

ON THE LINIARIZATION OF EXPERIMENTAL HYSTERETIC LOOPS

TUDOR SIRETEANU¹, MARIUS GIUCLEA¹, OVIDIU SOLOMON²

In this paper is presented a linearization method developed on the basis of the experimental hysteresis loop, by considering the most general case, when there is no mathematical model associated with the hysteretic behavior. The parameters of linear model are determined for a predicted dominant frequency component in the response spectrum of the mechanical structure equipped by the studied hysteretic device. The performances of proposed method are illustrated on the basis of an experimental loop corresponding to a seismic protection device which is used in base isolation of buildings.

1. INTRODUCTION

The behaviour of materials, structural elements or vibration isolators is described by hysteretic loops that are treated in a unified manner by a single nonlinear differential equation with no need to distinguish different phases of the applied loading pattern. In practice, the Bouc-Wen model [1, 2] is mostly used within the following inverse problem approach: given a set of experimental input–output data, how to adjust the Bouc-Wen model parameters so that the output of the model matches the experimental data. Usually, the experimental data are obtained by imposing cyclic relative motions between the mounting ends on the testing rig of a sample material, structural element or vibration control device and by recording the evolution of the developed force versus the imposed displacement. Various methods where developed to identify the model parameters from the experimental data of periodic vibration tests: analytical approaches, for example [3] and different methods based on genetic algorithms for extended versions of Bouc-Wen model, [4, 5, 6, 7].

One of the most efficient techniques for approximating non-linear models within the operating domain is the linearization method, both in deterministic and stochastic systems. An important advantage of this approach is, unlike other methods, it can readily be used to deal with complex systems having many degrees of freedom and with complex types of excitations. There are many studies about

¹ Institute of Solid Mechanics – Romanian Academy, Const. Mille, 15, RO-010141.

² Romanian-American University, Expozitiei Bvd., 1B, RO-012101.

statistical (or equivalent) linearization, [8, 9, 10] that have proved the efficiency of this approach. In the linearization techniques literature the linear model is obtained by taking as reference the non-linear equation which models the hysteretic loop.

This paper presents a linearization method that is developed only on the basis of experimental hysteretic loop, considering the general case, when there is no mathematical model associated to the hysteretic behavior. The linear model parameters are determined for a predicted dominant frequency component in the response spectrum of the mechanical structure equipped by the considered hysteretic device.

2. MATHEMATICAL MODEL

In this paper is considered known the dimensionless experimental hysteretic loop in the coordinates ξ and Φ where $\xi = \xi_m \sin(\omega_{\text{exp}} t)$ is the imposed cyclic displacement and $\Phi = \Phi(t)$ the force (Fig. 1).

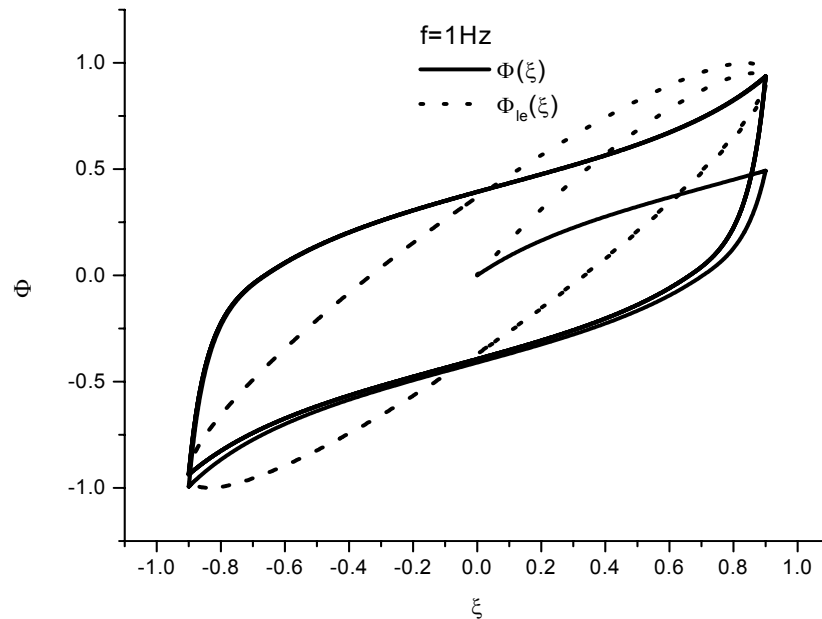


Fig. 1 – Hysteretic loops: experimental (—) and linear equivalent (- - -).

The maximum value of hysteretic force is denoted by Φ_m and the dissipated energy per cycle associated to the experimental loop is given by $E_d = \oint \Phi(\xi) d\xi$, that is the area of surface enclosed by the hysteretic loop.

