INFLUENCE OF INTERNAL DAMPING ON CABLE VIBRATION

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Abstract. The problem of vibration control of overhead line conductors subjected to laminar transverse wind, which induce stationary vibrations by Kármán effect, is of high importance due to the consequences on these structures lifetime and service. We consider the conductor (cable) model as Euler-Bernoulli beam with the author condition that detaches the conductor model of the beam model with viscous, hysteretic or dry friction internal damping hypothesis. The original analytical expression of the free vibration modes and the resonance frequencies equation for the cable with clamped extremities were produced. The analytical expression of the dissipated energies of the internal forces of the cable and other analytical results are performed by the authors. A possible equivalence between the internal damping coefficients of the cable models, using the described damping hypothesis, is analyzed.

Key words: cable vibration, modelling and control, Euler-Bernoulli beam, internal damping.

1. INTRODUCTION

We consider the cable model derived from the Euler-Bernoulli beam with viscous, hysteretic or dry friction internal damping [1–12]. The analytical expression of the free vibration modes and the resonance frequencies equation for the cable with clamped extremities are produced using our hypothesis of the cable imposed to Euler-Bernoulli beam, essentially for accurate identification of the cable model parameters. The property of any Euler-Bernoulli beam model to be substituted, for sufficient high frequencies, by our cable model, is underlined. Some recent studies have been performed in our domain of interest [13, 14]. Our experimental research was performed on a specialized stand endowed with the overhead conductor using clamped extremities, alone or with a choice of Stockbridge dampers, mounted on the extreme zones of the span. The resonance frequencies and vibration modes are identified theoretically and also experimentally, on the conductor in the stand. The analytical aspects on the internal damping terms influence versus frequency, in the cable models, are discussed. A possible equivalence between the internal damping coefficients of the cable models, using the described damping hypothesis, is studied, equalizing analytical

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expressions of corresponding damping energies of the cable. The formulas of equivalence between two possible hypotheses of internal damping coefficients of cable are performed and analyzed.

2. MATHEMATICAL MODEL OF CABLE

The following equation is considered:

$$m_l \frac{\partial^2 w_i(x,t)}{\partial t^2} = -\left(c_i^{H*} + k_i\right) w_i(x,t) - \left(c_i^V + \frac{c_i^H}{\omega_i}\right)\frac{\partial\, w_i(x,t)}{\partial\, t} + \\ + T\frac{\partial^2 w_i(x,t)}{\partial\, x^2} - EI\,\frac{\partial^4 w_i(x,t)}{\partial\, x^4} + q(x,t). \tag{1}$$

Equation (1) describes the behavior of the cable, excited by the force $q = q(x,t)$, applied transversally on the cable, whose actions in the point of abscissa *x*, at the time *t*, are described by: the constant coefficient c_i^V (in the case of viscous damping hypothesis); by the constant coefficient of the form c_i^H / ω_i^{VH} (in the case of hysteretic damping hypothesis); and by the coefficient c_i^{H*} (in the case of dry friction (Coulomb) damping hypothesis). The coefficient c_i^{H*} is piecewise constant, as a function of time *t*, and the sign is such that the sign of the damping force $c_i^{H*} w_i(x,t)$ to be opposite to that of the velocity $\dot{w}_i(x,t) = \partial w_i(x,t)/\partial t$, at any time *t*. Other explicit expression of the dry friction force is $c_i^{H1}\left|w_i(x,t)\right| sign(\dot{w}_i(x,t))$, where c_i^{H1} is constant [8]. The first expression of dry friction force is deduced, in our case, taking into account the properties that the functions $w_i(x,t)$ and $\partial w_i(x,t)/\partial t$ are continue and with separable variables. We denote by ω_i^{VH} the circular frequency of order i for damped free vibration, by $f_i^{VH} = \omega_i^{VH} / \pi / 2$ the resonance frequency of order i for damped free vibration, by m_L the mass unit length of the cable, by EI the bending rigidity of the cable, by T the tension in the cable, by k_i the rigidity coefficient of the cable, by $y_i(x,t)$ the corresponding vertical displacement of the cable for vibration mode of order i, and by L the span length of the cable.

