AN IDENTIFICATION PROBLEM FROM INPUT-OUTPUT DATA

ANTONIO S. GLIOZZI[1], LIGIA MUNTEANU[2], TUDOR SIRETEANU[2], VETURIA CHIROIU[2]

Abstract. This paper discusses the identification problem of the time-varying properties of a hysteretic system, associated with a given motion equation. The motion equation describes the behavior of a shear-beam building with *N* degree of freedom. The restoring force is portraying by a Bouc-Wen model. The relationship between input and output data is represented by using the cnoidal method.

Key words: hysteretic system, Bouc-Wen model, shear-beam building, identification problem, cnoidal method.

1. PROBLEM STATEMENT

The identification of dynamic systems from input-output data is one of the most important topics in applied mechanics, control engineering, chemical and biological sciences. If the dynamical system is known in terms of the motion equation and physical parameters, its response under dynamic loadings can be accurately predicted. For the linear systems, the identification problem from the input-output analysis is by now well established.

The identification problem is more complicated for a nonlinear system. Extensive work has been done in the last time in the identification of nonlinear systems area [1]. An interesting identification procedure is presented in [2] for a class of bilinear oscillators, using higher order frequency response functions derived from Volterra series under harmonic excitation. The author approximated the form of the restoring force by nonlinear polynomials and obtained a closed form relation between the polynomial coefficients and the bilinear stiffness parameter.

A wavelet multiresolution technique is proposed in [3] to identify the time-varying parameters of hysteretic structures. The relationship between input and output data is represented by using the wavelet method. In [4] a genetic algorithm is developed to identify the Bouc-Wen model parameters from the experimental data of periodic loading tests. It is considered an extension of classical model in order to increase its capacity to approximate experimental loops. Within this

[1] Politecnico of Torino, Physics and Structural Eng. Dept., Corso Duca degli Abruzzi 24, Torino 10129, Italy
[2] Institute of Solid Mechanics of Romanian Academy, Ctin Mille 15, 010141 Bucharest

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context, a hysteretic model was proposed initially by Bouc in 1971 and generalized by Wen in 1976. Since then, the Bouc-Wen model has been extensively used in the areas of civil and mechanical engineering [5,6].

This paper is inspired from [3] and [4]. The restoring force is portraying by a Bouc-Wen model, but the relationship between input and output data is represented by using the cnoidal method.

Let us begin with the general nonlinear physical system with N degree of freedom, in particularly, a shear-beam building is governed by the equation

$$L_i(\ddot{\mathbf{x}},\dot{\mathbf{x}},\mathbf{x},\mathbf{r},\mathbf{f}) \equiv M_{ij}\ddot{x}_j(t) + r_i(\dot{\mathbf{x}}(t),\mathbf{x}(t),\boldsymbol{\theta}(t)) + f_i(t) = 0\,,\ i,j = 1,2,3,...,N\,, \tag{1}$$

where $\mathbf{M} = \mathrm{diag}(m_1, m_2,..., m_N)$ is the mass matrix which is assumed to be available, $\mathbf{x} = [x_1, x_2,..., x_N]^{\mathrm{T}}$ is the structural displacement vector relative to the ground, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the corresponding velocity and acceleration vectors, respectively, $\boldsymbol{\theta}(t)$ is the time-varying parameter vector that models the structural behavior, $\mathbf{r} = [r_1 - r_2, r_2 - r_3,..., r_{N-1} - r_N]^{\mathrm{T}}$ is a nonlinear restoring force vector which is a function of the response state $\dot{\mathbf{x}}, \ddot{\mathbf{x}}$ and $\boldsymbol{\theta}$, while $f_i(t) = \mathbf{IM}f(t)$ is the input excitation, with $\mathbf{I}$ the identity vector and $f(t)$ the ground acceleration.

The restoring forces r_i, $i = 1,2,3,...,N$ characterize a hysteretic behaviour of the system. The restoring forces r_i are assumed to be described by the Bouc-Wen model [7-11]

$$\begin{aligned} Q_i(\dot{\mathbf{r}},\mathbf{r},\dot{\mathbf{x}},\mathbf{x}) \equiv \dot{r}_i - c_i(\ddot{x}_i - \ddot{x}_{i-1}) + k_i(\dot{x}_i - \dot{x}_{i-1}) - \beta_i\,|\dot{x}_i - \dot{x}_{i-1}|\,r_i\,|r_i|^{n_i-1} - \\ -\gamma_i\,|\dot{x}_i - \dot{x}_{i-1}|\,|r_i|^{n_i} = 0, \end{aligned} \tag{2}$$

where c_i, $i = 1,2,3,...,N$ is the viscous damping coefficient, k_i is the equivalent stiffness coefficient, β_i and γ_i are the shape parameters, and n_i governs the smoothness of the force-displacement curve. In addition we note that $\ddot{x}_{-1} = \dot{x}_{-1} = 0$.

