ON A SHAPE IDENTIFICATION PROBLEM

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Abstract. An inverse method using a genetic algorithm and a quasi-Newton method is presented in this paper. We consider the problem of identifying a finite elastic air-filled hollow body of unknown shape embedded in a compressible, inviscid homogeneous fluid, based on the concept of time reversal invariance. A nonlinear model describes the acoustical signature of the scatterer. Numerical simulation is carried out for several examples to illustrate the efficiency of the method.

Key words: shape identification problem, inverse method, genetic algorithm, quasi-Newton method, elastic air-filled hollow body, acoustical signature of the scatterer.

1. INTRODUCTION

The inverse problem is an important subject in mechanics. Inverse problems are dealing with the determination of the mechanical systems with unknown material properties from the knowledge of the responses to given excitations on its boundary [1–4]. The problem for an infinite air-filled hollow cylinder imbedded in water is analyzed in literature [5–7]. The inverse problem for identifying a finite elastic air-filled hollow body of unknown location and shape immersed in a compressible, inviscid, homogeneous fluid is a difficult problem and to our knowledge it has not been studied in very details in literature.

An inverse analysis method using a genetic algorithm and a quasi-Newton method is presented in this paper. We consider the problem of identifying a finite elastic air-filled hollow body of known location and unknown shape embedded in water. The resulting nonlinear least-squares problem is to be solved using as a first step a genetic algorithm. Starting from the parameters thus obtained with the genetic algorithm, more precise inverse analysis is carried out by the second step of the gradient quasi-Newton algorithm. The direct approach is developed first. A scenario of measuring and producing the instantaneous pressure field is described using a large array of piezoelectric transducers for a time reversal mirror. A reflected target in an incident field serves as a source of a scattered field. The dimensions of the obstacle are larger than wavelength and the scattered field is

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composed of the geometrical specular waves that would be obtained if the target were perfectly rigid, and the surface and volume waves around and inside the scatterer. The contribution to the scattered field of all symmetrical and anti-symmetrical ellipsoidal surface waves (of the Lamb and Rayleigh waves type) which define the acoustical signature of the scatterer are analyzed and detected.

2. DIRECT PROBLEM

We consider the problem of an elastic air-filled hollow body Ω with boundary Γ immersed in a compressible, inviscid homogeneous fluid. Let Ω^+ be the complement of Ω filled with fluid. If u denotes the displacement and $\Xi[u]$ the stress of the body then

$$\Xi[u] = \mu(\nabla u + (\nabla u)^{\mathrm{T}}) + \lambda\,\mathrm{div}\,uI\,, \tag{1}$$

where μ and λ are functions of position x. If ρ_E is the density, the equation of motion is

$$\rho_E u_{tt} = \mathrm{div}\,\Xi[u]\,. \tag{2}$$

Suppose the fluid is close to an equilibrium state with constant density and zero velocity. Thus the density $\rho = \rho_0 + \rho_1$, $\rho_1 << 1$. The equation of state for the pressure is

$$p = f(\rho_0 + \rho_1) \approx f(\rho_0) + f'(\rho_0)\rho_1 = p_0 + c^2\rho_1\,, \tag{3}$$

where c is the speed of sound in the fluid. If v is the velocity ($v << 1$) then the equations of motion are

$$\rho_0 v_t - \mathrm{grad}\,p = 0\,, \quad \rho_{1t} + \rho_0 \mathrm{div} v = 0\,, \quad \rho_1 = \frac{1}{c^2}p\,. \tag{4}$$

In terms of pressure we have

$$\frac{1}{c^2}p_{tt} = \Delta p\,, \quad v_t = -\frac{1}{\rho_0}\mathrm{grad}\,p\,. \tag{5}$$

The solid-fluid interaction problem involves solving (2) in Ω and (5) in Ω^+. Both equations provide the strong property of being invariant under time reversal.

