A PREISACH MODEL FOR THE ANALYSIS OF THE HYSTERETIC PHENOMENA

VASILE PREDA[1], MARIUS FLORINEL IONESCU[2], VETURIA CHIROIU[3], TUDOR SIRETEANU[3]

Abstract. In the field of the hysteretic behavior of systems, the displacement drift, force relaxation and non-closure of the hysteretic loops for systems subjected to short input signals, are investigated by using a Preisach model. The results of fitting the Bouc-Wen model used to imitate experimental data, are presented and discussed.

Key words: hysteretic behaviour, Preisach model, Bouc-Wen model, displacement drift, force relaxation.

1. INTRODUCTION

For a material in which the intrinsic order parameters of the hysteretic behavior may interact with the structural order parameter of the medium, the interplay of competing interactions stabilizes multiple topologically phases separated by sharp transitions in the hysteretic curves.

Hysteresis refers to systems that have memory, where the effects of the current input to the system are experienced with a certain delay in time. Such systems are not linear, the specific nonlinearities being modeled as feedback models for freeplay/backlash: the Bouc-Wen or Duhem models for friction, the Preisach models for smart materials, and so on. The non-equilibrium behavior of such systems is an intensely studied field in mechanical engineering, electromagnetism, structural biology, geology, and even financial analysis. The slow and complex time dependence of various parameters is a characteristic of such systems [1–3].

Several aspects of this non-equilibrium dynamics have already been described in great detail for shape memory alloys and spin glasses [4]. In [5], the origin of the reversal-field memory is explained due to the existence of a macroscopic number of symmetric clusters of spins associated with local spin-

[1] University of Bucharest, Faculty of Mathematics and Computer Science, str. Academiei 14, 010014 Bucharest
[2] "Ion Neculce" High-School, str. Ion Neculce 2, Bucharest
[3] Institute of Solid Mechanics of the Romanian Academy, Ctin Mille 15, 010141 Bucharest

Rev. Roum. Sci. Techn. – Méc. Appl., Tome 55, Nº 3, P. 243–254, Bucarest, 2010

reversal symmetry of the Hamiltonian. The Dahl, LuGre, and Maxwell-slip friction models are investigated as Duhem hysteresis models in [6].

According to the theory of dynamic systems, the critical conditions where the non-equilibrium dynamics begins to occur are mathematically correspondent to the singular points, which are special points at which all the partial derivatives simultaneously vanish. The topological approach to analyze the hysteretic behavior of the airfoil flow is reported in [7]. The approach is based on the topological invariant rules of singular points under topological mapping. Fig.1 displays a reversal constitutive curve (solid line) with the same value of the strain $\varepsilon = 0.0175$, observed for the copper wires at the unloading-load cycle [8]. Such singularities create non-analyticity in the behavior at a particular reversal point related to the history of the system. The reversal curves are minor loops, obtained by reversal from ascending or descending branch of the major loops (dotted line).