The unknown vector $\boldsymbol{\theta}$ is composed of the Bouc-Wen model parameters

$$\theta_i(t) = [c_i(t), k_i(t), \beta_i(t), \gamma_i(t)]^{\mathrm{T}}\,,\quad i = 1,2,3,...,N\,. \tag{3}$$

A new alternating procedure is used for the identification of time-varying parameters $\theta_i(t)$ and n_i. A cnoidal representation for the input-output relationship and a suitable parametric identification technique based on a genetic algorithm are proposed.

The input-output relationship is expressed as a cnoidal representation [12]

$$\tilde{x}(t) = \sum_{k=1}^{p} \alpha_k \mathrm{cn}^2[k\omega t; \tilde{m}_k]\,, \tag{4}$$

where the sub-indice $i = 1,2,...,N$ for the solution $\tilde{x}_i(t)$ is omitted, ω is the frequency, α_k, $k = 0,1,...,p$, are constants, and $0 \le \tilde{m}_k \le 1$, $k = 1,2,3,...,p$, is the modulus of the Jacobi elliptic function cn (the cnoidal function).

The parameters $\theta_i(t)$ and n_i, $i = 1,2,...,N$, as well as the unknowns ω, α_k and m_k, $k = 1,2,3,...,p$, are identified by minimizing the error function

$$E = \sum_{j=1}^{q} | x_j^{\text{mes}} - \tilde{x}_j | + \sum_{i=1}^{N} L_i^2 + \sum_{k=1}^{N} Q_k^2 , \tag{5}$$

where x_j^{mes}, $j = 1,2,...,q$ are the q measured displacements, $\tilde{x}_j$ are the analytical representation (4) corresponding to each measurement data, L_i is defined by (1) and Q_k by (2). The minimization technique is based on a genetic algorithm [4, 13, 14].

2. JUSTIFICATION OF THE CNOIDAL REPRESENTATION

The cnoidal functions are much richer than the trigonometric or hyperbolic functions. Since the original paper by Korteweg and DeVries, it remains an open question [15]: "*if the KdV linearised equation can be solved by an ordinary Fourier series as a linear superposition of sine waves, can the KdV equation itself be solved by a generalization of Fourier series which uses the cnoidal wave as the fundamental basis function?*"

The modulus m can be varied to obtain a sine or cosine function $(\tilde{m} \cong 0)$, a Stokes function $(\tilde{m} \cong 0.5)$ or a solitonic function, sech or tanh $(\tilde{m} \cong 1)$ parameter of the Jacobi elliptic functions.

The justification of the cnoidal representation (4) requires brief information necessary to describe the cnoidal method. The arc length of the ellipse is related to the integral

$$E(z) = \int_0^z \frac{\sqrt{(1-\kappa^2 x^2)}\mathrm{d}x}{\sqrt{(1-x^2)}},$$

with $0 < \kappa < 1$. Another elliptical integral is given by

$$F(z) = \int_0^z \frac{\mathrm{d}x}{\sqrt{(1-x^2)(1-\kappa^2 x^2)}} .$$

The integrals $E(z)$ and $F(z)$ are Jacobi elliptic integrals of the first and the second kinds. Legendre is the first who works with these integrals, being followed by Abel (1802–1829) and Jacobi (1804–1851). Inspired by Gauss, Jacobi discovered in 1820 that the inverse of $F(z)$ is an elliptical double-periodic integral $F^{-1}(\omega)=\mathrm{sn}(\omega)$. Jacobi compared the integral

$$v=\int_0^{\varphi}\frac{\mathrm{d}\varphi}{(1-\tilde{m}\sin^2\varphi)^{1/2}},$$

where $0\le\tilde{m}\le 1$, to the elementary integral

$$w=\int_0^{\varphi}\frac{\mathrm{d}t}{(1-t^2)^{1/2}},$$

and observed that the last integral defines the inverse of the trigonometric function sin if we use the notations $t=\sin\theta$ and $\psi=\sin w$. He defines a new pair of inverse functions, namely, $\mathrm{sn}\,v=\sin\varphi$, $\mathrm{cn}\,v=\cos\varphi$. These are two of the Jacobi elliptic functions, usually written as $\kappa(\varepsilon)=\frac{2}{\lambda}\frac{\partial^2}{\partial t^2}\log G(\varepsilon)$, $\varepsilon=k\omega t+\phi$, and $\mathrm{cn}(v,m)$ to denote the dependence on the modulus $\tilde{m}$. The angle φ is called the amplitude $\varphi=\mathrm{am}\,u$. We also define the Jacobi elliptic function $\mathrm{dn}v=(1-\tilde{m}\sin^2\varphi)^{1/2}$.