Firstly, we search the stabilized free transverse vibrations of the cable without damping and with clamped extremities for the resonance frequencies. The free vibrations without damping are of standing waves form:

$$w_r(x,t) = w_r(x)\sin(\omega_r t + \varphi)\,. \tag{2}$$

In formula (2), $\omega_r = 2\pi f_r$ and f_r is the resonance frequency of the cable and φ is the phase angle between the initial impulse and displacement. The specific notations, used in the paper, provided the condition $m_L\omega_r^2 - k_r > 0$, are defined below

$$\alpha^2 = \frac{TL^2}{EI}, \quad \beta_r^4 = \frac{(m_L\omega_r^2 - k_r)L^4}{EI}, \quad \delta_r = \left[\frac{\alpha^2}{2} + \left(\frac{\alpha^4}{4} + \beta_r^4\right)^{1/2}\right]^{1/2},$$

$$\varepsilon_r = \left[-\frac{\alpha^2}{2} + \left(\frac{\alpha^4}{4} + \beta_r^4\right)^{1/2}\right]^{1/2}, \quad \xi = \frac{x}{L}.$$

The expressions α, β_r, δ_r, ε_r verify the relationships:

$$\delta_r^2 - \varepsilon_r^2 = \alpha^2, \quad \delta_r\,\varepsilon_r = \beta_r^2, \quad \delta_r^4 - \alpha^2\delta_r^2 - \beta_r^4 = 0, \quad \varepsilon_r^4 + \alpha^2\varepsilon_r^2 - \beta_r^4 = 0. \tag{3}$$

We search the solution of (1) using the condition, performed by the cable wire, which represent our hypothesis that detaches the cable model of the beam model [11], [12]:

$$\mathrm{e}^{-\delta_r} \approx 0. \tag{4}$$

For the clamped cable analyzed in this paper, the equation of resonance frequencies is:

$$\alpha^2 \sin\,\varepsilon_r - 2\,\beta_r^2 \cos\,\varepsilon_r = 0. \tag{5}$$

The analytical expression of the free vibration modes for undamped vibrations of the cable, in the case of clamped boundary conditions, is as follows:

$$\begin{aligned} w_r(x) = C_r \Big\{ & \mathrm{e}^{-\delta_r\xi} + \frac{\delta_r}{\varepsilon_r}\sin\,\varepsilon_r\xi - \cos\,\varepsilon_r\xi - \\ & - \frac{\delta_r}{\varepsilon_r}\,\mathrm{e}^{\delta_r(\xi-1)}\sin\,\varepsilon_r + \mathrm{e}^{\delta_r(\xi-1)}\cos\,\varepsilon_r \Big\}. \end{aligned} \tag{6}$$

The factor C_r, $r = 1, 2, \ldots$ is a constant.

It can be verified that any Euler-Bernoulli beam model can be substituted by our cable model for sufficient high frequencies because $\delta_r \xrightarrow[r]{} \infty$ and thus $\mathrm{e}^{-\delta_r} \xrightarrow[r]{} 0$.

We specify below the particular solutions of the cable model for equation (1) with $q(x,t)=0,\ \ c_i^V=0,\ c_i^H=0,\ c_i^{H*}=0$, solutions that are also particular solutions of the beam model.

$$w_{1r}(x)=\mathrm{e}^{-\delta_r\xi},\quad w_{2r}(x)=\mathrm{e}^{\delta_r\xi},\quad w_{3r}(x)=\sin\varepsilon_r\xi,\quad w_{4r}(x)=\cos\varepsilon_r\xi\,. \tag{7}$$

The identities from (3) can be used to justify the particular solutions (7).

The vibration mode of undamped vibration, expressed by relations (2) and (6), is a solution of equation (1), where $q(x,t)=0,\ c_i^V=0,\ \ c_i^H=0,\ c_i^{H*}=0,$ because $w_r(x)$, from (6), is a linear expression of the particular solutions from (7). The vibration mode (2) verifies also the imposed boundary conditions.