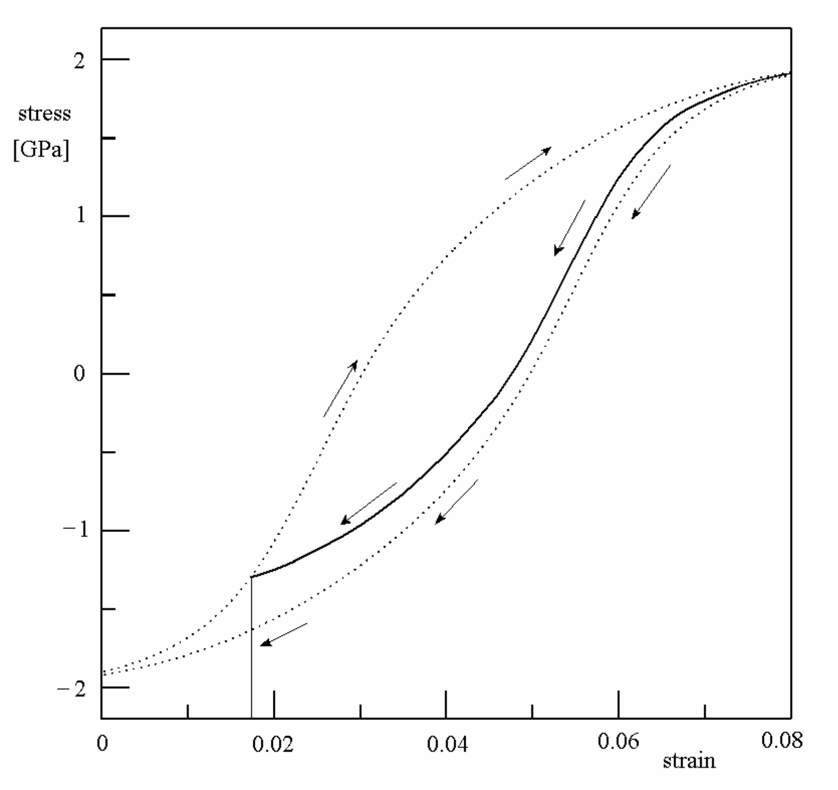

Fig. 1 – Reversal constitutive curve (solid line) and major hysteretic loop (dotted line).

The hysteretic loops which exhibit the displacement drift, force relaxation and non-closure are investigated in [9] by using a Bouc-Wen model. The unrealistic behavior of the Bouc-Wen model with respect to short input signals was eliminated by inserting a stiffening factor into the hysteretic differential equation.

In order to understand the microscopic origin of the reversal effect, the Preisach model can be used [10–13]. The friction contact models between two vibrating units (molecules or grains) are investigated in many papers [14, 15]. When the friction force exceeds a positive value, the contact interface starts to slip towards the positive displacement direction. The friction force remains equal to the varying slip load until the contact interface sticks again. The transition between slip and stick depends on the tangential relative displacement and on the variable normal load, which may decrease to reduce the slip load so that the occurrence of the transition can be postponed to some instant after the reversion of displacement. So, the moment when the interface changes back to the stick do not correspond to the moment when the displacement reverses its direction. During the cycle of motion the contact normal load may vanish and cause the separation [16–18].

In this paper, a Preisach model is used to describe the hysteretic behavior of a system with one degree of freedom subjected to short input signals.

To imitate the experimental data, consider a system obeying to the law

$$M\ddot{x}(t) + r(\dot{x}, x, \theta) + Mg(t) = 0\,, \tag{1}$$

where M is the mass, x is the displacement relative to the ground, $\dot{x}$ and $\ddot{x}$ are the corresponding velocity and acceleration, respectively, θ is the set of parameters that models the structural behavior, r is a nonlinear restoring force and $g(t)$ is the short unloading-reloading ground acceleration. The restoring force r characterizes the hysteretic behavior of the system and we consider that it can be described by the Bouc-Wen model

$$\dot{r} = c\ddot{x} - k\dot{x} + \beta\,|\,\dot{x}\,|\,r\,|\,r\,|^{n-1} + (1 - 2H(\dot{x}r)R(x,r))\gamma\,|\,\dot{x}\,||\,r\,|^{n}\,, \tag{2}$$

where c is the viscous damping coefficient, k is the equivalent stiffness coefficient, β and γ are the shape parameters, and n governs the smoothness of the force-displacement curve, $R(x,r) \in [0,1]$ is a stiffening factor and $H(\cdot)$ is the Heaviside function defined as

$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x \le 0. \end{cases}$$

The term $2H(\dot{x}r)R(x,r)$ was proposed in [9] within the idea that the Heaviside function can help the unloading branches to remain identical to those of the classical model [19,20]

$$\dot{r} = c\ddot{x} - k\dot{x} + \beta\,|\,\dot{x}\,|\,r\,|\,r\,|^{n-1} + \gamma\,|\,\dot{x}\,||\,r\,|^{n}\,. \tag{3}$$

Also, the stiffening factor R controls the transition between loading (reduced) stiffness and unloading (increased) stiffness, when loading or reloading.

For $R = 0$, (2) reduces to (3). For $R = 1$, the loading stiffness becomes equal to that of unloading at the same point.

