ON THE DYNAMIC BEHAVIOR OF ELASTIC FRAME STRUCTURES WITH BASE IMPOSED HARMONIC DISPLACEMENT

TUDOR SIRETEANU[1], GHEORGHE GHIȚĂ[1], VIOREL ȘERBAN[2]

Abstract. The dynamic response of an elastic shear frame structure to imposed lateral harmonic displacement of the base slab is analyzed within frequency ranges placed below and above natural frequencies. It is shown that between two successive frequencies of the natural vibration modes there are input frequencies for which the lateral displacements of certain structure levels are practically negligible and therefore behave like nodes. These nodes are different from those displayed by the natural vibration modes. This behavior is a direct consequence of the modification of phase shift between the lateral displacements of base and upper levels when the input frequency is swept below or above the natural frequencies of structure. This dynamic behavior leads to different distributions of lateral seismic forces along the structure height. The mid-storey collapse of medium-rise buildings cause by earthquakes could be possibly explained by this effect.

Key words: shear frame structure, frequency response function, lateral vibration nodes, inter-storey drift, mid-storey damage.

1. INTRODUCTION

Inspection of damaged buildings after major earthquakes reveals in many cases mid-storey collapse of medium-rise buildings [1]. Usually, this phenomenon is attributed to the structural dynamic loads produced by excitation of a higher vibration mode. However, for medium-rise buildings (around 10 stories) the above vibration modes are unlikely to be excited by the seismic ground motion. Generally, even the frequency of the second vibration mode of such buildings is too high (about 3 times greater than the frequency of the first vibration mode) to be excited by the ground motion, due to the spectral content of the seismic input (especially in case of slow earthquakes). In addition, analyses were carried out for multi-degree of freedom (MDOF) models of various distributions of lateral strength, in order to investigate the reason of such behavior [2, 3]. The vibration of a slender uniform cantilever beam with imposed harmonic motion at the clamped

[1] Institute of Solid Mechanics – Romanian Academy, Const. Mille, 15, RO-010141
[2] SITON – Subsidiary of Technology and Engineering for Nuclear Projects, Măgurele, Bucharest

end was investigated both analytically and experimentally, showing different vibration patterns when the beam vibration is excited bellow or above the first natural frequency [4, 5]. While bellow resonance, the harmonic motions of beam base and top are in phase, above resonance these motions are out of phase, displaying a node which shifts toward the free end with the increase of driving frequency. For same top absolute displacement, both bending moment and shear force along the beam are bigger above resonance than bellow resonance. Moreover, above resonance the shear force has a maximum value at the node position. In this paper, a similar dynamic behavior is described for an elastic shear frame structure modeled by a MDOF system. The analysis of modal amplification factors in the frequency intervals between resonant frequencies reveals the existence of a set of input frequencies for which the absolute displacements of different stories are practically negligible. Therefore, the structure displays more nodes than those corresponding to the natural modes of vibration, leading to a more complex distribution of dynamic loads along the structure height that must be considered in the assessment of lateral strength distribution.

2. ANALYTICAL MODEL

The analytical model of a five-storey building is the MDOF system shown in Fig. 1. Only lateral motion is considered, the building being treated as a shear frame structure. The mass of each story of the structure is considered to be concentrated at the level of the slab. These concentrated masses are connected by linear springs and viscous dampers to represent structural stiffness and damping for displacements in the elastic region. We assume that the distribution of mass, stiffness and damping is uniform. Therefore, each level is a single degree of freedom oscillating system with mass m, stiffness coefficient k and damping coefficient c.