In the case of damped free vibrations described by equation (1), one searches the solution of the form $w_i(x,t)=X_i(x)\ T_i(t)$, where $X_i(x)$ defines a vibrating mode of order *i*, described in (6). Deduced from equations (1), the equation for the unknown function $T_i(t)$ is as follows:

$$\begin{gathered}\frac{\mathrm{d}^2T_i(t)}{\mathrm{d}t^2}+2c_i^{VH}\ \frac{\mathrm{d}T_i(t)}{\mathrm{d}t}+c_i^{\Omega H*}T_i\ (t)=0,\\ c_i^{VH}=c_i^V/m_L/2+c_i^H/m_L/\omega_i^{VH}/2,\qquad c_i^{\Omega H*}=\omega_i{}^2+c_i^{H*}/m_L-k_i/m_L.\end{gathered} \tag{8}$$

The equation (8) is deduced using the relationship:

$$EI\frac{\mathrm{d}^4X_i(x)}{\mathrm{d}x^4}-T\frac{\mathrm{d}^2X_i(x)}{\mathrm{d}x^2}=(\omega_i^2\ m_L-k_i)\,X_i(x)\,. \tag{9}$$

In equation (8), ω_i is the circular frequency of free undamped vibration of the cable and ω_i^{VH} is the circular frequency of free damped vibration of the cable.

The characteristic equation attached to equation (8) is as follows:

$$Z_i{}^2+2c_i^{VH}Z_i+c_i^{\Omega H*}=0\,, \tag{10}$$

In the case $c_i^{\Omega H*}\le\left(c_i^{VH}\right)^2$, equivalent with $\omega_i{}^2\le\left(c_i^{VH}\right)^2-c_i^{H*}/m_L+k_i/m_L$, we find a solution that does not describe our physical model. It is necessary to take into account the following inequality $\omega_i{}^2>\left(c_i^{VH}\right)^2-c_i^{H*}/m_L+k_i/m_L$, which can be expressed in the form:

$$\omega_i{}^2+c_i^{H*}/m_L-k_i/m_L>\left(c_i^{VH}\right)^2. \tag{11}$$

In the case above, the solution is as follows:

$$T_i(t) = \mathrm{e}^{-c_i^{VH} t}\left\{C_{1\,i}\sin\omega_i^{VH} t + C_{2\,i}\cos\omega_i^{VH} t\right\},$$
$$\omega_i^{VH} = \left\{\omega_i^{\,2} + c_i^{H*}/m_L - k_i/m_L - \left(c_i^{VH}\right)^2\right\}^{1/2}, \quad i = 1,2,... \tag{12}$$

If the initial conditions for the searched solution $w_i(x,t) = X_i(x)\ T_i(t)$ (with fixed index *i*) of equation (1), where $q(x,t) = 0$, are chosen as $w_i(x_0,t_0) = D_{0i}$, $\frac{\partial w_i}{\partial t}(x_0,t_0) = V_{0i}$, where $X_i(x)$ is a vibrating mode, defined by formula (6), then we take the expression (13) of $w_i(x,t),\ i = 1,2,...$

$$w_i(x,t) = \frac{X_i(x)}{X_i(x_0)}\mathrm{e}^{-c_i^{VH}(t-t_0)}\left\{\left(c_i^{VH}\frac{D_{0i}}{\omega_i^{VH}} + \frac{V_{0i}}{\omega_i^{VH}}\right)\sin\omega_i^{VH}(t-t_0) + \right.$$
$$\left. + D_{0i}\cos\omega_i^{VH}(t-t_0)\right\}, \tag{13}$$
$$\omega_i^{VH} = \left\{\omega_i^{\,2} + c_i^{H*}/m_L - k_i/m_L - \left(c_i^{VH}\right)^2\right\}^{1/2}.$$

From (13), for $t_0 = 0$ and $v_0 = 0$, we can write:

$$w_i(x,t) = X_i(x)\frac{D_{0i}}{X_i(x_0)}\mathrm{e}^{-c_i^{VH} t}\left\{\frac{c_i^{VH}}{\omega_i^{VH}}\sin\omega_i^{VH} t + \cos\omega_i^{VH} t\right\}. \tag{14}$$