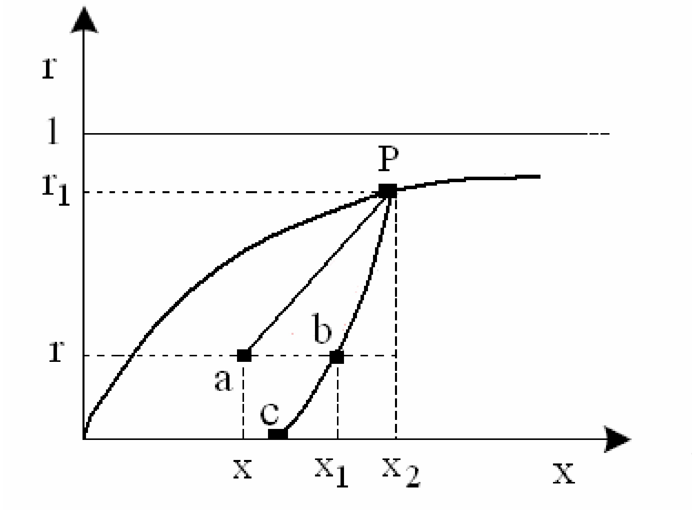

Fig. 2 – The definition of R.

We use the Charalampakis and Koumousis definition of R (see [9] and Fig. 2)

$$R = H(r_1 - r)H(x_1 - x)\left(\frac{x_2 - x_1}{x_2 - x}\right)^{\kappa}. \tag{4}$$

As shown in Fig. 2, P is a reversal point. During reloading, the current state is represented by the point a for $0 \le r < r_1$. The point b corresponds to the unloading path. As a approaches b from the left, $R \to 1$. When a and b coincide, then $R = 1$ and loading follows the unloading path exactly. The unloading path $P - c$ can not be crossed. When $r > r_1$ or $x > x_1$, the stiffening effect disappears due to the Heaviside function. Parameter κ controls the intensity of stiffening to the left of the unloading path. The stiffening is closed to the unloading path for increased values of κ, and diminished every where else. It was observed in [9] that $1 \le \kappa \le 2$ is the best for identifying the realistic hysteretic loops.

Therefore, the set of parameters θ of the Bouc-Wen model described by (1), (2) and (4), is

$$\theta(t) = [c, k, \beta, \gamma, \kappa], \tag{5}$$

for a given n.

2. THE PREISACH MODEL

Consider a hysteretic transducer which is characterized by an input $u(t)$ and an output $f(t)$. Its input-output relationship is a multi branch nonlinearity for which a branch-to-branch transition occurs after each input extremum. The input is determined by the interaction of the transducer with the rest of the system. For this reason, a mathematical model is needed to detect and accumulate input extrema and to choose the appropriate branch of the hysteretic nonlinearity with respect to the accumulated history [10, 11, 18].

The Preisach model, which describes a hysteretic operator with nonlocal (global) memory, implies mapping of input $u(t)$ on output $f(t)$ in the form

$$f(t) = \iint_{\alpha\geq\beta} P(\alpha,\beta) G_{\alpha\beta} u(t)\,\mathrm{d}\alpha\mathrm{d}\beta\,, \tag{6}$$

where $G_{\alpha\beta}$ is an elementary hysteretic operator – a rectangular loop shown in Fig. 3.

Numbers α and β correspond *up* and *down* switching values of input, respectively $+1$ and -1 are two possible output values. Therefore, in the Preisach model a dynamic system is described as a collection of independent two state ± 1 switching units. As the input $u(t)$ is monotonically increased, the ascending branch *abcde* is followed. When decreased, the descending branch *edfba* is traced. The function $P(\alpha,\beta)$ is named the Preisach function. It is assumed $\alpha \geq \beta$, which is quite natural in the physical point of view. Thus, the integration in (6) is performed over the right triangle in (α,β) plane, with the line $\alpha = \beta$ being the hypotenuse and point $(\alpha_0, \beta_0 = -\alpha_0)$ being the triangular vertex. The value of $\alpha_0 > 0$ is defined by the largest extremum value of the input function $u(t)$.

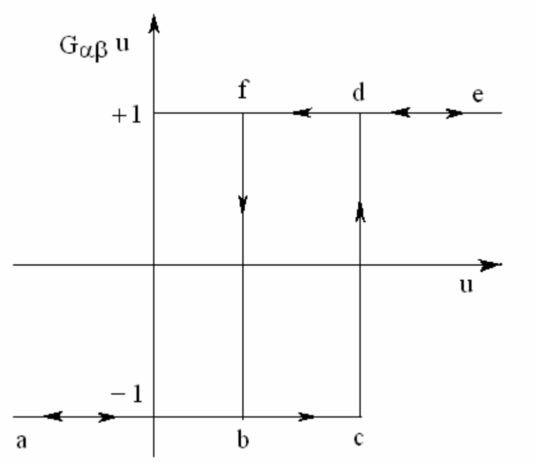

Fig. 3 – Elementary hysteretic operator.