The system of equations of motion is

$$\begin{cases} \ddot{y}_1 + \omega_0^2 y_1 - \omega_0^2 y_2 + 2\zeta_0\omega_0\dot{y}_1 - 2\zeta_0\omega_0\dot{y}_2 = -\ddot{x}_0 \\ \ddot{y}_i - \omega_0^2 y_{i-1} + 2\omega_0^2 y_i - \omega_0^2 y_{i+1} - 2\zeta_0\omega_0\dot{y}_{i-1} + 4\zeta_0\omega_{i0}\dot{y}_i - 2\zeta_0\omega_0\dot{y}_{i+1} = 0 \\ i = 2,3,4 \\ \ddot{y}_n - \omega_0^2 y_4 + 2\omega_0^2 y_5 - 2\zeta_0\omega_0\dot{y}_4 + 4\zeta_0\omega_0\dot{y}_5 = 0, \end{cases} \tag{1}$$

where

$$y_i = x_i - x_{i-1}, \quad x_i = \sum_{k=1}^{i} y_k + x_0, \quad i = 1,2,...,5, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \zeta_0 = \frac{c}{2m\omega_0}. \tag{2}$$

In the above equations, $x_0(t) = X_0 \sin \omega t$ is the harmonic displacement, applied to the structure base, $x_1,...,x_5$ are the absolute lateral displacements of structure slabs with respect to the equilibrium position (Oz) and y_1, y_2, y_3, y_4, y_5 are the inter-storey drifts (Fig. 1).

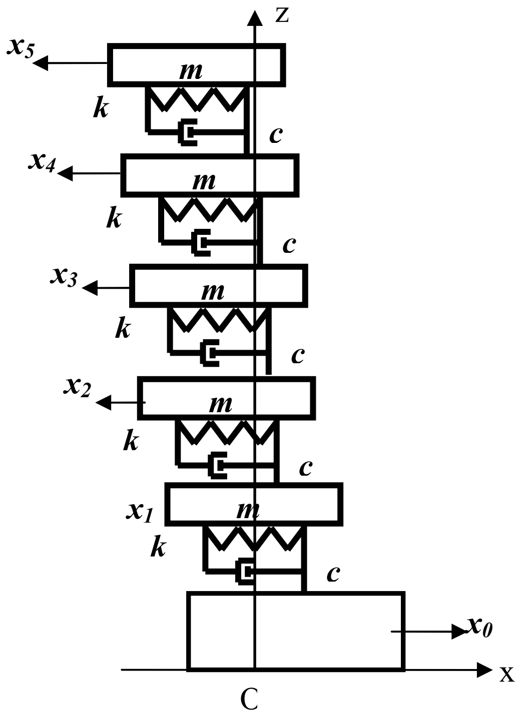

Fig. 1 – MDOF model of structure.

Let us denote by $H_{y_i}(\nu, \zeta_0)$, $i = 1,...,5$, where $\nu = \omega/\omega_0$, the frequency response functions of the relative displacements. In order to determine these functions, one must introduce in equation (1) the complex form of input and output

$$x_0(t) = X_0 \mathrm{e}^{\mathrm{i}\nu\omega_0 t}, \; y_i(t) = H_{y_i}(\nu, \zeta_0) X_0 \mathrm{e}^{\mathrm{i}\nu\omega_0 t} \tag{3}$$

and the following system of linear equations is then obtained

$$\mathbf{A}(\nu, \zeta_0)\mathbf{H}_y(\nu, \zeta_0) = \mathbf{B}, \tag{4}$$

where

$$\mathbf{A}(\nu,\zeta_0)=\begin{bmatrix}\nu^2-a & a & 0 & 0 & 0\\ a & \nu^2-2a & a & 0 & 0\\ 0 & a & \nu^2-2a & a & 0\\ 0 & 0 & a & \nu^2-2a & a\\ 0 & 0 & 0 & a & \nu^2-2a\end{bmatrix},\quad a=1+2\mathrm{i}\zeta_0\nu,$$

$$\mathbf{H}_y(\nu,\zeta_0)=\left[H_{y_1}(\nu,\zeta_0)\ H_{y_2}(\nu,\zeta_0)\ H_{y_3}(\nu,\zeta_0)\ H_{y_4}(\nu,\zeta_0)\ H_{y_5}(\nu,\zeta_0)\right]^{\mathrm{T}},$$

$$\mathbf{B}=\left[-\nu^2\ 0\ 0\ 0\ 0\right]^{\mathrm{T}}. \tag{5}$$

Taking into account the relationships (2), the frequency response functions of the absolute displacements can be written as

$$H_{x_i}(\nu,\zeta_0)=\sum_{k=1}^{i}H_{y_i}(\nu,\zeta_0)+1,\ i=1,2,...,5. \tag{6}$$

