BRANCHED SYSTEMS WITH NONLINEAR KINEMATICS – COMPUTER-AIDED ANALYSIS & DESIGN

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Abstract. This paper presents a computer-aided modeling way of the branched systems with nonlinear kinematics that make possible the design and the analysis, as well. Being very complex systems, the elasto-dynamic study of the real branched systems with nonlinear kinematics require a treatment with special tools. The paper shows how the computational mechanics allowed us to create these necessary study tools.

Key words: elasto-dinamic systems, design, mathematica.

1. INTRODUCTION

The elasto-dynamic behavior simulation of multi-branched systems comes to give an answer in *the conflicting requirements between high speed and high accuracy in the systems assembly task*. On one hand, in order to improve the industrial productivity, it is required to reduce the weight and/or to increase the speed of operation and it is very desirable to build flexible systems. On the other hand, for fast-working machines, the flexibility leads to a limited stiffness of certain sections of the shaft and the transmission rods, limitation that *must* be taken into consideration at the design stage. The assumption of a rigid body system does not hold, yet, in that case.

Since the branched systems are widely found, we conducted an extended study to characterization and modeling the elasto-dynamic behavior of this class of multi-body systems generally named branched systems. The term “branched systems” refers to the mechanical systems with a number of branches activated and controlled by the driving links of a common driver, taking into consideration the flexibility of the elements (the "rigid body system" assumption does not hold) and the *essentially nonlinear* position functions of the driver mechanisms (one in each branch) in their real form, without any linearization.

At the same time, for real systems requiring, on one hand, a large number of branches – which leads to a large number of degrees of freedom (DOF) and, on the

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other hand, essentially nonlinear position functions, this study becomes highly complex. Usually, there are two trends: (1) to treat the mechanical systems with a large number of DOF solely as linear systems, or (2) to tackle the nonlinearity for only a few DOF. This work pays attention to both aspects dealing with the study of the elasto-dynamic behaviour of branched systems with a large number of branches, by considering the nonlinearities of their mathematical model.

Developed in the welcoming frame of Computational Mechanics, our study was based on the computer-aided (CA) approach synergistically completed by the Mathematica®[9] software symbolic calculus capabilities. As a result, we achieved a CA way to obtain the mathematical model in a symbolic form for any number of branches (and, by consequence, for any number of DOF). This mathematical model is represented by a system of second order nonlinear differential equations with non-constant coefficients. The number of the system equations, their complexity and high nonlinearity make the numerical treatment as the unique way to solve it (section 3).

Even if these numerical solutions give us enough data for the analysis stage, they can not be used for the design (not in the classical sense). So, we have to turn over a new way.

This paper proposes such a CA way to attain the design for branched systems described by nonlinear mathematical models with numerical solution. The method is applicable to very different kinds of systems. So, we study the feed drive system (modeled as a 1-BM with non-constant components) (Eisinger Borcia *et al.* [4]), the automatic milling systems (considered a combination of two interacting 1-BM's) (Eisinger Borcia *et al.* [5]), the valve train systems (described as a three-branched system) (Eisinger Borcia [1]).

2. THE MATHEMATICAL MODEL

In the following, we will use the abbreviation ***n-BM*** for a multi-body branched system with *n* branches, with a general scheme, as is shown in Fig. 1. Any system with a camshaft driver (Fig. 2) is an immediate application.

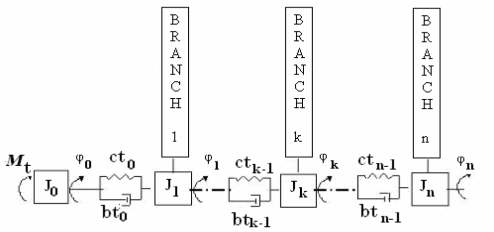

Fig. 1 – A general scheme of an n-BM.

Fig. 2 – Camshaft driver for a 16-BM.

In order to give an appropriate answer, in the case of n-BM, to the classical questions of any dynamical research:

(i) What is the proper dynamic model for a real n-branched system?