There is a one-to-one correspondence between $G_{\alpha\beta}$ operators and points (α,β) of the triangle. The triangle (Fig. 4) is called a limiting triangle support for the Preisach function, since the Preisach function $P(\alpha,\beta)$ is assumed to vanish outside the triangle. The interface between two parts of the triangle is a staircase line $L(t)$ whose vertices have (α,β) coordinates that are local input maxima and minima at previous instants of time.

If the input is increasing, the line of $L(t)$ is horizontal; if it is decreasing, it is vertical. At any instant of time the triangle is subdivided into two sets: a positive $A^+(t)$ consisting of points (α,β) for which $G_{\alpha\beta}u(t)=1$, and a negative set $A^-(t)$ consisting of points (α,β) for which $G_{\alpha\beta}u(t)=-1$, separated by $L(t)$. Thus, equation (6) can be rewritten as

$$f(t)=\iint_{A^+(t)} P(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta - \iint_{A^-(t)} P(\alpha,\beta)\,\mathrm{d}\alpha\mathrm{d}\beta . \tag{7}$$

The model has the following property: each local input maximum wipes out the vertices of $L(t)$ whose α coordinate are below this maximum, and each local minimum wipes out the vertices whose β coordinates are above this minimum. In other words, the Preisach model stores the alternating series of dominant input extrema, while the other extrema are wiped out (Fig. 5). The wiping out of vertices is equivalent to the erasing of the history associated with these vertices. The major hysteretic curve must be defined, at a point h, by the integral (7). Secondary curves are defined by both the primary curve from which it departs, and the point at which it departs from its parent curve, *i.e.* by two values h_1 and h_2.

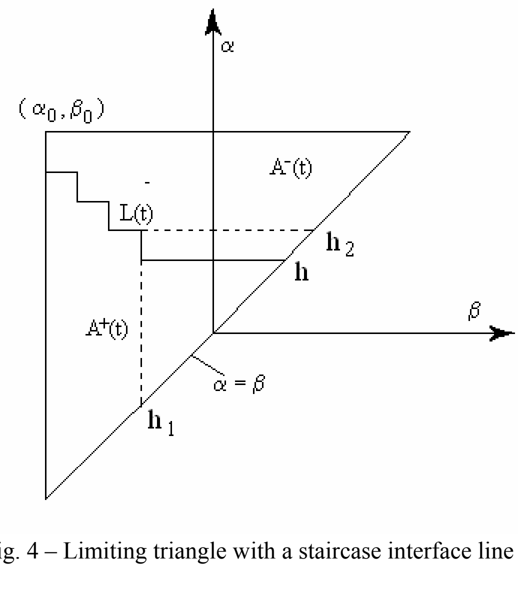

Fig. 4 – Limiting triangle with a staircase interface line $L(t)$.

All hysteretic loops corresponding to the same extremum values of input are congruent (Fig. 6). Such minor loops, obtained by reversal from ascending or descending branch of the major loops, can only be shifted relatively to each other along the output axes.

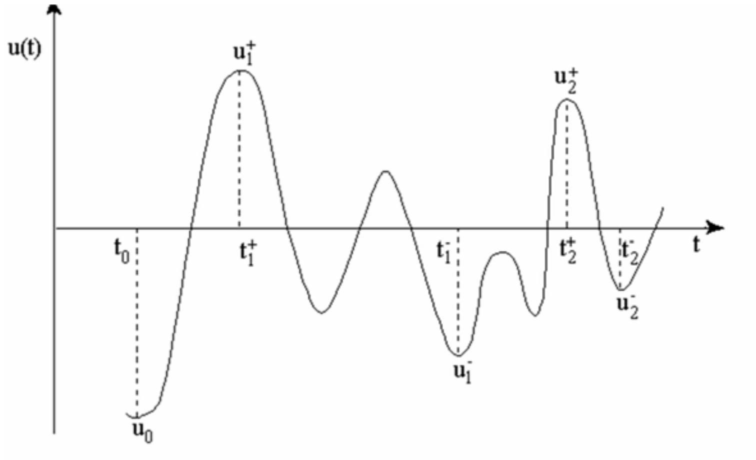

Fig. 5 – Preisach model stores only alternating series of dominant input extrema.