The analytical forms of these frequency response functions are very complicated for $\zeta\neq 0$. A substantial simplification is obtained for the undamped system. As the usual values of fraction of critical damping of building structures are very small ($0.02\leq\zeta_0\leq 0.05$), the determination of frequency response functions for undamped system provides very useful information about the structure dynamic behavior. By solving system (4) for $\zeta_0=0$, one obtains the frequency response functions of the inter-storey drifts $y_i(t),\ i=1,...,5$ for the undamped system:

$$H_{y_i}(\nu,0)=\frac{(-1)^i P_i(\nu)}{\det\mathbf{A}(\nu,0)},\ i=1,2,...,5,$$

$$P_1(\nu)=\nu^2\left(\nu^8-8\nu^6+21\nu^4-20\nu^2+5\right),\ P_2(\nu)=\nu^2\left(\nu^6-6\nu^4+10\nu^2-4\right), \tag{7}$$

$$P_2(\nu)=\nu^2\left(\nu^4-4\nu^2+3\right),\ P_3(\nu)=\nu^2\left(\nu^4-4\nu^2+3\right),\ P_4(\nu)=\nu^2\left(\nu^2-2\right),$$

$$P_5(\nu)=\nu^2,\ \det\mathbf{A}(\nu,0)=\nu^{10}-9\nu^8+28\nu^6-35\nu^4+15\nu^2-1=0.$$

Introducing (7) in (6), yields

$$H_{x_i}(\nu,0)=\frac{(-1)^i Q_i(\nu)}{\det\mathbf{A}(\nu,0)},\ i=1,2,...,5,$$

$$Q_i(\nu)=\nu^8-7\nu^6+15\nu^4-10\nu^2+1,\ Q_2(\nu)=\nu^6-5\nu^4+6\nu^2-1, \tag{8}$$

$$Q_3(\nu)=\nu^4-3\nu^2+1,\ Q_4(\nu)=\nu^2-1,\ Q_5(\nu)=1.$$

The natural frequencies of the undamped system are given by $\omega_i = \nu_i \omega_0,\ i = 1, 2, \ldots, 5$, where ν_i are the positive roots of the characteristic equation $\det \mathbf{A}(\nu, 0) = 0$:

$$\nu_1 = 0.286,\ \nu_2 = 0.839,\ \nu_3 = 1.31,\ \nu_4 = 1.685,\ \nu_5 = 1.919\,. \tag{9}$$

The amplification factors $\left|H_{x_i}(\nu, 0)\right|,\ i = 1, 2, \ldots, 5$, of absolute lateral displacements, are plotted in Figs. 2–6.

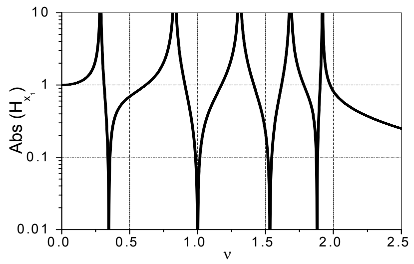

Fig. 2 – Amplification factor of 1st storey.

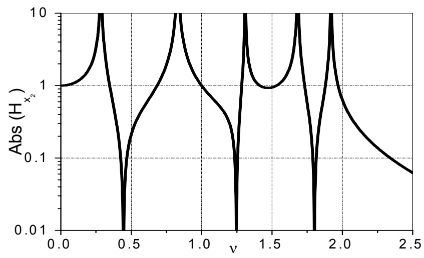

Fig. 3 – Amplification factor of 2nd storey.

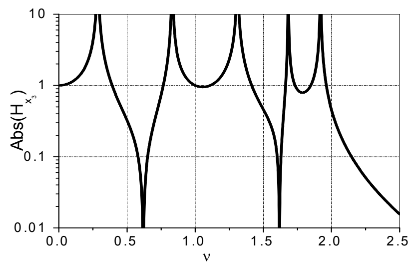

Fig. 4 – Amplification factor of 3rd storey.