For $\tilde{m}=0$, we have

$$v=\varphi,\quad \mathrm{cn}(v,0)=\cos\varphi=\cos v,$$
$$\mathrm{sn}(v,0)=\sin\varphi=\sin v,\quad \mathrm{dn}(v,0)=1,$$

and for $\tilde{m}=1$

$$v=\mathrm{arcsech}(\cos\varphi),\quad \mathrm{cn}(v,1)=\mathrm{sech}v,$$
$$\mathrm{sn}(v,1)=\tanh v,\quad \mathrm{dn}(v,1)=\mathrm{sech}v.$$

The functions $\mathrm{sn}\,v$ and $\mathrm{cn}\,v$ are periodic functions with the period

$$\int_0^{2\pi}\frac{\mathrm{d}\varphi}{(1-\tilde{m}\sin^2\varphi)^{1/2}}=4\int_0^{\pi/2}\frac{\mathrm{d}\varphi}{(1-\tilde{m}\sin^2\varphi)^{1/2}}.$$

The later integral is the complete elliptic integral of the first kind

$$K(m)=\int_0^{\pi/2}\frac{\mathrm{d}\varphi}{(1-\tilde{m}\sin^2\varphi)^{1/2}}.$$

The period of the function $\mathrm{dn}\,v$ is $2K$. For $\tilde{m}=0$ we have $K(0)=\pi/2$. For increasing of $\tilde{m}$, $K(\tilde{m})$ increases monotonically

$$K(\tilde{m})\approx\frac{1}{2}\log\frac{16}{1-\tilde{m}}.$$

Thus, this periodicity of $\operatorname{sn}(v,1)$ and $\operatorname{cn}(v,1)=\operatorname{sech}v$ is lost for $\tilde{m}=1$, so $K(\tilde{m})\to\infty$. Some important algebraic and differential relations between the cnoidal functions are given below

$$\operatorname{cn}^2+\operatorname{sn}^2=1,\quad \operatorname{dn}^2+\tilde{m}\operatorname{sn}^2=1,\quad \frac{\mathrm{d}}{\mathrm{d}v}\operatorname{cn}=-\operatorname{sn}\operatorname{dn},$$
$$\frac{\mathrm{d}}{\mathrm{d}v}\operatorname{sn}=\operatorname{cn}\operatorname{dn},\quad \frac{\mathrm{d}}{\mathrm{d}v}\operatorname{dn}=-\tilde{m}\operatorname{sn}\operatorname{cn},$$

where the argument v and parameter $\tilde{m}$ are the same throughout relations.

Now, consider the function $\wp(t)$ introduced by Weierstrass (1815–1897) in 1850, which verifies the equation

$$\dot{\wp}^2=4\wp^3-g_2\wp-g_3, \tag{6}$$

where the superimposed point means differentiation with respect to t.

If e_1,e_2,e_3 are real roots of the equation $4y^3-g_2y-g_3=0$ with $e_1>e_2>e_3$, then (6) can be written under the form

$$\dot{\wp}^2=4(\wp-e_1)(\wp-e_2)(\wp-e_3), \tag{7}$$

with

$$g_2=2(e_1^2+e_2^2+e_3^2),\quad g_3=4e_1e_2e_3,\quad e_1+e_2+e_3=0.$$

Introducing

$$\Delta=g_2^3-27g_3^2,$$

when $\Delta>0$, equation (7) admits the elliptic Weierstrass function as a particular solution, which is reducing in this case to the Jacobi elliptic function cn

$$\wp(t+\delta';g_2,g_3)=e_2-(e_2-e_3)\operatorname{cn}^2(\sqrt{e_1-e_3}\,t+\delta'), \tag{8}$$

where δ' is an arbitrary real constant. If we impose initial conditions to (7)

$$\wp(0)=\theta_0,\quad \wp'(0)=\theta_{p0},$$

then a linear superposition of cnoidal functions (8) is also a solution for (6)

$$\kappa=2\sum_{k=1}^{n}\alpha_k\operatorname{cn}^2[\omega_k t;\tilde{m}_k]. \tag{9}$$

For a generalized Weierstrass equation with a polynomial of n degree in $\theta(t)$ given by

$$\dot{\kappa}^2=P_n(\kappa), \tag{10}$$

the functional form of solutions is determined by the zeros of the right-hand side polynomial. For biquadratic polynomial $n=4$ we can have four real zeros, two real and two purely imaginary zeros, four purely imaginary zeros or four genuinely complex zeros. For $n=5$ the functional form of solutions depends also on the zeros of the polynomial. For all cases the solutions are expressed in terms of Jacobi elliptic functions, the hyperbolic and trigonometric functions.

Osborne [15] discussed the cnoidal method for integrable nonlinear equations that have periodic boundary conditions, in particular for the KdV equation. Munteanu and Donescu [12] have extended this method to nonlinear partial differential equations that can be reduced to Weierstrass equations of the type (10).