The Preisach function $P(\alpha,\beta)$ can be determined as follows. Starting from the state of negative saturation, let the input be increased to some value α. The output follows the ascending branch of the major loop, and at input $u = \alpha$ has the value f_α. If the input is subsequently decreased to some value β, the output follows the corresponding reversal (transition) curve, as in Fig. 7.

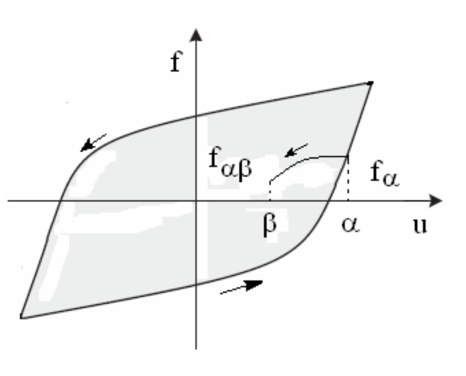

Fig. 6 – Congruency property of the Preisach model.

Denoting the output value at $u = \beta$ by $f_{\alpha\beta}$, then from the limiting triangle it follows

$$F(\alpha,\beta)=f_{\alpha\beta}-f_{\alpha}=-2\int_{\beta}^{\alpha}\left(\int_{\beta}^{\alpha}P(\alpha',\beta')\mathrm{d}\alpha'\right)\mathrm{d}\beta', \tag{8}$$

where by differentiation with respect to β and α, respectively, we have

$$P(-\alpha,-\beta)=P(\alpha,\beta),\quad P(\alpha,\beta)=-\frac{1}{2}\frac{\partial^2 F(\alpha,\beta)}{\partial\alpha\partial\beta}. \tag{9}$$

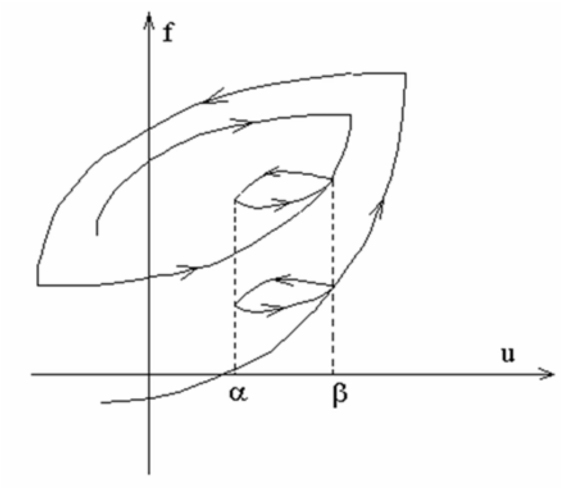

Fig. 7 – First order transition curve obtained by input reversal from $u=\alpha$ to $u=\beta$.

3. **RESULTS OF FITTING**

To imitate the experimental data, the Bouc-Wen parameters $\theta=[c,k,\beta,\gamma,\kappa]$ are chosen to be

$$c=0.07\ \text{kNs/m},\ \ k=25\ \text{kN/m},\ \ \beta=2,\ \ \gamma=1,\ \ \kappa=1.5.$$

The parameter n is kept constant ($n=2$). The mass of the system is $M=120$ kg. The simulated experimental data are obtained by randomly multiplying by $\varepsilon=\{10^{-3},10^{-2}\}$ the results obtained by numerically solving of the equations (1), (2) and (4). The results are shown in Fig. 8. We see that the data exhibits multiple reversal points.

A random short input signal is chosen to be applied to the system (Fig. 9). The problem consists in fitting of the model parameters $\theta=[c,k,\beta,\gamma,\kappa]$ by using the Preisach model.

The central result of the Preisach modeling is presented in Fig. 10. This is a specific response to short input signals. In Fig. 10 we see that the displacement drift, the force relaxation and non-closure of hysteretic loops are present. We remember that the displacement drift appears when cycled between two unequal forces, the force relaxation appears when cycled between two unequal displacements (Fig. 11). We must specify that such responses exhibit multiple reversals of small amplitude and are experimentally put into evidence [9, 21].