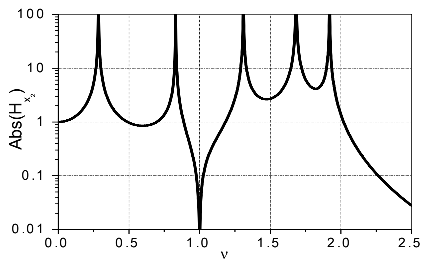

Fig. 5 – Amplification factor of 4th storey.

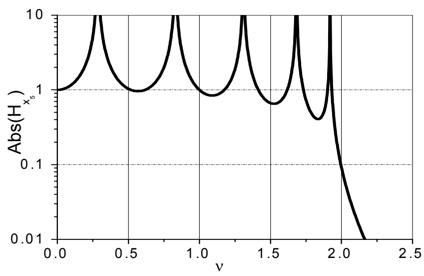

Fig. 6 – Amplification factor of 5th storey.

These graphs show very clearly that between natural frequencies of structure, there are frequencies at which the amplification factors of lateral absolute displacements display very low values, with the exception of fifth floor. Therefore, there are specific input frequencies at which the stories 1–4 behave like nodes, which are different from the nodes displayed by the natural vibration modes 2–5.

The number of these pseudo-nodes decreases by one (from four to zero) with the increase of storey height. Another important piece of information, provided by these plots, concerns the input frequencies for which the amplitude of lateral motion of one or more stories is equal to the amplitude of base imposed motion, *i.e.* The value of frequency response functions is 1. The base and storey motions can be in phase or out of phase if the input frequency is below or above the resonance frequencies.

3. NUMERICAL SIMULATION RESULTS

For numerical simulation, the values of structure parameters ω_0 and ζ_0 were chosen such as the frequency and the fraction of critical damping for the first vibration mode of the entire structure to be $f_1 = 1.2\text{Hz}$ (T_1=0.83s) and $\zeta_1 = 0.02$, respectively. Therefore the undamped frequency of the SDOF system, corresponding to one storey, is $\omega_0 = \omega_1/\nu_1 = 2\pi f_1/\nu_1 = 26.45\ \text{rad/s}$. By analyzing the free motion of the 5$^{\text{th}}$ storey in the first vibration mode for different values of ζ_0, the desired value of ζ_1 was obtained for $\zeta_0 = 0.075$ (see the free vibration plot shown in Fig. 7).

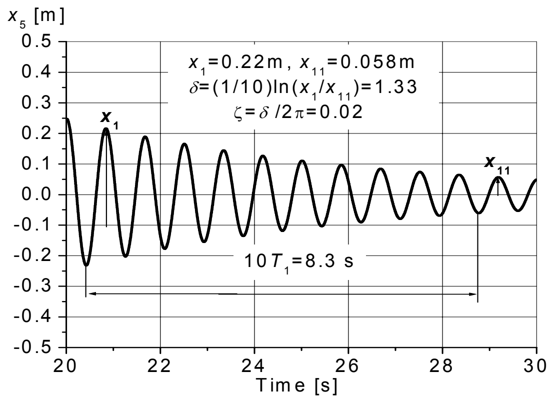

Fig. 7 – Free motion of the 5$^{\text{th}}$ storey in the 1$^{\text{st}}$ vibration mode.

Figs. 8–11 show the time histories of base and stories absolute displacements, obtained by numerical simulation for the following set of input frequencies (periods):

- bellow first vibration mode frequency:

$$f_{01} = 0.93\ \text{Hz} < f_1 = 1.2\ \text{Hz},\ T_{01} = 1.08\ \text{s} > T_1 = 0.83\ \text{s};$$

- and between first and second vibration modes frequencies:

$$f_{12} = 1.45 \text{ Hz}, T_{12} = 0.69 \text{ s}; f_{13} = 1.86 \text{ Hz}, T_{13} = 0.54 \text{ s};$$
$$f_{14} = 2.59 \text{ Hz} < f_2 = 3.52 \text{ Hz}, T_{14} = 0.39 \text{ s} > T_2 = 0.28 \text{ s}.$$

These values were chosen such as to minimize the amplification factors of the absolute displacements of stories 1, 2 and 3.