The following notation and formulas are used in the relation (14):

$$c_i^{VH}/\omega_i^{VH} = \mathrm{ctan}(\alpha_i), \quad \alpha_i = \mathrm{arcctan}(c_i^{VH}/\omega_i^{VH}), \quad \alpha_i \in (0,\pi/2),$$
$$\sin^2(\alpha_i) = (\omega_i^{VH})^2/(\omega_i^2 + c_i^{H*}/m_L - k_i/m_L), \tag{15}$$
$$\cos^2(\alpha_i) = (\mathrm{c}_i^{VH})^2/(\omega_i^2 + c_i^{H*}/m_L - k_i/m_L).$$

Hence, the form deduced for the function $T_i(t),\ i = 1,2,...$ is:

$$T_i(t) = \frac{D_{0i}(\omega_i^2 + c_i^{H*}/m_L - k_i/m_L)^{1/2}}{X_i(x_0)\,\omega_i^{VH}}\ \mathrm{e}^{-c_i^{VH} t}\sin(\omega_i^{VH} t + \alpha_i). \tag{16}$$

The function $\frac{\mathrm{d}T_i(t)}{\mathrm{d}t}$, deduced using (14), has the form:

$$\frac{\mathrm{d}T_i(t)}{\mathrm{d}t} = -\frac{D_{0i}(\omega_i^2 + c_i^{H*}/m_L - k_i/m_L)}{X_i(x_0)\,\omega_i^{VH}}\,\mathrm{e}^{-c_i^{VH}t}\sin(\omega_i^{VH}t)\,. \quad (17)$$

Between the parameters of the mathematical model of the cable, the following condition of compatibility arises:

$$(\omega_i^{VH})^2 = \left\{\omega_i^{\,2} + c_i^{H*}/m_L - k_i/m_L - \left(c_i^V/m_L/2 + c_i^H/m_L/\omega_i^{VH}/2\right)^2\right\}. \quad (18)$$

The condition of compatibility has the following form:

$$4m_L^2(\omega_i^{VH})^4 - [4m_L^2\omega_i^2 + 4m_L c_i^{H*} - (c_i^V)^2](\omega_i^{VH})^2 + 2c_i^V c_i^H \omega_i^{VH} + (c_i^H)^2 = 0\,. \quad (19)$$

Another form of the above condition is:

$$\begin{aligned}(\omega_i^{VH})^2(c_i^V)^2 + 2(c_i^H\omega_i^{VH})c_i^V + 4m_L^2(\omega_i^{VH})^4 - \\ -(4m_L^2\omega_i^2 + 4m_L c_i^{H*})(\omega_i^{VH})^2 + (c_i^H)^2 = 0.\end{aligned} \quad (20)$$

The condition of real value for the damping coefficient c_i^V, derived from (20), is:

$$\left(\omega_i^{VH}\right)^2 \le \omega_i^{\,2} + c_i^{H*}/m_L - k_i/m_L\,. \quad (21)$$

The condition (21) is fulfilled for any compatible value ω_i^{VH} because it is justified through the condition (18). The viscous damping coefficient is positive, so that, from the equation (20), the following condition is also performed:

$$\frac{c_i^V}{c_i^H} > 1 - \frac{1}{\omega_i^{VH}}\,. \quad (22)$$

The infinite set of coefficients c_i^V, c_i^H, c_i^{H*}, $i = 1,2,...$ (with possibility of repetition) is supposed to be bounded.

The formulas (13) and (16) specify that the influence of the hysteretic and dry friction damping are negligible for vibration mode, if this mode is sufficiently high, because $c_i^{VH} \approx c_i^V/m_L/2$ and $\omega_i^{VH} \approx \omega_i$, for sufficiently high frequencies. However, the influence of viscous damping is maintained. This property of the cable is confirmed experimentally as well.