Suppose now that the motion equation (1) can be reduced to

$$\frac{\mathrm{d}\kappa_i}{\mathrm{d}t}=F_i(\kappa_1,\kappa_2,...,\kappa_n),\ \ i=1,...,n,\quad n\geq 3\,, \tag{11}$$

with $x\in\mathbb{R}^n$, $t\in[0,T]$, $T\in\mathbb{R}$, where F may be of the form

$$F_i=\sum_{p=1}^{n}a_{ip}\kappa_p+\sum_{p,q=1}^{n}b_{ipq}\kappa_p\kappa_q+\sum_{p,q,r=1}^{n}c_{ipqr}\kappa_p\kappa_q\kappa_r+$$
$$+\sum_{p,q,r,l=1}^{n}d_{ipqrl}\kappa_p\kappa_q\kappa_r\kappa_l+\sum_{p,q,r,l,m=1}^{n}e_{ipqrlm}\kappa_p\kappa_q\kappa_r\kappa_l\kappa_m+...\,,$$

with $i=1,2,...,n$, and $a,b,c...$ constants.

The system of equations (11) has the remarkable property that it can be reduced to Weierstrass equations of the type (10).

In the following, we present the cnoidal method, suitable to be used for equations of the form (11). To simplify the presentation, let us omit the index i and note the solution by $\kappa(t)$.We introduce the function transformation

$$\kappa=2\frac{\mathrm{d}^2}{\mathrm{d}t^2}\log\Theta_n(t)\,, \tag{12}$$

where the theta function $\Theta_n(t)$ is defined as

$$\Theta_1=1+\exp(\mathrm{i}\omega t+B_{11})\,,$$

$$\Theta_2=1+\exp(\mathrm{i}\omega t+B_{11})+\exp(2\mathrm{i}\omega t+B_{22})+\exp(3\omega+B_{12})\,,$$

$$\Theta_3=1+\exp(\mathrm{i}\omega t+B_{11})+\exp(2\mathrm{i}\omega t+B_{22})+\exp(3\mathrm{i}\omega t+B_{33})+\exp(3\omega+B_{12})+$$
$$+\exp(4\omega+B_{13})+\exp(5\omega+B_{23})+\exp(6\omega+B_{12}+B_{13}+B_{23}),$$

and

$$\Theta_n = \sum_{M\in(-\infty,\infty)} \exp(\mathrm{i}\sum_{k=0}^{n} kM_k\omega t + \frac{1}{2}\sum_{k<j}^{n} B_{kj}M_iM_j),$$

$$\exp B_{ij} = \left(\frac{\omega_i - \omega_j}{\omega_i + \omega_j}\right)^2, \quad \exp B_{ii} = \omega^2 .$$

Further, we write the solution (12) under the form

$$\kappa(t) = 2\frac{\partial^2}{\partial t^2}\log\Theta_n(\varepsilon) = \kappa(\varepsilon), \tag{13}$$

for $\varepsilon = \omega t + \phi$. The first term $\kappa(\varepsilon)$ represents, as above, a linear superposition of cnoidal waves. Indeed, after a little manipulation and algebraic calculus, we have

$$\kappa = \sum_{l=1}^{n}\alpha_l\left(\frac{2\pi}{K_l\sqrt{\tilde{m}_l}}\sum_{k=1}^{\infty}\left(\frac{q_l^{k+1/2}}{1+q_l^{2k+1}}\cos(2k+1)\frac{\pi l\omega t}{2K_l}\right)^2\right). \tag{14}$$

In (14) we recognize the expression (4) [17, 18] with

$$q = \exp\left(-\pi\frac{K'}{K}\right), \quad K = K(\tilde{m}) + \int_0^{\pi/2}\frac{\mathrm{d}u}{\sqrt{1-\tilde{m}\sin^2 u}},$$

$$K'(\tilde{m}_1) = K(\tilde{m}), \quad \tilde{m} + \tilde{m}_1 = 1.$$

As a result, the cnoidal method yields to solutions consisting of a linear superposition expressed as (4).

3. RESULTS

The proposed identification problem is tested for three-story building subjected to a random excitation, modeled as a lumped-mass shear-beam model displayed in Fig. 1. The motion equation results from (1)

$$\mathbf{L} \equiv \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{r}(\dot{\mathbf{x}}(t),\mathbf{x}(t),\boldsymbol{\theta}(t)) + \mathbf{MI}f(t) = 0, \tag{15}$$

where $\mathbf{M} = \mathrm{diag}(m_1, m_2, m_3)$ is the mass matrix, $\mathbf{x} = [x_1, x_2, x_3]^{\mathrm{T}}$ is the displacement vector relative to the ground, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are the corresponding velocity and acceleration vectors, respectively, $\boldsymbol{\theta}(t)$ is the time-varying parameter vector defined by (3), $\mathbf{r} = [r_1 - r_2,\ r_2 - r_3,\ r_3]^{\mathrm{T}}$ is the nonlinear restoring force vector and $f_i(t) = \mathbf{IM}f(t)$ is the input excitation with $f(t)$ the random ground acceleration. The restoring force vector $\mathbf{r}$ can be written under the form

$$\mathbf{r} \equiv -\mathbf{M}\ddot{\mathbf{x}} - \mathbf{MI}f. \tag{16}$$

When $f(t)$ and the floor acceleration are known, the restoring forces can be obtained from (16). We suppose that the restoring forces are defined by the Bouc-Wen model (2).