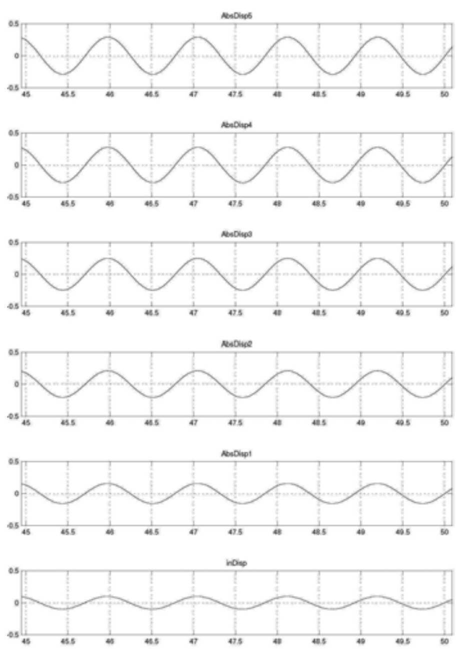

Fig. 8 – Output below 1st resonance.

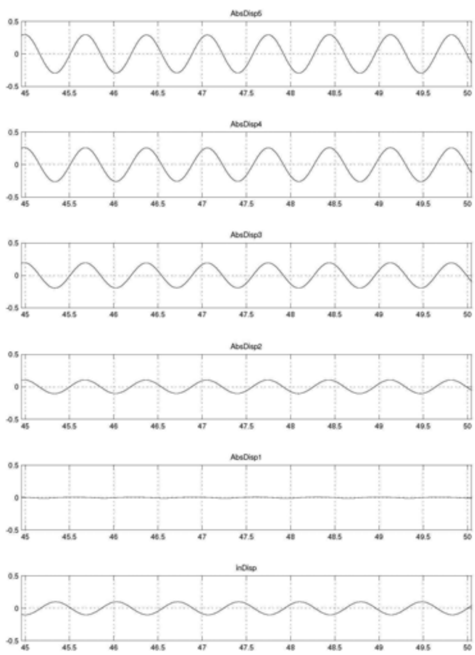

Fig. 9 – Output above 1st resonance – node at 1st storey.

These graphs show very clearly that below first resonance, the absolute displacements of base and stories are in phase. Above the first mode frequency, for certain input frequency values, sensibly smaller than the second mode frequency, the first three stories can behave like nodes. The absolute displacements of stories below and above the nodes are in opposite phase. Moreover, when the third storey becomes a node, the base and first stories have equal absolute displacements, *i.e.* the five-storey structure behaves like a four-storey one, with same base displacement. The plots presented in Figs. 12 and 13 advocate once more these assertions.

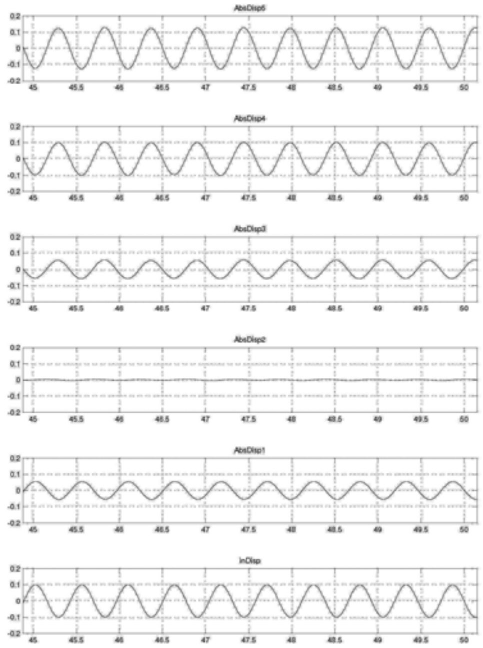

Fig. 10 – Output below 2nd resonance – node at 2nd storey.