In the next section is presented a linearization method and studied its performance using an experimental loop of a seismic protection device – Elastomeric Bearings (BIS) – used for base isolation of buildings [11].

In the proposed approach, the linear equivalent hysteretic force is given by:

$$\Phi_{le} = \alpha \xi + \delta z, \quad (1)$$

where $\alpha \geq 0$ and $\delta > 0$ and the variable z satisfies the differential equation (as in [9]):

$$\dot{z} = az + b\dot{\xi} + c\xi, \quad (2)$$

with $a < 0$ (derived from the stability condition) and $[a] = T^{-1}$, $[c] = T^{-1}$ and b is a dimensionless parameter.

Next, the frequency response function corresponding to the input $\xi(t)$ and output $\Phi_{le}(t)$ is denoted by $H(\omega)$.

In order to determine this function the imposed displacement and velocity are written as:

$$\xi(t) = \xi_m e^{i\omega t}, \quad (3)$$

$$\dot{\xi}(t) = i\omega \xi_m e^{i\omega t}. \quad (4)$$

As $\Phi_{le}(t) = H(\omega) \xi_m e^{i\omega t}$, then by using equation (1), one can obtain:

$$z(t) = \frac{H(\omega) - \alpha}{\delta} \xi_m e^{i\omega t}, \quad (5)$$

$$\dot{z}(t) = i\omega \frac{H(\omega) - \alpha}{\delta} \xi_m e^{i\omega t}. \quad (6)$$

Therefore, the relation number (2) becomes:

$$i\omega \frac{H(\omega) - \alpha}{\delta} = a \frac{H(\omega) - \alpha}{\delta} + bi\omega + c \quad (7)$$

and the following form of $H(t)$ is yielded

$$H(\omega) = \frac{(\alpha + b\delta)\omega^2 + a(a\alpha - c\delta)}{\omega^2 + a^2} - i\delta\omega \frac{ab + c}{\omega^2 + a^2}, \quad (8)$$

with the absolute value

$$|H(\omega)| = \frac{\sqrt{[(\alpha + b\delta)\omega^2 + a(a\alpha - c\delta)]^2 + \delta^2\omega^2(ab + c)^2}}{\omega^2 + a^2}, \quad (9)$$

and the phase

$$\theta(\omega) = \tan^{-1} \frac{-\delta\omega(ab + c)}{(\alpha + b\delta)\omega^2 + a(a\alpha - c\delta)}, \quad (10)$$

having

$$\sin \theta(\omega) = -\frac{\delta\omega(ab + c)}{\sqrt{[(\alpha + b\delta)\omega^2 + a(a\alpha - c\delta)]^2 + \delta^2\omega^2(ab + c)^2}}. \quad (11)$$

Next the following notations are considered:

$$u_1 = \alpha + b\delta, u_2 = (a\alpha - c\delta), v = \frac{u_2}{u_1}. \quad (12)$$

Consequently, the absolute value of frequency response function, the phase and $\sin \theta(\omega)$ can be written:

$$|H(\omega)| = u_1 \sqrt{\frac{\omega^2 + v^2}{\omega^2 + a^2}}, \quad (13)$$

$$\theta(\omega) = \tan^{-1} \frac{(v - a)\omega}{\omega^2 + av}, \quad (14)$$

$$\sin \theta(\omega) = \frac{(v - a)\omega}{\sqrt{(\omega^2 + a^2)(\omega^2 + v^2)}}. \quad (15)$$

In this case, the energy dissipated per cycle by linear equivalent hysteretic force has the form:

$$E_{le}(\omega) = \oint \Phi(\xi) d\xi = \pi |H(\omega)| \xi_m^2 \sin \theta(\omega). \quad (16)$$

From the relations (15) and (16), since $E_{le} > 0$, one can derive:

$$\sin \theta(\omega) > 0 \Leftrightarrow a < v \Leftrightarrow \frac{v}{a} < 1. \quad (17)$$

This method assume known a frequency, denoted ω_{ext} , which is chosen by analyzing the frequency range of interest.

