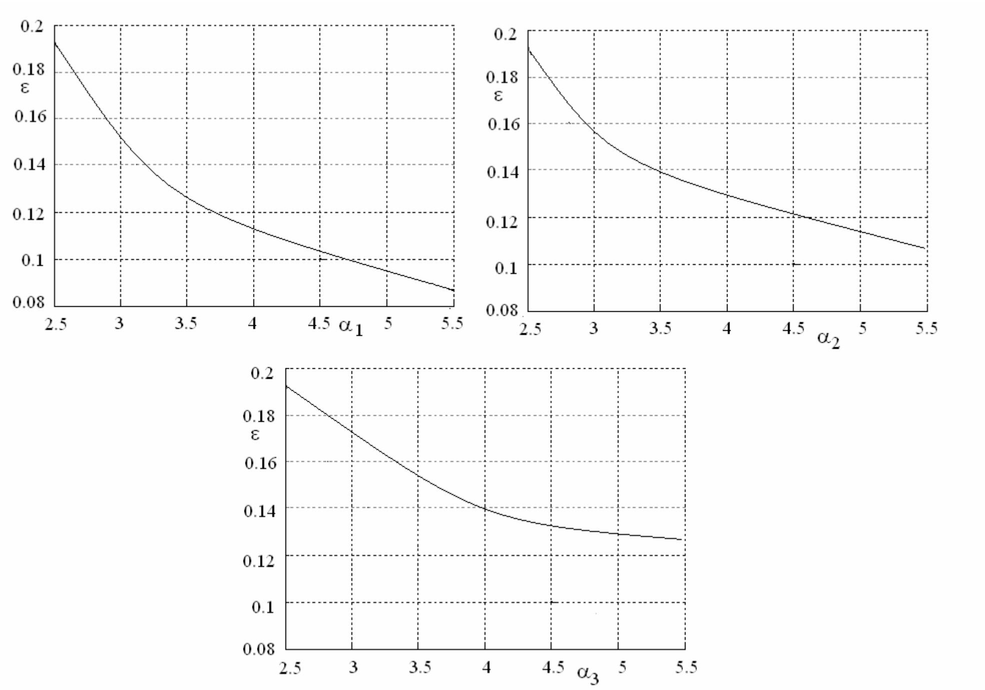

Fig. 6 – The optimal energy transfer in the plane of parameters (ε, α_i), $i = 1,2,3$.

4. CONCLUSIONS

In this paper, the energy exchange between a beam and a nonlinear end attachment is analyzed, for an external sine excitation applied on the assembly. It is studied the transition of the attachment motion from the pass band to the resonance capture regime. The reported results indicate that the nonlinear attachment is able to extract energy from the beam in a multi-frequency fashion, through simultaneous dynamic interactions of multiple modes of the nonlinear attachment with multiple modes of the linear system. The nonlinear attachment acts, in essence, as a nonlinear energy sink.

As shown in [4] and [14], the physics of the energy pumping/resonance capture phenomenon in a nonconservative system can be understood and explained by studying the energy dependence of the nonlinear free periodic solutions (nonlinear normal modes [15]) of the corresponding conservative system that is obtained when all nonconservative forces are eliminated. The enhancement of the energy pumping properties of the system should involve proper design of the topological structure of the nonlinear normal modes of the underlying conservative system.

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REFERENCES

1. GILSON, C.R., RATTER, M.C., *Three-dimensional three-wave interactions: a bilinear approach*, J. Phys. A: Math. Gen., **31**, *1*, pp. 349-367, 1998.
2. MUNTEANU, L., DONESCU, Şt., *Introduction to Soliton Theory: Applications to Mechanics*, Book Series "Fundamental Theories of Physics", Kluwer Academic Publishers, 2004.
3. MANEVITCH, L.I., GENDELMAN, O., MUSIENKO, A.I., VAKAKIS, A.F., BERGMAN, L., *Dynamic interaction of a semi-infinite linear chain of coupled oscillators with a strongly nonlinear end attachment*, Physica D: Nonlinear Phenomena, **178**, *1-2*, pp. 1-18, 2003.
4. VAKAKIS, A.F., MANEVITCH, L.I., GENDELMAN, O., BERGMAN, L., *Dynamics of linear discrete systems connected to local, essentially nonlinear attachments*, J. Sound Vibration, **264**, *3*, pp. 559-577, 2003.
5. VAKAKIS, A.F., GENDELMAN, O., *Energy pumping in nonlinear mechanical oscillators: Part II – Resonance capture*, J. Appl. Mech., **68**, *1*, pp. 42-48, 2001.
6. VAKAKIS, A.F., MANEVITCH, L.I., MUSIENKO, A.I., KERSCHEN, G., BERGMAN, L.A., *Transient dynamics of a dispersive elastic wave guide weakly coupled to an essentially nonlinear end attachment*, Wave Motion, **41**, *2*, pp. 109-132, 2005.
7. QUINN, D., RAND, R.H., *The dynamics of resonant capture*, Nonlinear Dynamics, **8**, *1*, pp. 1-20, 1995.
8. QUINN, D., *Resonance capture in a three degree-of-freedom mechanical system*, Nonlinear Dynamics, **14**, *4*, pp. 309-333, 1997.
9. GOODMAN, R.H., HOLMES, P.J., WEINSTEIN, M.I., *Interaction of sine-Gordon kinks with defects: phase space transport in a two-mode model*, Physica D: Nonlinear Phenomena, **161**, *1-2*, pp. 21-44, 2002.
10. THOMPSON, D.J., JONES, C.J.C., WATERS, T.P., FARRINGTON, D., *A tuned damping device for reducing noise from railway track*, Applied Acoustics, **68**, *1*, pp. 43-57, 2007.
11. THOMPSON, D.J., *The theory of a continuous damped vibration absorber to reduce broad-band wave propagation in beams*, University of Southampton, Institute of Sound and Vibration Research Dynamic Group, ISVR Technical Memorandum no. 968, 2007.
12. TEODORESCU, P.P., CHIROIU, V., *On the nonlinear resonance wave interactions*, National Conference on Mechanics of Solids, Bucharest, September 10-12, 2009.
13. MIKHLIN, Y., PIERRE, C., AVRAMOV, K., KOZMIN, A., *Nonlinear normal modes and transfer of energy in systems containing essentially nonlinear elements*, 2nd International Conference on Nonlinear Normal Modes and Localization in Vibrating Systems, Samos, Greece, June 19-23, 2006.
14. VAKAKIS, A.F., *Designing a linear structure with a local nonlinear attachment for enhanced energy pumping,* Meccanica, **38**, *6*, pp. 677-686, 2003.
15. VAKAKIS, A.F., MANEVITCH, L.I., MIKHLIN, Y.V., PILIPCHUK, V.N., ZEVIN, A.A., *Normal modes and localization in nonlinear systems*, Wiley Interscience, New York, 1996.