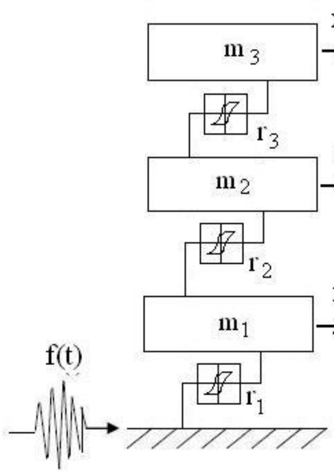

Fig. 1 – A model for a 3-story building.

The random input signal is displayed in Fig. 2. Fig. 3 presents the output time histories x_i, $i = 1,2,3$, which are numerically computed from (1) and (2) with the following constant Bouc-Wen parameters:

$$\begin{gathered} m_1 = m_2 = m_3 = 125\,\text{kg}, \quad c_1 = c_2 = c_3 = 0.07\,\text{kNs/m}, \\ k_1 = k_2 = k_3 = 24\,\text{kN/m}, \\ \beta_1 = 2.05, \ \beta_2 = 1.97, \ \beta_3 = 1.93, \ \gamma_1 = 1.12, \ \gamma_2 = 1.02, \ \gamma_3 = 0.98, \\ n_1 = n_2 = n_3 = 2. \end{gathered} \tag{17}$$

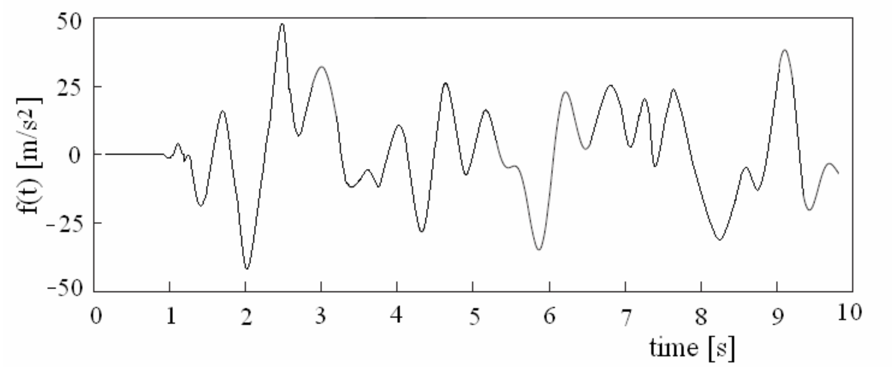

Fig. 2 – The random input signals.

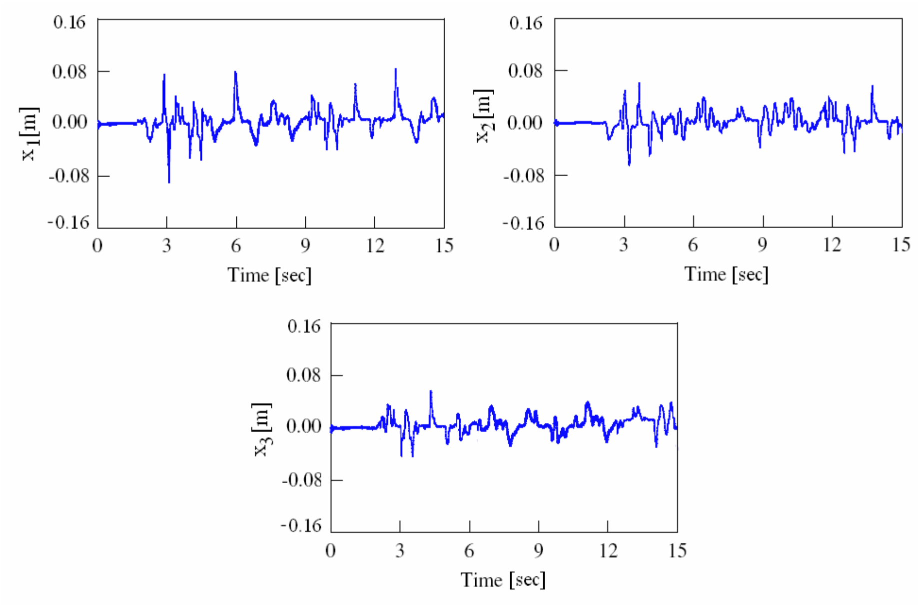

Fig. 3 – The output time histories computed from (1) and (2).