(ii) What is an appropriate mathematical description for the dynamic model?

the appropriate framework of basic concepts must be established.

The study of the n-BM was developed in the general assumptions from dynamics of the flexible multi-body systems, with explicitly reference to the following two:

(i) *The damping assumption* – the friction is proportional to the relative speed between two neighboring inertial bodies; the energy dissipation in the elastic element is proportional to the relative speed of the masses connected by the element.

(ii) *The bending assumption* – the bending deflection is defined as the vertical (only) displacement of the neutral axis of the deformed beam (Eisinger Borcia [3]).

To automatically obtain the mathematical model for an n-BM we used special terms and notions (Eisinger Borcia [3]) like dynamic model, vibratory element, typical branch, descriptor matrix that will be briefly presented in the following. Generally, every specific mechanic element can be schematically described either as a continuous model or as a lumped model with a single mass or a finite number of concentrated masses. In this work we chose the *lumped model*. Consequently, the *dynamic model* of an n-BM is represented as a collection of vibratory elements in parallel or/and in series connection. The *vibratory element* is a generic name for a mechanical item that provides all the necessary information to describe the punctual dynamical behavior by three components: a kinematical one – Π; an inertial one – J or m and a deformability one – c (optionally with damping, b) (Eisinger Borcia [3]). The type and dimension of the inertial and deformability components are to be chosen according to the motion. The kinematical properties in a certain point are represented by the kinematical analog (often called the position function) that describes the kinematical ratio between adjacent elements if it differs from 1 (see, Vulfson II [7] and [8]).

Our *CA*-method to obtain the mathematical model for a n-BM started to the following observation: under the assumption that the n-MB develops only rotation and translational motion and the main shaft driver undergoes bending (according to assumption *ii*), the dynamical model can be represented as a series and/or parallel connection of vibratory elements that can be grouped in four subsystems (S1– S4) and in each subsystem, the same equation of motion is respected:

$$[\mathbf{M}_i]\ddot{\mathbf{q}}_i + [\mathbf{B}_i]\dot{\mathbf{q}}_i + [\mathbf{C}_i]\mathbf{q}_i = \mathbf{F}_i\,, \tag{1}$$

where $i = 1, 4$ is the subsystem number.

The subsystems S1 and S4 deal with the driver (which serves as a programming tool for the whole system), undergoing mainly torsion and bending deformations, and vibrations. The subsystems S2 and S3 deal with the branches-each branch representing a chain of mechanical elements (for details, see, Eisinger Borcia [2] and [6]).

Since a real system can lead to a dynamical model that contains more than one vibratory element in a branch, on one hand, and, on the other hand, not all the subsystems S1-S4 are necessarily present, we introduced the term *typical branch* to indicate a branch composed of four vibratory elements – one from each subsystem S1-S4 and shown in Fig. 3. Now, *the dynamical model of an n-BM can be redefined as a series connection of typical and/or atypical branches* (Fig. 4).

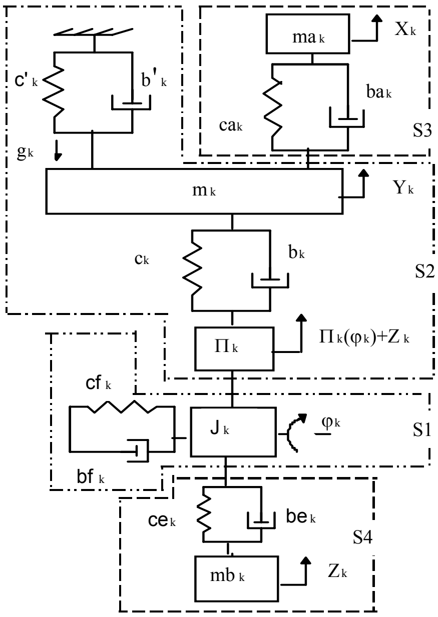

Fig. 3 – A typical branch.