The conditions for obtaining the linear model parameters are:

– $\sin \theta(\omega)$ is maximum for the given frequency ω_{ext} , that is

$$\frac{d \sin \theta}{d \omega}(\omega_{\text{ext}}) = 0 ; \quad (18)$$

– frequency response function for ω_{ext} equals $\frac{\Phi_m}{\xi_m}$:

$$|H(\omega_{\text{ext}})| = \frac{\Phi_m}{\xi_m} ; \quad (19)$$

– energy dissipated per cycle by linear equivalent hysteretic force at ω_{ext} equals E_d :

$$E_{le}(\omega_{\text{ext}}) = E_d . \quad (20)$$

The relation (18) implies

$$\omega_{\text{ext}}^2 = a \nu , \quad (21)$$

with the corresponding maximum value of $\sin \theta(\omega)$:

$$[\sin \theta(\omega)]_{\text{max}} = \sin \theta(\omega_{\text{ext}}) = \frac{\nu - a}{|\nu + a|} . \quad (22)$$

One can observe, as $a < 0$ (stability condition) then using the equation (21) is obtained $\nu < 0$.

An equivalent form of the relation (19) is

$$u_1 \sqrt{\frac{\omega_{\text{ext}}^2 + \nu^2}{\omega_{\text{ext}}^2 + a^2}} = \frac{\Phi_m}{\xi_m} . \quad (23)$$

By using the relation (19), (20) and (22) one can derive ($a < 0$ and $\nu < 0$):

$$\frac{\nu - a}{|\nu + a|} = \frac{E_d}{\pi \xi_m \Phi_m} \Leftrightarrow \frac{-\nu + a}{\nu + a} = \frac{E_d}{\pi \xi_m \Phi_m} . \quad (24)$$

Therefore, one has the parameters α , δ , a , b , c and three relations (21), (23) and (24). As is readily seen from (13), (16) and (23), the amplification factor $|H(\omega)|$ and dissipated energy $E_{le}(\omega)$ do not depend on the parameters α and δ . Hence,

in the framework of the proposed linearization method one can assume the particular values $\alpha = 0$ and $\delta = 1$.

Consequently, in this section is proposed the following: *equivalent linearization algorithm*.

1. The value of ω_{ext} is chosen as function of the frequency range of interest.
2. From the relations (21) and (24) the values of a and υ are obtained:

$$a = -\omega_{\text{ext}} \sqrt{\frac{\mu+1}{\mu-1}}, \quad \upsilon = -\omega_{\text{ext}} \sqrt{\frac{\mu-1}{\mu+1}}. \quad (25)$$

where $\mu = \frac{\pi \xi_m \Phi_m}{E_d} > 1$.

3. The values of b and c are determined by the following relations:

$$u_1 = \frac{\Phi_m}{\xi_m} \sqrt{\frac{\mu+1}{\mu-1}} \quad (\text{using (23)}), \quad u_2 = \upsilon u_1, \quad b = \frac{u_1 - \alpha}{\delta} \quad \text{and} \quad c = \frac{\alpha a - u_2}{\delta}.$$

3. APPLICATION OF LINEARIZATION METHOD

The efficiency of the proposed linearization method is illustrated for the hysteretic loop presented in figure 1, determined by normalization of experimental data. The parameters of the considered loop, involved in the identification of linear equivalent model, are: $E_d = 1.187$, $\xi_m = 0.9$, $\Phi_m = 0.95$. The dominant frequency of the component of an oscillating system equipped by hysteretic device is assumed to be $\omega_{\text{ext}} = 2\pi \text{ rad} \cdot \text{s}^{-1}$ (frequency $f_{\text{ext}} = 1 \text{ Hz}$). Then, by applying the previous equivalent linearization algorithm one can find: $a = -10.1$, $b = 1.67$ and $c = 8.08$. In figure 2 are represented the experimental and the predicted hysteresis loops corresponding to the developed linear model for three frequencies 0.5, 1 and 1.5 Hz. In order to show the quality of linear approximation, in figure 3 are depicted the relative errors associated to the dissipated energy, $E_{\text{le}}(\omega)$:

$$\text{er}E_{\text{le}}(\omega) = \frac{|E_{\text{le}}(\omega) - E_d|}{E_d}. \quad (26)$$

On the graph is pointed out the frequency range where the relative error is less than 20%, which is the interval $[0.7; 4] \text{ Hz}$.

