A SPECIAL CLASS OF DRIP MEDIA WITH HYSTERESIS

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Abstract. Since the interaction phenomenon for waves of arbitrary shape and amplitude is a property of the transmitting medium rather than of the particular wave profiles, Seymour and Varley named as DRIP the media that do not remember the interaction process. In such media the profile of the interacting waves is affected by the interactions with hysteresis. The waves exhibit the solitonic features, but in contrast to solitons, the waves distort as they propagate by an amount that is altered by the interaction. The hysteretic effect is to alter the arrival time of their fronts at any point.

Key words: hysteresis operator, DRIP media, interacting waves, Preisach model, soliton theory.

1. INTRODUCTION

Despite the rapid development in the soliton theory, there are still unanswered questions at the elementary level. The solitonic equations with hysteresis require a clarification of the erratic and unpredictable properties of dynamical systems. The soliton solution show a remarkable survivability under conditions where one might normally expect such a feature to be destroyed [1]. In spite of this property, the nonlinear interfaces between different media substantially affect the solitons propagation. Fermi, Pasta and Ulam studied in 1955 [2, 3] the oscillations of a heterogeneous string which is governed by nonlinear wave equations of the form [1]:

$$y_{tt} = A^2(y_x, y_t) y_{xx} , \tag{1}$$

$$\frac{\mathrm{d}A}{\mathrm{d}y_x} = A^{3/2}(\mu + \nu A) , \tag{2}$$

where $y(x,t)$ is the physical displacement, $A(y_x, y_t)$ a positive function representing the local speed of propagation and μ, ν the material constants; x ranges from $-\infty$ to $+\infty$. In particular, when $A = A(x)$, the equation (1) is applied

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to all physical systems essentially involving only one space-dimension and onetime-dimension. For the transverse vibrations of a string, we have $A^2 = T/m$, where T is the constant tension and m the mass per unit length function of x; for the compressional vibrations of an isotropic laminated elastic solid in which the density and elastic constants are functions of x only, $A^2 = (\lambda + 2\mu)/\rho$, and for the transverse vibrations of a laminated solid, $A^2 = \mu/\rho$.

In 1982 Seymour and Varley [4] studied the equation (1) and have shown that, for certain functions $A(y_x)$ that satisfies (2), the solutions of (1) are simple waves whose profiles can be specified arbitrarily, that have the properties: they interact between themselves like solitons, being unaffected by the interaction, but in contrast to solitons, distort as they propagate by an amount that is not affected by the interaction. Since the interaction phenomenon for waves of arbitrary shape and amplitude is a property of the transmitting medium rather than of the particular wave profiles, Seymour and Varley named these media DRIP media (media that do not remember the interaction process). In such media the profile of the interacting waves is affected by the interactions with hysteresis.

The hysteretic scenario of reflections and transmissions represents a new class of nonlinear systems where the hysteresis is associated with the chaos theory [5,6]. The existence of an infinite number of conservation laws is an important link to the symmetric groups of differential equations consisted of those transformations of the variables which leave the system invariant.

Hysteresis is a nonlinear phenomenon that appears in various dynamical systems including piezoceramic and magnetostrictive actuators, ferromagnetic materials, electronic relay circuits, shape memory alloys, and so on. The non-differentiable and often unknown hysteresis properties of actuators cause inaccuracies and oscillations in the system responses that may lead to instability of the closed loop system.

The concept of *hysteresis operator*, acting in spaces of time dependent functions was developed in 1970s by Krasnoselskii and Pokrovskii [7]. In connection with nonlinear PDEs with hysteresis, the nonlinear semigroup theory represents a widely used tool for solving nonlinear PDEs with hysteresis [8–11].

Relevant results concerning the hysteresis operators analyzed in connection with the PDEs with hysteresis and the nonlinear semigroup theory results from a nonlinear semigroup theory are found in [12–17].

In this paper, the hysteretic interaction phenomenon for waves of arbitrary shape and amplitude in DRIP media is investigated by means of a Preisach model.

2. DISCONTINUOUS HYSTERESIS OPERATORS

The hysteretic operators may be discontinuous with loops which may exhibit singularities such as the displacement drift, force relaxation, non-closure and other [18–21]. To simplify the analysis, let us consider a system whose state is characterized by two scalar variables, u and w, confined to a set $L \subset R^2$. Assume that $\forall t \in [0,T]$, the function $w(t)$ depends on the previous evolution of u (memory effect) and on the initial state w_0, as follows:

$$w(t) = A(u, w_0)(t)\,, \quad \forall t \in [0, T], \tag{3}$$

where $A(u, w_0)$ is a memory operator defined in a Banach space of time-dependent functions. If $(u(0), w_0) \in L$ then $A(u, w_0)(0) = w_0$. The memory operator is causal, $\forall (u_1, w_0), (u_2, w_0) \in \mathrm{D}(A)$ with $u_1 = u_2$ in [0, T], then $A(u_1, w_0)(t) = A(u_2, w_0)(t)$.