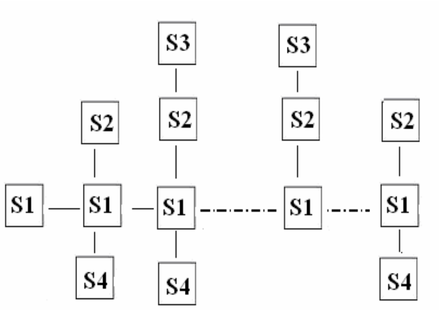

Fig. 4 – A general scheme of the dynamic model of an n-BM that contains only vibratory elements from subsystems S1-S4.

In order to automatically generate the equations of motion of an n-BM, we introduced a binar description, the *descriptor matrix of an n-BM* that describes in a symbolic way its dynamic model: each column describes a branch of the n-BM and each row, a subsystem. The component *kj* of the descriptor matrix represents the number of vibratory elements from the subsystem *k* that exist in the branch *j*–1. The first column of *DE* always describes the motor as a vibratory element of subsystem S1. For example the descriptor matrix of an n-BM from Fig. 4 is:

$$\mathrm{DE} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \cdot & \cdot & \cdot & 1 & 1 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & 1 & 1 \\ 0 & 1 & 1 & \cdot & \cdot & \cdot & 0 & 1 \end{bmatrix}.$$

Based on the descriptor matrix, we obtain automatically the mathematical model for an n-BM, using the notations from Fig. 3, as the following set of second order nonlinear differential equations:

$$J_0\ddot{\varphi}_0 - bf_1\,\dot{q}_{1;1} - cf_1\,q_{1;1} = \mathbf{M}_0,$$

$$\begin{aligned}
& J_k\sum_{i=1}^{k}\ddot{q}_{2;i} + bf_k\,\dot{q}_{1;k} - bf_{k+1}\,\dot{q}_{1;k+1} + cf_k\,q_{1;k} - cf_{k+1}\,q_{1;k+1} = \\
& = (c_k q_{2;k} + b_k\dot{q}_{2;k})\Pi'_k, && k = 1,\dots,n_1 \\
& m_k\ddot{q}_{2;k} + (b_k + b_k^{'})\dot{q}_{2;k} + c_k q_{2;k} = ba_k\dot{q}_{3;k} + ca_k q_{3;k} - \\
& -(g_k + m_k\ddot{Y}_k + b_k^{'}\dot{Y}_k + c_k^{'}Y_k), && k = 1,\dots,n_2 \\
& ma_k\ddot{q}_{3;k} + ba_k\dot{q}_{3;k} + ca_k q_{3;k} = -ma_k(\ddot{q}_{2;k} + \ddot{Y}_k), && k = 1,\dots,n_3 \\
& mb_k\ddot{q}_{4;k} + \sum_{i=1}^{k}\sum_{j=1}^{k} be_{ij}\dot{q}_{4;j} + \sum_{i=1}^{k}\sum_{j=1}^{k} ce_{ij}q_{4;j} = c_k q_{2;k} + b_k\dot{q}_{2;k}, && k = 1,\dots,n_4
\end{aligned} \tag{2}$$

where $Y_k = q_{2;k} + q_{4;k} + \Pi_k(\sum_{i=0}^{k} q_{1;i}))$, $k = 1,\dots,n_s$, $s = 1,\dots,4$.

The solution, the *generalized coordinates* $\mathbf{q}_k$, $k = 1,\dots,n$, represents the difference between the real motion and the ideal motion which will be obtained if the system is treated as a rigid one.

If the motor rotation φ_0 is a known time function, the equation $(2)_1$ allows us to determinate the drive active momentum, $\mathbf{M}_0$ after solving the last n equations.

If $\mathbf{M}_0$ is a known time function, the equations $(2)_{2\text{-}5}$ forms a set of *n* non-linear second-grade differential equations sufficient to determine the *n* generalized coordinates.