The continuity of traction condition is given by

$$(\Xi[u^-](n))\cdot n = -p^+, \quad (\Xi[u^-](n))\times n = 0\,, \tag{6}$$

where the minus and plus denote interior and exterior limits on Γ and n is the outward unit normal. Since tangential motion of the obstacle will not produce tangential motion of a non-viscous fluid we have the continuity only of the normal component of velocity

$$u_{tt}^{-} \cdot n = v_t^{+} \cdot n = -\frac{1}{\rho_0} p_n^{+} . \tag{7}$$

A time reversal method consists in an array of transducers which transforms a wave emerging from a source whose position is unknown into a wave converging on the source. The time reversal mirror transmits a wide incident wave and senses the wave reflected by the obstacle (Fig. 1).

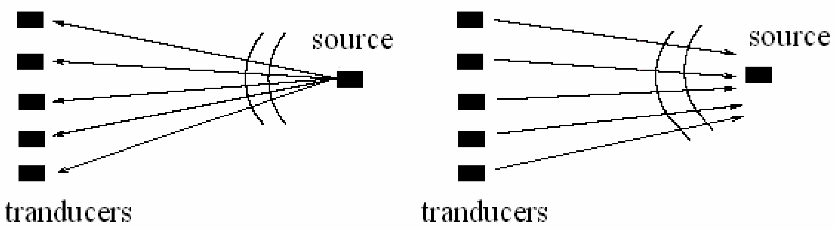


Fig. 1 – Time reversal in a homogeneous medium.

The incident fluid motion is given by a pressure $\mathrm{Re}(p^0(x)\mathrm{e}^{\mathrm{i}\omega t})$ and we look for the corresponding steady state period motion

$$\begin{aligned} u(x,t) &= \mathrm{Re}(U(x)\exp(\mathrm{i}\omega t), \\ p(x,t) &= \mathrm{Re}((p^0(x)+P(x))\exp(\mathrm{i}\omega t), \end{aligned} \tag{8}$$

where P is the scattered field in Ω^+ which satisfies the radiation condition. The surface wave propagating around the hollow body is generated under a certain angle of incidence θ with respect to the normal to the surface which satisfies $c_1 \sin\theta = c$, where c_1 is the phase velocity of the surface wave. The direct problem is solved by using the integral representations for the solution in the infinite exterior region occupied by the fluid. The problem is reduced to one defined only over the finite region occupied by the solid, with associated nonlocal boundary conditions [5, 8–11].

3. INVERSE PROBLEM

The unknown source is searched as a n-ellipsoid characterized by ten design parameters d_i, $i = 1,...,10$: arbitrary center coordinates x_G, y_G, z_G, principal axes a,b,c and principal directions defined by Euler angles ξ,ψ,ζ. The surface Γ is defined as the image of the unit n-sphere S [8]

$$x_1^n + x_2^n + x_3^n = 1, \tag{9}$$

through an affine transformation

$$y = (Y_1, Y_2, Y_3) \in S \to \ y = (y_1, y_2, y_3) \in \Gamma, \tag{10}$$

expressed in terms of the rotation $r_{ij} = r_{ij}(\xi, \psi, \zeta)$ which transforms the coordinate axes into the principal axes of the ellipsoid

$$\begin{aligned} y_1 &= x_G + r_{11} a Y_1 + r_{12} b Y_2 + r_{13} c Y_3, \\ y_2 &= y_G + r_{21} a Y_1 + r_{22} b Y_2 + r_{23} c Y_3, \\ y_3 &= z_G + r_{31} a Y_1 + r_{32} b Y_2 + r_{33} c Y_3. \end{aligned} \tag{11}$$

Thus, the unit sphere and the unit cube are respectively transformed into ellipsoids and boxes, with arbitrary center, size and orientation. The unknown shape is sought so as to achieve a best fit between the measured and computed values of the exterior pressure field

$$\begin{aligned} &\min J(p) \\ &J(p) = \frac{1}{2}\int | p(y) - \overline{p}(y) |^2 \,\mathrm{d}C, \end{aligned} \tag{12}$$

where C is the surface on which the maximum pressure field is measured by using the time reversal mirror method.