The formula (13) can also be used for performing the objective function referred to unknown parameters EI, c_i^H, c_i^{H*}, c_i^V, expressed in theoretically and experimentally ways. We use cable displacements and the weighted least square method to identify the specified parameters. The expression of the objective function is:

$$f(EI, c_i^V, c_i^H, c_i^{H*}) = \sum_{i,x,t} w_i^{\text{cof}} \left(w_i{}^c(x,t) - w_i{}^{\exp}(x,t) \right)^2 . \tag{23}$$

Theoretical displacements $w_i{}^c(x,t)$ and experimental displacements $w_i{}^{\exp}(x,t)$ are used in expression (23), theoretically calculated or measured in some points of abscises x, for some moments of time and for some frequencies too, in the domain of interest, and where $w_i^{\text{cof}} = 1/(w_i{}^{\exp}(x,\ t))^2$, which assures a dimensionless objective function.

The model of cable, defined by equation (1), was used for the parameters determination only for the hypothesis of viscous free damping ($c_i^H = 0,\ c_i^{H*} = 0$). The searched parameters are the bending rigidity EI of the cable and the coefficient of viscous damping c^V, referred to some frequencies in the domain of interest. The experimental values are measured using an experimental stand with single overhead cable [15–18]. The span is $L = 33\,\text{m}$, the ends of the cable are well fixed, $T = 1333\,\text{N}$ and $m_L = 0.757\,\text{kg/m}$. The used frequencies, according to experimental values, are $f_9 = 11.89\,\text{Hz}$, $f_{15} = 19.5\,\text{Hz}$ and $f_{19} = 24.98\,\text{Hz}$. The points considered are $x_1 = 0.089\,\text{m}$, $x_2 = 2.0\,\text{m}$ and $x_3 = 16.2\,\text{m}$. The moments of time correspond to the main values of the displacements. The values of the bending rigidity ($EI = 40\,\text{N}\cdot\text{m}^2$) and of viscous damping ($c^V = 2.675\,\text{N}\cdot\text{s/m}$) are determined by the minimization of the function (23), adapted to this case.

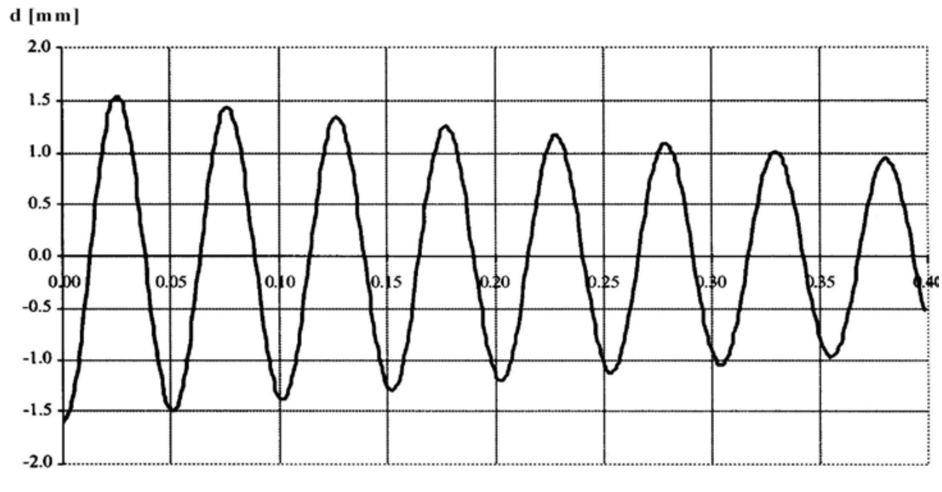

Fig. 1 – The diagram of damped displacement for resonance frequency.

The diagram of the damped displacement of the cable for the resonance frequency of $19.5\,\text{Hz}$, in the point of abscissa $x = 16.2\,\text{m}$ on the span of overhead cable is plotted in Fig. 1.

4. CONCLUSIONS

The original analytical considerations concerning the definition of the cable in viscous, hysteretic or dry friction internal damping hypothesis, using our cable model detached from the Euler-Bernoulli beam model, permit us to perform the analytical vibration modes of the cable and the control of these vibrations.

We remark analytically that there is the possibility for the conductor cable to consider simultaneously the influence of viscous, hysteretic and dry friction internal damping of the cable, but the hysteretic and dry friction damping are negligible for a sufficiently high vibration mode of the cable while the influence of viscous damping is maintained.

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