If the time-related functions for the active momentum $\mathbf{M}_0$ or the movement φ_0 are unknown explicitly, but an implicit relationship between $\mathbf{M}_0$ and φ_0 (a dynamic characteristic of the engine) is known, this will be attached to the $(n+1)$ differential equations of the systems (2), thus being allowed the determination of the $(n+2)$ unknowns – the $(n+1)$ generalized coordinates $\mathbf{q}_k$ ($k = 0,\dots,n$, $q_0 = \varphi_0$) and the active momentum, $\mathbf{M}_0$.

In all these three cases, the equations system (2) is a set of non-linear second-grade differential equations sufficient to determine the n generalized coordinates.

3. THE COMPUTER-AIDED ANALYSIS

By analysis of a n-MB we mean the determination of the elasto-dynamic behavior of each inertial point of the system.

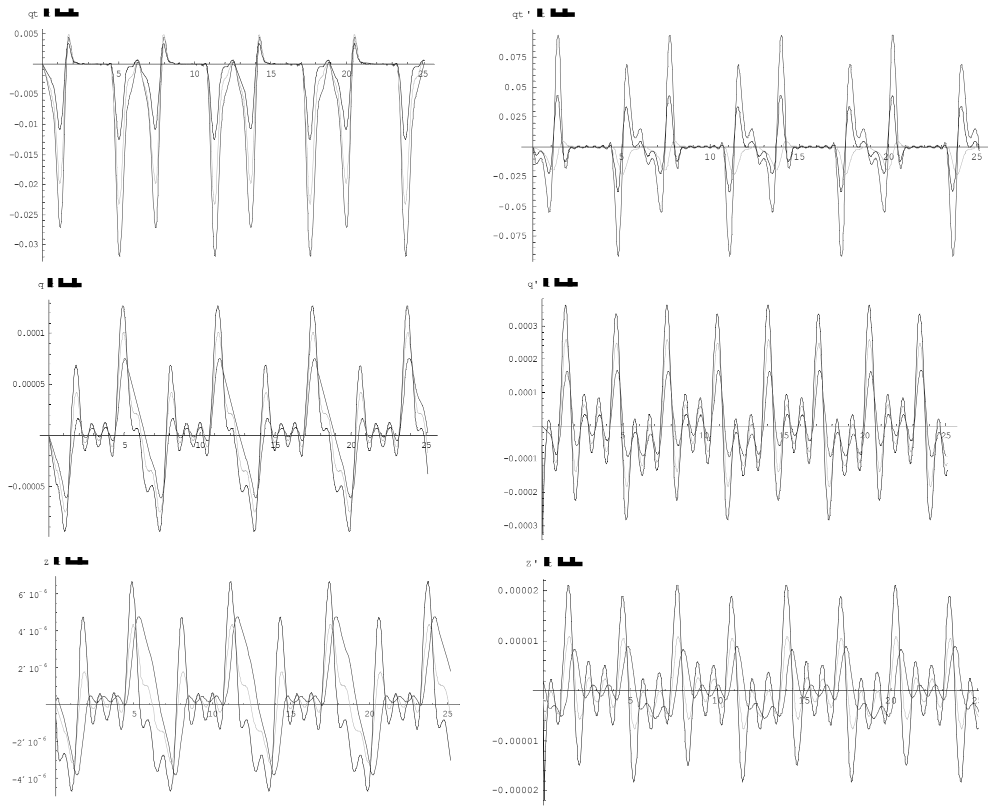

Fig. 3 – Comparative dynamic perturbations and perturbed speeds for 3 branches:
a) the main shaft at torsion (S1 in Fig. 3); b) the main mass (S2 in Fig. 3);
c) the main shaft at bending (S4 in Fig. 3).

Solving the mathematical model with the appropriate set of input data we obtain the dynamic perturbations (the dynamic perturbated speeds and dynamic perturbated accelerations, also) for each inertial point in numeric and/or graphic form. For example, in Figs. 5a-c is shown the comparative data for 3 branches.