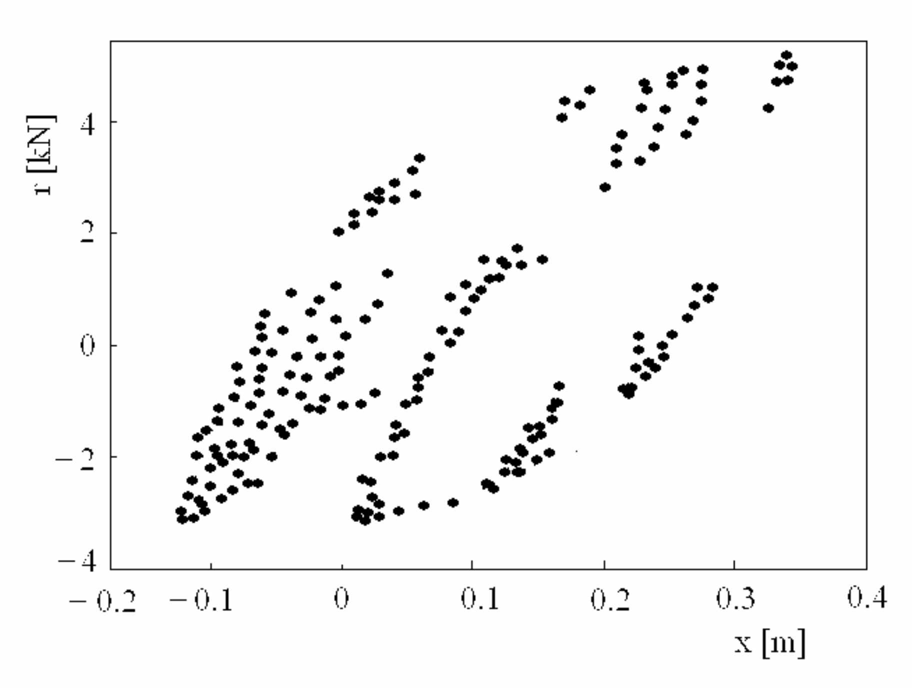

Fig. 8 – The imitated experimental data.

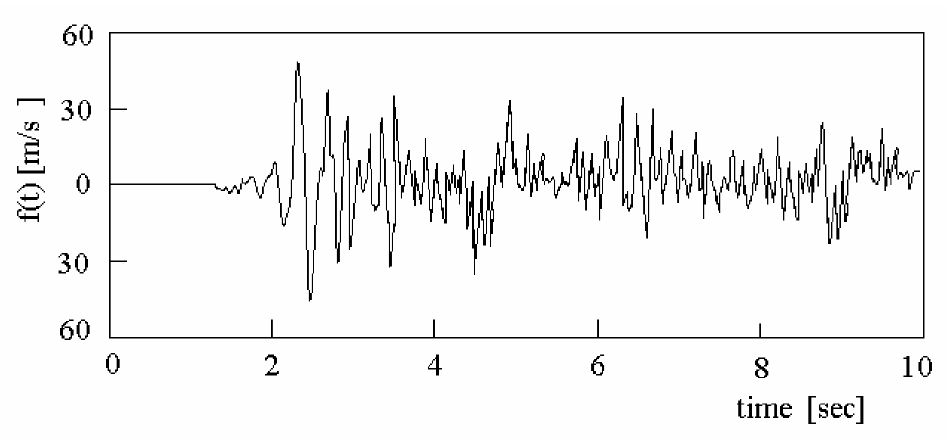

Fig. 9 – The random input signals.

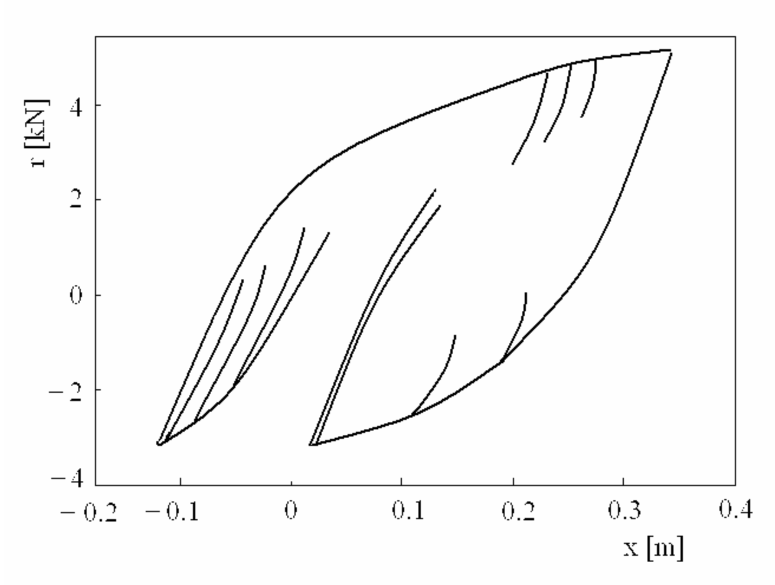

Fig. 10 – The results obtained by the Preisach model.