We test the identification procedure of the system parameters by using the input-output relationship expressed as a cnoidal representation (4). The unknown parameters are

$$c_i(t),\ k_i(t),\ \beta_i(t),\ \gamma_i(t),\ n_i,\ i=1,2,3,\quad \alpha_k,\ \tilde{m}_k,\ k=1,2,...,p,\quad \omega.$$

These unknown are computed by minimizing the error function (5) with $q=300$

$$E=\sum_{j=1}^{300}|x_j^{\text{mes}}-\tilde{x}_j|+\sum_{i=1}^{3}L_i^2+\sum_{k=1}^{3}Q_k^2.$$

The identification procedure gives the following results for $p=1$:

$$c_1=c_2=c_3=0.0723\text{kNs/m},\quad k_1=k_2=k_3=24{,}223\text{kN/m},$$
$$\beta_1=2{,}0543,\ \beta_2=1{,}977,\ \beta_3=1{,}934,\ \gamma_1=1{,}127,\ \gamma_2=1{,}022,\ \gamma_3=0.981,$$
$$n_1=n_2=n_3=2{,}02,\quad \tilde{m}_1=0.87,\quad \alpha_1=0.55,\quad \omega=0.41.$$

For $p=2$, more accurate results are obtained:

$$c_1=c_2=c_3=0.0702\text{kNs/m},\quad k_1=k_2=k_3=24{,}009\text{kN/m},$$
$$\beta_1=2{,}0511,\ \beta_2=1{,}970,\ \beta_3=1{,}930,\ \gamma_1=1{,}122,\ \gamma_2=1{,}020,\ \gamma_3=0.980,$$
$$n_1=n_2=n_3=2,\ \tilde{m}_1=0.87,\ \tilde{m}_2=0.94,\ \alpha_1=0.55,\ \alpha_2=0.57,\ \omega=0.4.$$

For $p=3$, the exact results (17) are obtained for $\tilde{m}_1 = 0.87$, $\tilde{m}_2 = 0.94$, $\tilde{m}_3 = 0.59$, $\alpha_1 = 0.55$, $\alpha_2 = 0.57$, $\alpha_3 = 0.43$ and $\omega = 0.399$.

Next, we consider the case of time varying Bouc-Wen parameters with the same random input signals displayed in Fig. 2. The parameters for all stories obey the relations

$$
\begin{gathered}
c = 0.0025t + 0.05, \quad k = 25 - 0.67t \ \text{for}\, 0 \le t \le 15, \\
\beta = \gamma, \ \beta = -1/3t + 2 \ \text{ for } \ 0 \le t \le 3, \\
\beta = 1/3t \ \text{ for } \ 3 \le t \le 6, \\
\beta = -1/3t + 4 \ \text{ for } \ 6 \le t \le 9, \\
\beta = 1/3t - 2 \ \text{ for } \ 9 \le t \le 12, \\
\beta = -1/3t + 6 \ \text{ for } \ 9 \le t \le 12.
\end{gathered} \tag{18}
$$

The other parameters are $m_1 = m_2 = m_3 = 125\,\text{kg}$ and $n_1 = n_2 = n_3 = 2$.

Fig. 4 shows plots of restoring forces r_i, $i = 1,2,3$, with respect to time. The output time histories are displayed in Fig. 5.

Identification results for Bouc-Wen time-varying parameters are plotted in Fig. 6, for $p=3$ and $\tilde{m}_1 = 0.91$, $\tilde{m}_2 = 0.88$, $\tilde{m}_3 = 0.63$, $\alpha_1 = 0.51$, $\alpha_2 = 0.53$, $\alpha_3 = 0.44$ and $\omega = 0.4$. A good agreement is observed. It is easily seen that the results are identically with (18).

In the following, the restoring forces were plotted as functions of relative displacements in Fig. 7. It is seen that the hysteretic behavior for first story is distinctively different than those of the last two stories.

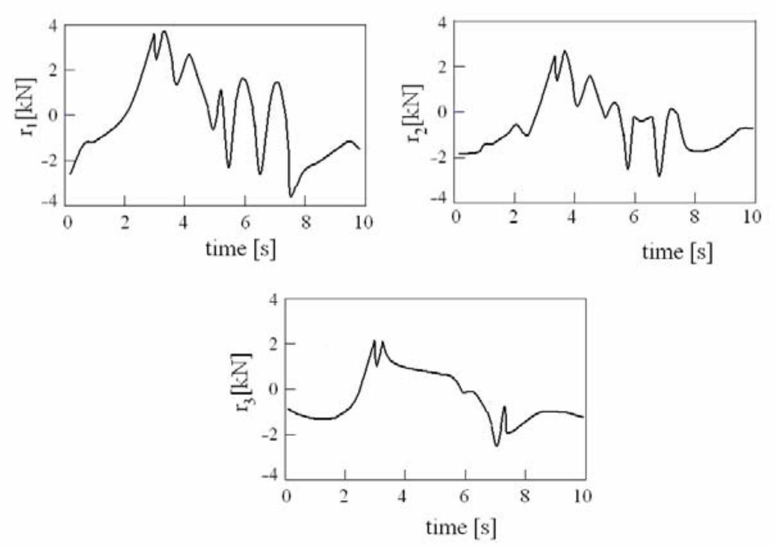

Fig. 4 – The time variation of the restoring forces.