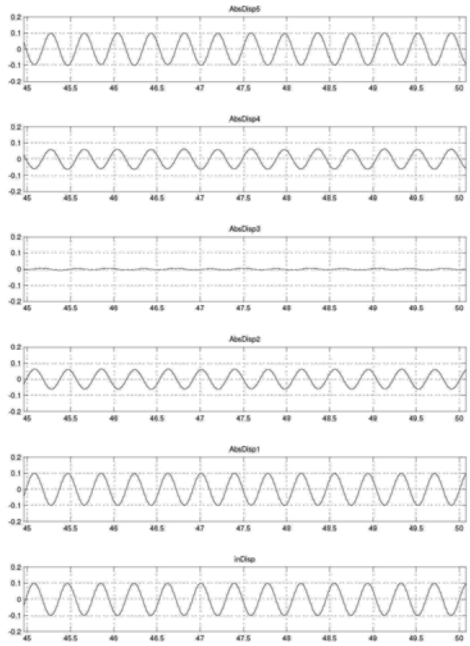

Fig. 11 – Output below 2nd resonance – node at 3rd storey.

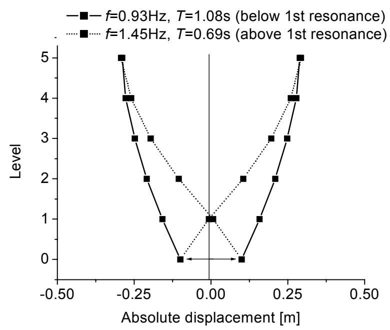

Fig. 12 – The structure output shape below and above 1st resonance (first storey is a node).

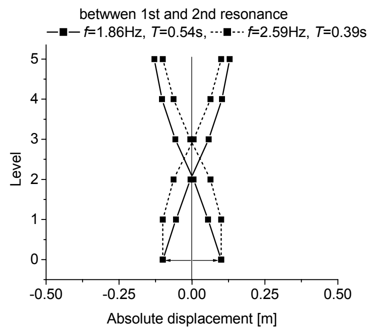

Fig. 13 – The structure output shape between 1st and 2nd resonance (2nd and 3rd stories are nodes).

Obviously, the above analysis can be easily extended for all frequency intervals, situated between the frequencies of the higher vibration modes, but is it less relevant for a frame structure with five stories, since the higher vibration modes are very unlikely to be exited by seismic motions.

4. EXPERIMENTAL RESULTS

Within the framework of an experimental program [6], carried out to assess the efficiency of semi-active control of structural vibration, a uniform structure with three stories has been constructed (Fig. 14) and tested for both harmonic and seismic base imposed displacement.

The frequency of the first vibration mode of structure is $f_1 = 1.17$ Hz ($T_1 = 0.85$ s). Fig. 15 shows the amplitude spectra of harmonic input and output obtained by measuring the lateral accelerations of structure base and slabs without supplementary damping.

The measured acceleration time histories, shown in Figs. 15 and 16, outline qualitatively the main features of dynamic behavior of elastic structures with base imposed displacement, discussed above:

- the change of phase shift between base and upper levels from 0 (below resonance) to 180° (immediately above resonance);
- node like behavior of the upper levels, displayed for driving frequencies much less than the frequency of the next vibration mode.