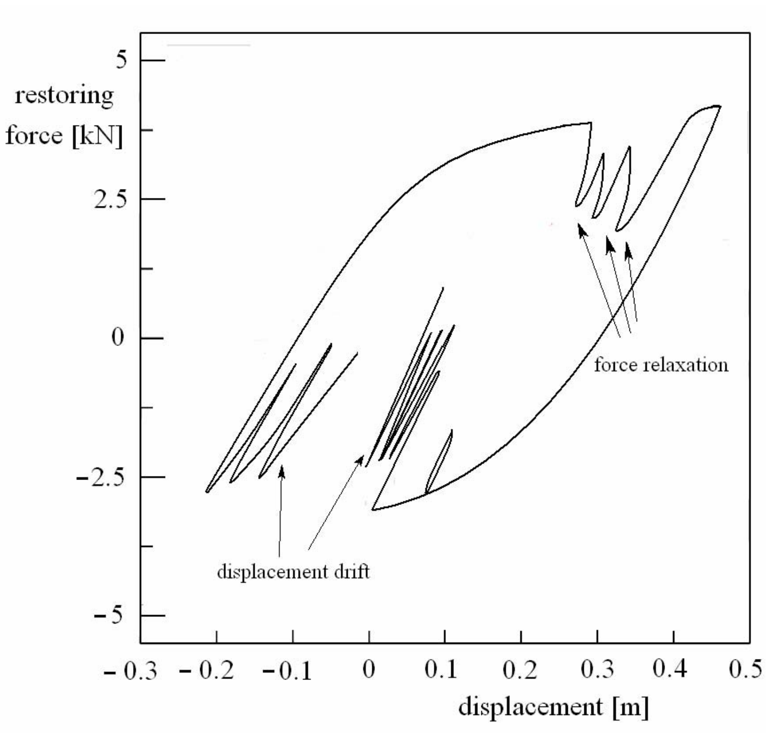

Fig. 11 – The displacement drift and force relaxation.

4. **CONCLUSIONS**

In this paper, a Preisach model is proposed to describe the hysteretic behavior of a system with one degree of freedom subjected to short input signals. This system exhibits displacement drift, force relaxation and non-closure of loops. The experimental data are imitated by using a Bouc-Wen model. Results of fitting the Bouc-Wen model are presented and discussed. Shown hysteretic properties assess the wide potentialities of the proposed model. The principal properties of this model are:

1. Each local maximum wipes out the vertices whose α coordinates are below this maximum and each local minimum wipes out the vertices whose β coordinates are above the minimum;
2. All hysteretic loops corresponding to the same extremum values of input are congruent.

These two properties constitute necessary and sufficient conditions for a hysteretic nonlinearity to be depicted by the Preisach model. The main advantage of the model is associated with its simple working, allowing describing of the hysteretic nonlinearities such as the displacement drift, the force relaxation and non-closure of hysteretic loops. The singularity which creates non-analyticity in the constitutive behavior at a particular reversal point related to the history of the system is a hallmark of many engineering systems.

Acknowledgements. The authors acknowledge the support from the National Authority for Scientific Research (ANCS, UEFISCSU), through PN-II research projects, code ID_247/2007 and ID_112/2007.