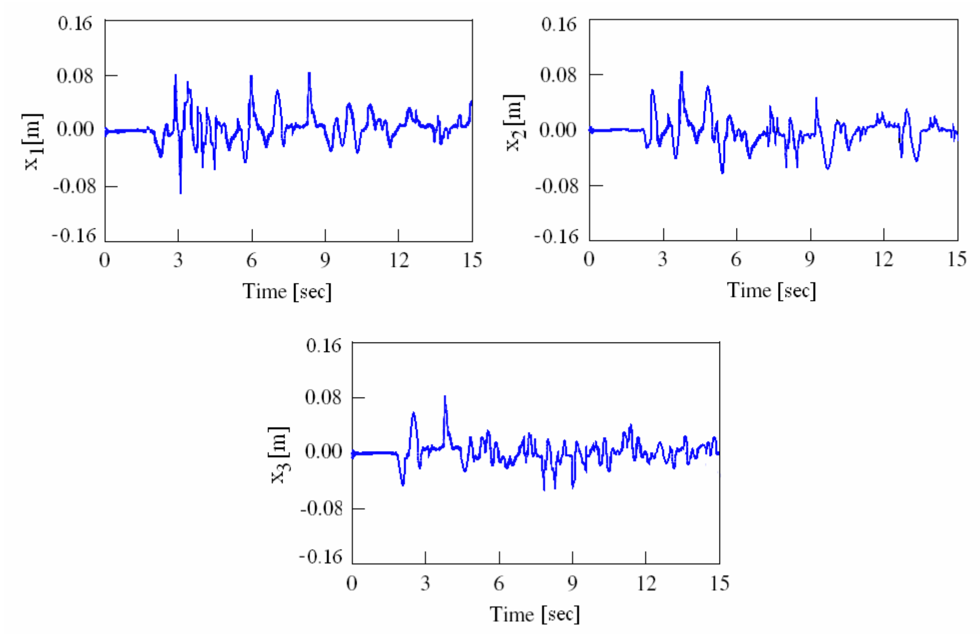

Fig. 5 – The output time histories computed from (1) and (2).

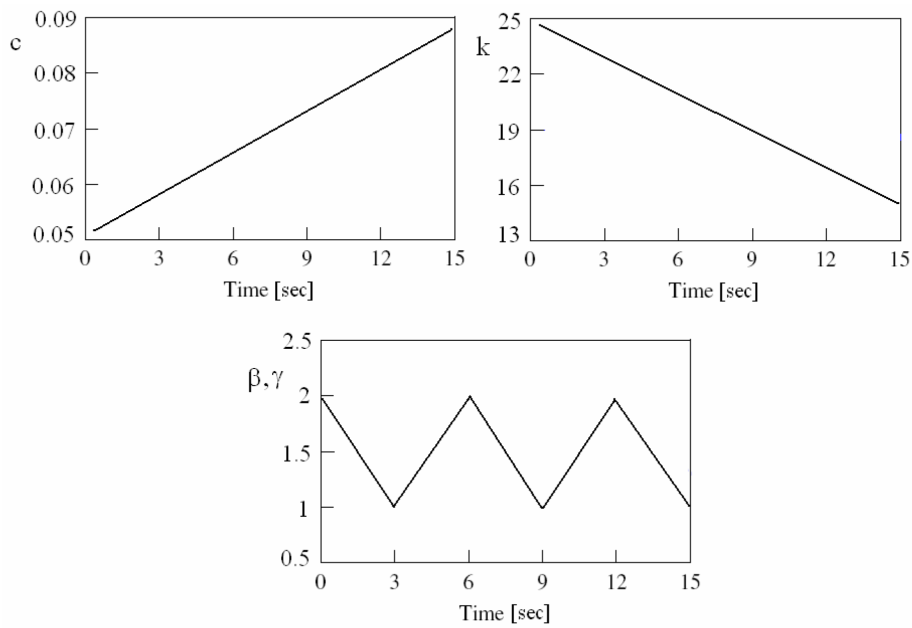

Fig. 6 – Identification results for Bouc-Wen time-varying parameters.