As the same n-ellipsoid can result from many combinations of Euler angles and permutations of principal axes, it is difficult to measure the accuracy of the identification of Γ by means of comparison of the identified parameters d_i, $i = 1,...,10$, with those defining the "true" Γ and used to compute the simulated data. Instead, the relative errors $\varepsilon_V, \varepsilon_A, \varepsilon_I$ for the volume, boundary area and geometrical inertia tensor (with respect to the fixed coordinates $Ox_1x_2x_3$) are computed.

The indicator ε_I is very sensitive to the orientation of Γ in space, together with the ratio J_n / J_0, where $J_n = J(\Gamma_n)$ and Γ_n is the current Γ after the n-th iteration of the minimization process.

Expressions of indicators $\varepsilon_V, \varepsilon_A, \varepsilon_I$ in terms of boundary integrals are as follows [8, 9]

$$\varepsilon_V = \frac{V(\Gamma_n)}{V(\Gamma)} - 1, \quad V(S) = \frac{1}{3}\int_S y_i n_i \mathrm{d}S_y, \tag{13}$$

$$\varepsilon_A = \frac{A(\Gamma_n)}{A(\Gamma)} - 1, \quad A(S) = \int_S \mathrm{d}S_y, \tag{14}$$

$$\varepsilon_I = \left(\frac{\sum_{1\le i,j\le 3} (I_{ij}(\Gamma_n) - I_{ij}(\Gamma))^2}{\sum_{1\le i,j\le 3} I_{ij}^2(\Gamma)} \right)^{1/2}, \quad I_{ij}(S) = \frac{1}{5}\int_S y_i y_j y_k n_k \mathrm{d}S_y \,. \tag{15}$$

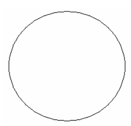
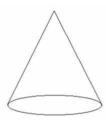
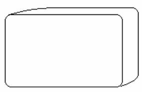
The resulting nonlinear least-squares problem is solved by two steps: a genetic algorithm and a quasi-Newton minimization algorithm. It is assumed that the design parameters are discretized into discrete values with given steps. A combination number is done for the discretized parameters to indicate a specific location and shape of the target. The genetic algorithm obtains an optimum initial guess for the inverse analysis by using the selection, the crossover and reproduction, the mutation and fluctuation operators. The alternation of generations is terminated when convergence is detected.

4. RESULTS

The numerical simulation is carried out for a number of three 3D targets to illustrate the efficiency of the two-step inverse proposed technique. The targets are air-filled hollow bodies, namely a sphere, a cone and a rectangle, respectively, with a known location. An array of 33 transducers is used to transforms a wave emerging from a source whose position is known into a wave converging on the source.

Table 1

Results

Object	(J_{fin}/J_0)	ε_V	ε_A	ε_I
	4.778×10^{-4} 3.345×10^{-5} 1.073×10^{-7}	8.643×10^{-2} 7.988×10^{-4} 6.763×10^{-7}	1.254×10^{-2} 9.874×10^{-4} 6.236×10^{-6}	8.994×10^{-3} 7.985×10^{-3} 5.187×10^{-5}
	4.336×10^{-3} 3.128×10^{-5} 1.537×10^{-6}	9.693×10^{-3} 8.835×10^{-3} 6.009×10^{-6}	1.933×10^{-2} 9.395×10^{-3} 5.364×10^{-5}	1.556×10^{-2} 1.475×10^{-2} 8.336×10^{-5}
	1.717×10^{-3} 2.338×10^{-4} 1.976×10^{-5}	14.224×10^{-3} 11.677×10^{-3} 8.356×10^{-5}	1.345×10^{-2} 8.587×10^{-3} 6.109×10^{-4}	1.293×10^{-2} 8.765×10^{-2} 6.643×10^{-4}

The values J_{final}/J_0, $\varepsilon_V, \varepsilon_A, \varepsilon_I$ are displayed in Table 1 in the case of non-perturbed data, for only quasi-Newton (first line), the genetic algorithm (second line), and two steps procedure (third line). The procedure may also be applied for unknown location and unknown size of the targets.