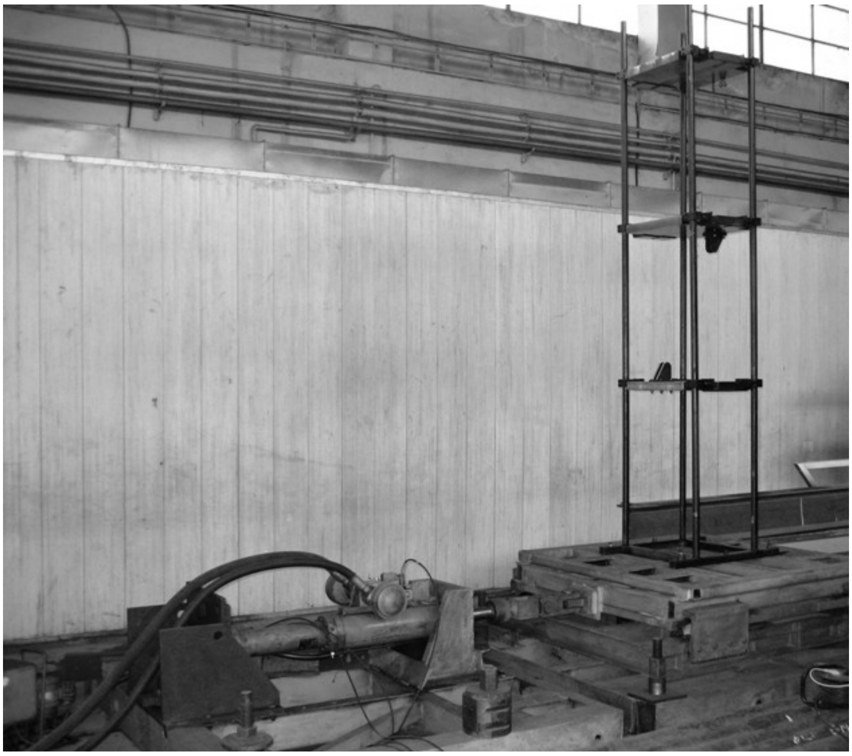

Fig. 14 – Elastic structure with three stories.

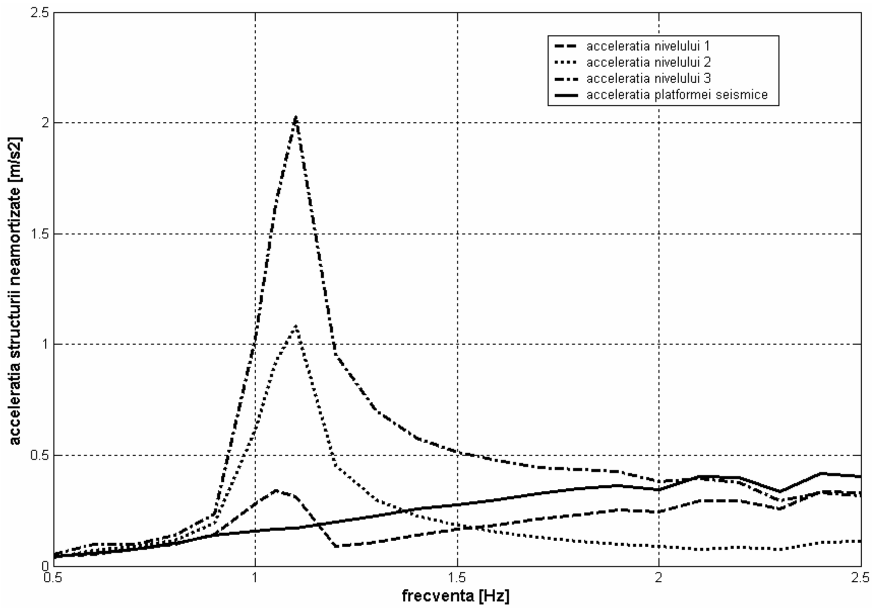

Fig. 15 – Acceleration amplitude spectra of elastic structure without additional damping.