4. THE COMPUTER-AIDED DESIGN

In our view, the design of an n-BM can be reach choosing the “best fit” by comparing a large enough set of solutions of the mathematical model. Since we have the possibility to obtain the mathematical model in a *symbolic* form (2) for every n-BM, we can solve it for any set of input data. The easy way to obtain the information allows us to repeat the calculation for a large number of *variants* (the variants differ by design or value of one parameter at least). If the quantity of input data and comparisons is large enough, we can get the combination of parameters that suits our needs: *the fulfillment of predefined criteria*. Since, for each run we get numerical solutions, that mean discreet numerical values for 2n time-functions that request a big storage memory, we need a special tool to compare the solutions. Therefore, we have to define measures that are to be calculated at each run and stored for further comparison. In our case the statistical measures were used.

The increased capability of computers to obtain, to save and to compare a large number of different cases synergistically combined with the Mathematica® [9] software symbolic calculus capabilities and the statistical representation of the results, allowed us to developed a tool for design optimization (“The State Matrix Strategy, a quasi-optimization tool” [6]) to find the values of design parameters which lead to an admissible solutions that fulfills a set of predefined criteria. For this, several terms and definitions have been introduced.

The *state matrix* is a $({}_mN \times {}_pN)$-matrix, where: ${}_mN$ is the number of the chosen measures and ${}_pN$ is the number of the critical points (the special points in which are to be computed different measures). Since the state matrix components could be: all kinds of numbers, matrix, functions and even messages, the word "matrix" is used here with an extended meaning. A point in the $({}_mN \times {}_pN)$-space is named *behavior point*. After each run, a behavior point is achieved. The set of behavior points obtained when all the design parameters cover their utilization domains defines the *behavior map*. The *utilization domain of a design parameter* is a predefined domain in which the parameter may vary without the state matrix exceeding given admissible limits.

Important to observe that the influence on the behavior of the studied model of *any* parameter (not only the numeric ones) can be observed and analyzed. For a n-BM, we can consider: design changes and/or number of constructive parameters; changes in the mass distribution, the damping level, the mean frequency; supplementary design conditions; changes in the dynamic model; any particular parameter relevant for the studied case.

Our “State Matrix Strategy” was developed as a CA-procedure that contains 8 main steps.

i. Define the *problem*: the correct selection of the dynamic model – by correct selection of the dynamic model we mean to choose the grade of approximation: taking (or not) into account the bending of the driving shaft;

taking (or not) into account the vibration damping effects; considering (or not) additional elements of inertia at any section of the dynamic model.

ii. Identify *all the parameters* that can change their value, determine their utilization domains and choose the appropriate *design parameters set*. For an n-BM we have up to 4n physical parameters.

iii. Establish the collection of the input data sets – each set describes a relevant combination of design parameters;

iv. Define the *objective* – a set of static and/or dynamic criteria that have to be fulfilled;

v. Define the *critical points* appropriate for the problem and for the objective. The results/measures obtained in these points have to be stored;

vi. Obtain the *state matrix* that carries the information about each acceptable variant;

vii. Obtain the *behavior map* by repeating step (vi.) for each input data set established at (iii.);

viii. Get (automatically) the *optimal combination* of values of the design parameters for which the objective is reached**.**

Apart from the determination of the numerical values for the physical parameters of the n-MB, this strategy can be use to obtain a large palette of knowledge about elasto-dynamic behavior of any given n-BM, like:

- sensitivity of the elasto-dynamic behavior to the dynamic model;
- sensitivity of the elasto-dynamic behavior to the damping level;
- analysis of the influence of varying the design parameters on the elasto-dynamic behavior;
- supplementary design conditions;
- mean frequency as function of the design parameters;
- optimal mass distribution.

5. CONCLUSIONS

In this work we have proposed a new computer-aided modelling way to study the elasto-dynamic behaviour of branched systems with nonlinear kinematics. A Mathematica®-based software developed to apply this method makes available the analysis and design as well. The core is the description of the mechanical system by a special type of dynamic model and a special type of matrix based on the observation that under the assumption that the n-MB develops only rotation and translational motion and the main shaft driver undergoes vertical bending, the dynamical model can be represented as a series and/or parallel connection of vibratory elements that can be grouped in four subsystems and in each subsystem, the same equation of motion is respected.

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