Received on August 9, 2010

REFERENCES

1. KATOK, A., HASSELBLATT, B., *Introduction to the modern theory of dynamical systems*, in: *Encyclopedia of Mathematics and its Applications* – Vol. 54 (edited by G.C. Rota), Cambridge Univ. Press, 1999.
2. SIRETEANU, T., GIUCLEA, M., MITU, A.M., *Identification of an Extended Bouc-Wen Model with Application to Seismic Protection through Hysteretic Devices*, Computational Mechanics, **45**, *5*, pp. 431-441, 2009.
3. MIELKE, A., ROUBICEK, T., *A Rate-Independent Model for Inelastic Behavior of Shape-Memory Alloys*, Multiscale Model. Simul., **1**, *4*, pp. 571-597, 2003.
4. KATZGRABER, H.G., PÁZMÁNDI, F., PIKE, C.R., LIU, K., SCALETTAR, R.T., VEROSUB, K.L., ZIMÁNYI, G.T., *Reversal-Field Memory in the Hysteresis of Spin Glasses*, Physical Review Letters, **89**, *25*, 2002.
5. OH, J.H., PADTHE, A.K., BERNSTEIN, D.S., RIZOS, D.D., FASSOIS, S.D., *Duhem Models for Hysteresis in Sliding and pre sliding Friction*, Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005, Seville, Spain, Dec. 12-15, 2005.

6. KOROBOV, A.I., EKONOMOV, A.N., *Singularities of nonlinear acoustic properties of copper wires in the field of plastic strains at a unloading-load*, XI Session of the Russian Acoustical Society, Moscow, Nov. 19-23, 2001, pp. 109-112.
7. CUI, T., LIAO, W., YU, D., *Topological Approach for Analyzing and Modeling the Aerodynamic Hysteresis of an Airfoil*, CMES: Computer Modeling in Engineering & Science, **45**, *3*, pp. 273-293, 2009.
8. DELSANTO, P.P., MUNTEANU, L., CHIROIU, V., Private communication, Politecnico di Torino, 2009.
9. CHARALAMPAKIS, A.E., KOUMOUSIS, V.K., *A Bouc-Wen model compatible with plasticity postulates*, Journal of Sound and Vibration, **322**, pp. 954-968, 2009.
10. PREISACH, F., *Uber die magnetische Nachwirkung*. Z. Physics, **94**, pp. 277-302, 1935.
11. MAYERGOYZ I.D., *Mathematical models of hysteresis,* Springer-Verlag, New-York, 1991.
12. KRASNOSELSKY, M., POKROVSKY, A., *Systems with hysteresis* (in Russian), Nauka, Moskva, 1983.
13. GUYER, R., MCCALL, K., BOITNOTT, G., HILBERT, L., PLONA, T., *Quantitative implementation of Preisach-Mayergoys space to find static and dynamic elastic moduli in rock*, J. of Geophysical Research, **102**, *B3*, p. 5281, 1997.
14. BADEA, T., CHIROIU, V., MUNTEANU, L., DONESCU, St., *A Preisach model of hysteretic behaviour of nonlinear mesoscopic elastic materials*, Rev. Roum. Sci. Techn. – Méc. Appl., **47**, *1-6*, pp. 83-96, 2002.
15. CHIROIU, V., BADEA, T., SCALERANDI, M., *A Preisach model of hysteresis for nonlinear mesoscopic materials*, Analele Ştiinţifice ale Universităţii "Ovidius" Constanţa, Seria Matematică, pp. 7-12, 2001.
16. MUNTEANU, L., DELSANTO, P.P., CHIROIU, C., DONESCU, St., *On the behaviour of nonlinear mesoscopic materials,* Analele Ştiinţifice ale Universităţii "Ovidius" Constanţa, Seria Matematică, pp. 41-46, 2001.
17. CHIROIU, V., DELSANTO, P.P., MIGNOGNA, R.B., MUNTEANU, L., SCALERANDI, M., *Modelling interfaces: analytical and numerical techniques*, Proc. of the 14th World Conference on Non-Destructive Testing, New-Delhi, India, pp. 2157-2160, 1996.
18. BADEA, T., NICOLESCU, C.M., *A Preisach model of hysteretic behavior of nonlinear mesoscopic elastic materials* (Ch. 1), in: *Topics in Applied Mech*anics – Vol. 1 (eds. V. Chiroiu and T. Sireteanu), Romanian Academy Publishing House, Bucharest, pp. 1-26, 2003.
19. GLIOZZI, A, MUNTEANU, L., SIRETEANU, T., CHIROIU, V., *An identification problem from input-output data*, Rev. Roum. Sci. Techn. – Méc. Appl., **55**, *3*, pp. 209–221, 2010.
20. CHANG, C.C., SHI, Y., *Identification of Time-varying Hysteretic Structures using Wavelet Multiresolution Analysis*, International Journal of Non-Linear Mechanics, **45**, pp. 21-34, 2010.
21. Pacific Earthquake Engineering Research Centre (PEER), *Strong ground motion database*, 2006.