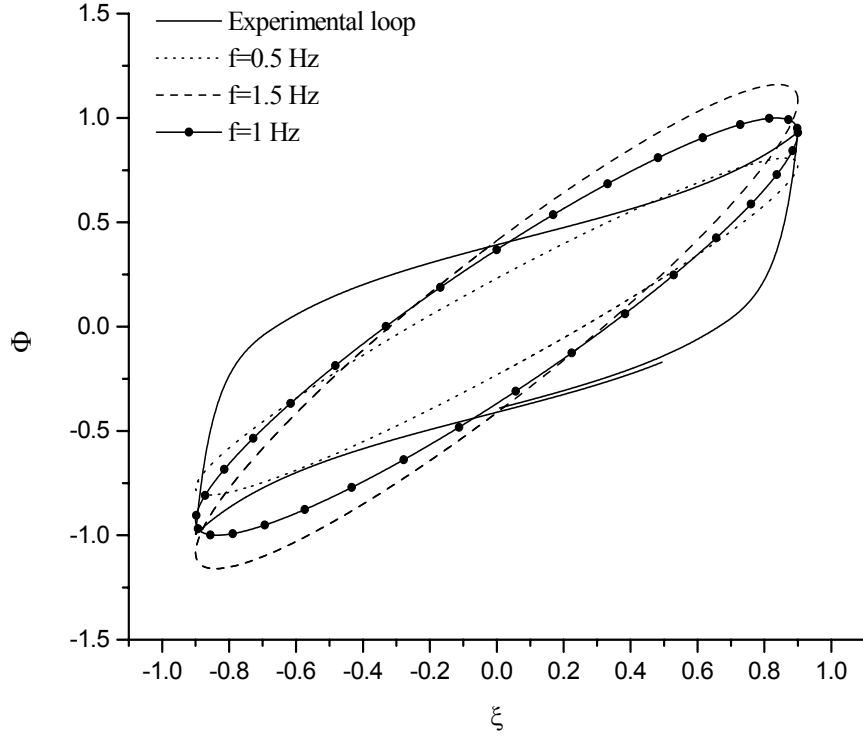


Fig. 2 – Hysteretic loops: experimental and linear equivalent for different frequencies.

The efficiency of the proposed linearization algorithm in terms of the approximation of dissipated energy by the hysteretic device is assessed by the relative error was computed for a frequency range centered in ω_{ext} . Usually, the linear viscous damping $\lambda \dot{\xi}$, equivalent to the energy dissipated by a hysteretic device, is determined from:

$$\lambda = \frac{E_d}{\pi \omega_{\text{ext}} \xi_m^2}. \quad (27)$$

In this case,

$$E_{\text{le}}(\omega) = \frac{\omega}{\omega_{\text{ext}}} E_d. \quad (28)$$

Figure 3 shows comparatively the relative errors of dissipated energy given by the proposed method and by the equivalent viscous damping.

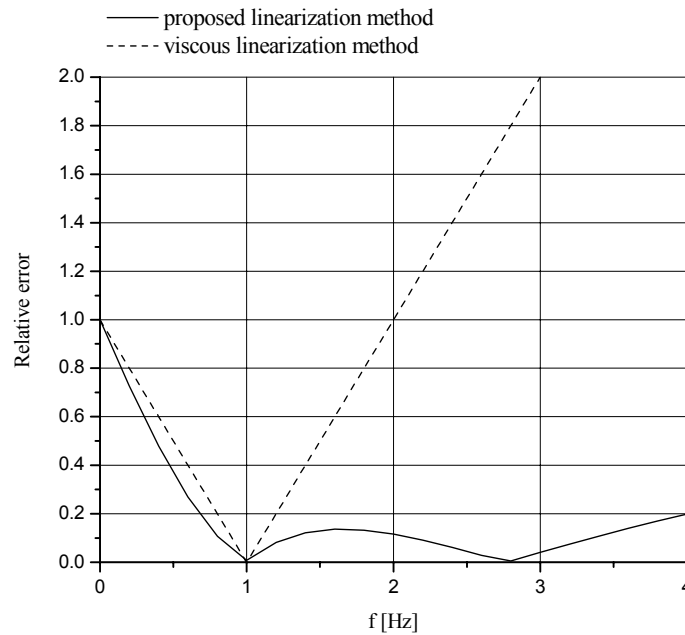


Fig. 3 – Relative errors of dissipated energy.

4. CONCLUSIONS

In this paper is presented an analytical method for the identification of a differential equivalent linear model to approximate experimental hysteretic loops. The linearization algorithm requires the determination of only three parameters which can be easily determined from the force-displacement plot: amplitude of imposed cyclic displacement, the maximum force developed by the tested device and the dissipated energy per cycle, *i.e.* the loop area. Since the parameters of the linear equivalent model depend inherently on the cycle frequency, the linearization algorithm is applied for a convenient choice of this frequency. The obtained results show that the dissipated energy per cycle, obtained for the proposed equivalent linear model, is less dependent on the cycle frequency than in the case of viscous equivalent model. Moreover, the hysteretic curves portrayed by the linear equivalent model are in good agreement with the experimental loop around the frequency used for parameters identification. If this frequency is a resonant

frequency of an oscillating system equipped by the considered hysteretic device, then the proposed linearization method will provide a better description of system response than the one predicted by viscous linearization.

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