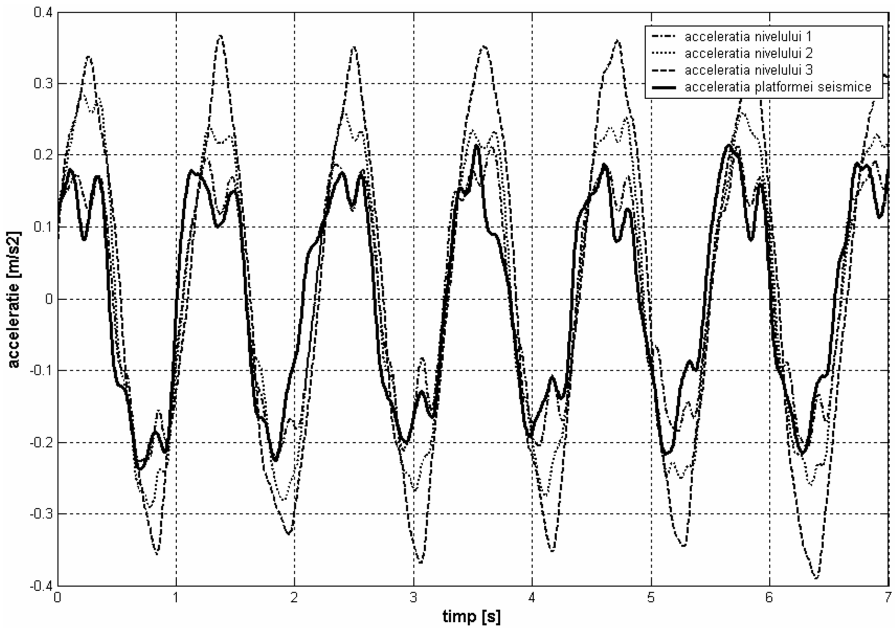

Fig. 16 – Time histories of accelerations measured for input frequency below resonance: $f_{01} = 0.9$ Hz $(T_{01} = 1.11\ \text{s})$.

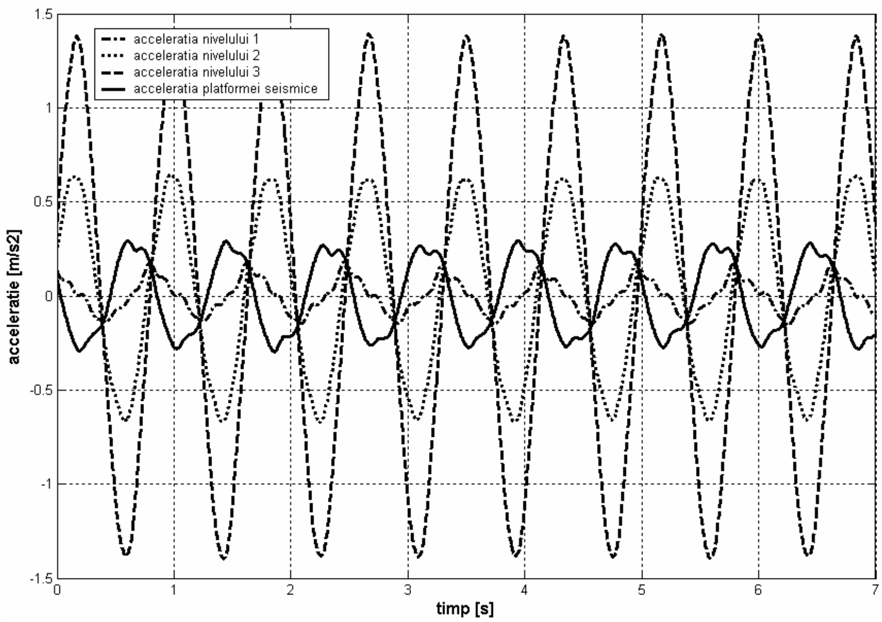

Fig. 17 – Time histories of accelerations measured for input frequency above resonance: $f_{12} = 1.2$ Hz ($T_{12} = 0.83$).

5. CONCLUSIONS

The dynamic response of an elastic frame structure with concentrated masses at slab levels to imposed lateral harmonic displacement of the base slab was analyzed, both analytically and experimentally, within frequency ranges placed below and above natural frequencies. It is shown that between two successive frequencies of the natural vibration modes there are input frequencies for which the lateral displacements of certain structure levels are practically negligible and, therefore, behave like nodes. These nodes are different from those normally displayed by the natural vibration modes. This effect is a direct consequence of the modification of phase shift between the lateral displacements of base and upper levels when the input frequency is swept below or above the natural frequencies of structure. This dynamic behavior leads to different distributions of lateral output dynamic loads (bending moments and shear forces) along the structure height for different input frequencies, which cannot be obtained by using an “equivalent” model with fixed base exited by lateral forces. The mid-storey damages or even collapse of medium-rise buildings, cause by earthquakes, could be possibly explained by this effect.

Received on June 11, 2010

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