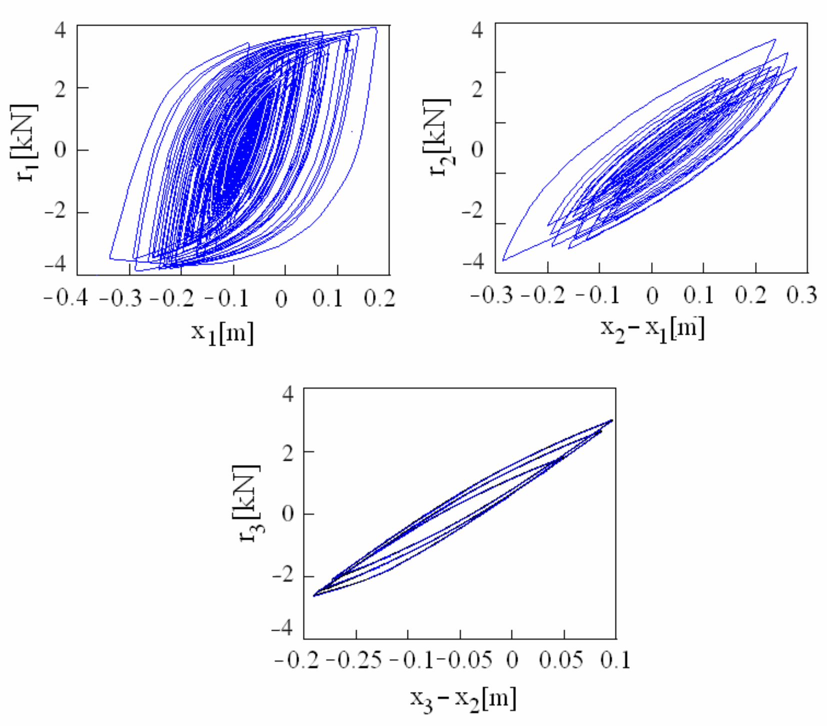

Fig. 7 – The hysteretic loops for restoring forces plotted as functions of relative displacements.

4. CONCLUSIONS

The paper has presented an identification method for the time-varying properties of a shear-beam building with N degree of freedom. The restoring force is portraying by a Bouc-Wen model. The method consists in exiting the hysteretic system with a random input signal and uses the output time histories to derive the parameters of the model. A cnoidal representation for the input-output relationship is proposed. The procedure provides the exact time evolution of the parameters in the absence of the disturbances. The implementation of the procedure has been illustrated by means of numerical simulation for a three-story building modelled as a lumped-mass shear-beam model.

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REFERENCES

1. KERSCHEN, G., WORDEN, K., VAKAKIS, A.F., GOLINVAL, J.C., *Past, Present and Future of Nonlinear System Identification in Structural Dynamics,* Mech. Sys. Signal Proc., **20**, pp. 505-592, 2006.
2. CHATTERJEE, A., *Identification and Parameter Estimation of a Bilinear Oscillator using Volterra Series with Harmonic Probing*, International Journal of Non-Linear Mechanics, **45**, pp. 12-20, 2010.
3. CHANG, C.C., SHI, Y., *Identification of Time-varying Hysteretic Structures using Wavelet Multiresolution Analysis*, International Journal of Non-Linear Mechanics, **45,** pp. 21-34, 2010.
4. GIUCLEA, M., SIRETEANU, T., MITU, A.M., *Use of Genetic Algorithms for Fitting the Bouc-Wen Model to Experimental Hysteretic Curves*, Rev. Roum. Sci. Techn – Méc. Appl., **54**, *1*, pp. 3-10, 2009.
5. IKHOUANE, F., RODELLAR, J., *Systems with Hysteresis. Analysis, Identification and Control using the Bouc-Wen Model*, John Wiley & Sons, Ltd., 2007.
6. MAYERGOYZ, I., *Mathematical Models of Hysteresis and their Applications*, Elsevier, 2003.
7. SMYTH, A.W., MASRI, S.F., CHASSIAKOS, A.G., CAUGHEY, T.K., *On-line Parametric Identification of MDOF Nonlinear Hysteretic Systems*, ASCE J. Eng. Mech., **125**, *2*, pp. 133-142, 1999.
8. SIRETEANU, T., GIUCLEA, M., SERBAN, V., MITU, A.M., *On the Fitting of Experimental Hysteretic Loops by Bouc-Wen Model*, Proceedings of the Annual Symposium of the Institute of Solid Mechanics SISOM, May 29-30, 2008.
9. SIRETEANU, T., GIUCLEA, M., MITU, A.M., *Identification of an Extended Bouc-Wen Model with Application to Seismic Protection Through Hysteretic Devices*, Computational Mechanics, **45**, *5*, pp. 431-441, 2009.
10. SIRETEANU, T., GIUCLEA, M., MITU, A.M., *An Analytical Approach for Approximation of Experimental Hysteretic Loops by Bouc-Wen Model*, Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **10**, *1*, pp. 43-54, 2009.
11. SIRETEANU, S., MITU, A.M., BALDOVIN, D., CHIROIU, V., *On the Forced Vibrations of a Cantilever Beam with Hysteretic Damping*, A IV-a Conferință Națională: Zilele Academice ale Academiei de Științe Tehnice din Romania, Iasi, Nov. 19-20, 2009.
12. MUNTEANU, L., DONESCU, Şt., *Introduction to Soliton Theory: Applications to Mechanics,* Book Series "Fundamental Theories of Physics", **143**, Kluwer Academic Publishers, 2004.
13. CHIROIU, V., CHIROIU, C., *Probleme inverse în mecanică*, Romanian Academy Publishing House, Bucharest, 2003.
14. CHIROIU, V., MUNTEANU, L., *Identification of Hysteretic Behavior of Materials by Using Genetic Algorithms*, 10th International Conference on Automation and Information (ICAI'09), Prague, March 23-25, 2009, pp. 46-51.
15. OSBORNE, A.R., *Soliton Physics and the Periodic Inverse Scattering Transform*, Physica D: Nonlinear Phenomena, **86**, *1-2*, pp. 81-89, 1995.