Results show a good convergence and accuracy, especially for the sphere. By using the two-step method, the performance is better. The proposed method could be successfully and efficiently applied for the identification of arbitrary sizes and locations of obstacles embedded in a homogeneous medium.

5. **CONCLUSIONS**

The results obtained in this paper show the efficiency of our approach for the problem of identifying a finite elastic air-filled hollow body of unknown location and shape embedded in a compressible, inviscid homogeneous fluid, based on the concept of time reversal invariance. The proposed technique is an inverse method based on the genetic algorithm and a quasi-Newton method. The unknown targets are searched as *n*-ellipsoids characterized by ten design parameters d_i, $i = 1,...,10$: arbitrary center coordinates x_G, y_G, z_G, principal axes a, b, c and principal directions defined by Euler angles ξ, ψ, ζ. Results show a good convergence and accuracy, especially for the sphere. Estimating the location and size of targets in the inhomogeneous media is more complicated. For solving the problem for the random media see [12].

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REFERENCES

1. BUI, H.D., TANAKA, M. (eds.), *Inverse Problems in Engineering Mechanics*, Balkema, Rottardam, Brookfield, 1994.
2. CHIROIU, V., CHIROIU, L., *Probleme inverse în mecanică*, Romanian Academy Publishing House, Bucharest, 2003.
3. CHIROIU, V., *Identification and inverse problems related to material properties and behaviour* (Chap. 4), in: *Topics in Applied Mechanics* – Vol. 1 (eds. V. Chiroiu, T. Sireteanu), Romanian Academy Publishing House, Bucharest, 2003, pp. 83-126.
4. CHIROIU, V., POIENARIU, M., GEORGESCU, M., *On the inverse problems in mechanics and the Lagrangian inverse problem*, Computer and Experimental Simulations in Engineering and Science (CESES), *5*, pp. 35-47, 2009.
5. PRADA, C., THOMAS, J.L., ROUX, P., FINK, M., *Acoustic time reversal and inverse scattering*, in: *Inverse Problems in Engineering Mechanics* (eds. Bui, Tanaka *et al.*), Balkema, Rotterdam, 1994, pp. 309-316.
6. PRADA, C., WU, F., FINK, M., *The iterative time reversal mirror: A solution to self-focusing in the pulse echo mode*, J. Acoust. Soc. Am., **90**, *2*, pp. 1119-1129, 1991.
7. POPA, D., MUNTEANU, E., MUNTEANU, L., CHIROIU, V., *On the shape reconstruction of 3D Stokes flows*, Proc. of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **10**, *3*, pp. 254-260, 2009.

8. BONNET, M., *Shape identification problems using boundary elements and shape differentiation*, Proc. of the 2-nd National Conference on Boundary and Finite Element Methods, ELFIN2, Sibiu, 1993.
9. CHIROIU, V., MUNTEANU, L., BADEA, T., NICOLESCU, C.M., *On a finger model actuated with shape memory alloy artificial muscles*, Symposium on “Active Control of Vibration and Noise 2003” ASME International Mechanical Engineering Congress and Exposition, Nov. 16-21, 2003, Washington, DC. Paper no. 41064.
10. CHIROIU, V., MUNTEANU, L., NICOLESCU, C.M., *Shape description of general 3D object using tactile sensing information*, AMSE: Advances in Modelling, Series B: Signal Processing and Pattern Recognition, **47**, *3*, pp.81-90, 2004.
11. CHIROIU, V., MUNTEANU, L., NICOLESCU, C.M., *A shape description model by using sensor data from touch,* Fourth Symposium on Multibody Dynamics and Vibration at the Nineteenth Biennial Conference on Mechanical Vibration and Noise ASME International Design Engineering Technical Conferences Chicago, Illinois September 2-6, 2003, Paper no. DETC2003/VIB-48337.
12. BORCEA, L., PAPANICOLAOU, G., TSOGKA, C., BERRYMAN, J., *Imaging and time reversal in random media*, Inverse Problems, **18,** pp. 1247-1279, 2002.