Generally, most hysteresis phenomena exhibit rate-dependent memory and as consequence, the rate-dependent effects are superposed to hysteresis. Equation (3) can exhibit singularities in a finite time. The generalized concept of weak solutions with allowed discontinuities must be defined in order to be able to obtain the solutions of a given PDEs [17].

A discontinuous hysteresis operator is the relay operator defined in the sense of Visitin [14]:

Definition of the Relay Operator (Visintin): Let us denote by $C_r^0([0, T])$ the space of continuous functions on the right in $[0, T]$. For any pair $\rho := (\rho_1, \rho_2) \in R^2$ the (*delayed*) *relay operator* is defined as

$$h_\rho : C^0([0, T]) \times \{-1, 1\} \to BV(0,T) \cap C_r^0([0, T])\,. \tag{4}$$

The operator h_ρ is not closed. For any $u \in C^0([0, T])$ and $\forall \xi \in \{-1, 1\}$, we set

$$X_t := \{\tau \in [0, t] : u(\tau) = \rho_1 \text{ or } \rho_2\}\,,$$

for any $t \in [0, T]$, and define the function $w = h_\rho(u, \xi) : [0, T] \to \{-1, 1\}$ as follows:

$$w(0) := \begin{cases} -1 & \text{if } u(0) \le \rho_1, \\ \xi & \text{if } \rho_1 < u(0) \le \rho_2, \\ 1 & \text{if } u(0) \ge \rho_2, \end{cases} \tag{5}$$

$$w(t) := \begin{cases} -1 \quad \text{if } X_t \neq \varnothing \text{ and } u(\max X_t) = \rho_1 \quad \forall t \in [0, T], \\ \quad 1 \quad \text{if } X_t \neq \varnothing \text{ and } u(\max X_t) = \rho_2, \end{cases} \tag{6}$$

with $w(t) = w(0)$ if $X_t = \varnothing$. The conditions (3) and (4) assure the uniqueness of w in [0, *T*]. The jumps of w up to 1 and down to -1 are illustrated in Fig. 1.

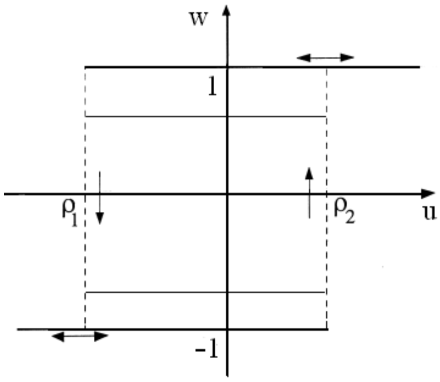

Fig. 1 – Relay operator.

In order to associate the relay operator to PDEs, it is of interest to deal with the closure of (4). The relay operator has to be extended by alloying w to have intermediate values between -1 and 1.

Therefore, for any $u \in C^0([T])$ and $\xi \in [-1, 1]$, a set $w \in k_\rho(u, \xi)$ is introduced if and only if w is measurable in $[0, T]$ and

$$w(0) = \begin{cases} -1 & \text{if } u(0) < \rho_1, \\ \{-1, \xi] & \text{if } u(0) = \rho_1, \\ \xi \ \text{if } \rho_1 < u(0) < \rho_2, \\ [\xi, 1] & \text{if } u(0) = \rho_2, \\ 1 & \text{if } u(0) > \rho_2, \end{cases} \tag{7}$$

$$w(t) := \begin{cases} -1 \ \text{if } u(t) < \rho_1, \\ [-1, 1] \quad \text{if } \rho_1 \leq u(t) \leq \rho_2, \\ 1 \quad \text{if } u(0) > \rho_2, \end{cases} \tag{8}$$

with additional remarks: (*i*) if $u(t) \neq \rho_1, \rho_2$ then w is constant in a neighbourhood of t; (*ii*) if $u(t) = \rho_1$ then w is nonincreasing in a neighbourhood of t; and if $u(t) = \rho_2$ then w is nondecreasing in a neighbourhood of t.

So, the function $w \in BV(0, T)$ exists for

$$k_\rho : C^0([0, T]) \times \{-1, 1\} \to P(BV(0, T)) . \tag{9}$$

3. DRIP MEDIA

Let Ω be a regular domain of R and set $Q := \Omega \times [0, T]$. We assume to have a measurable field $\rho : \Omega \to (P, \mu)$, where μ is the Preisach measure. The pair of admissible thresholds of the relay operator forms the Preisach halfsplane

$$P := \left\{ \rho = (\rho_1, \rho_2) \in R^2 : \rho_1 < \rho_2 \right\} .$$

The Preisach model is working with linear combinations of delayed relays with different thresholds and the same input. The combination of 4 relay operators is shown in Fig. 2. The equation (1) can be written as a system of first order equations with hysteresis:

$$(u + w)_t = A^2(e,u)e_x , \quad u_x = e_t \text{ in } Q ,$$

$$w(0) = w_0 , \quad u(0) = u_0 , \tag{10}$$

or under the form

$$(u + w)_t + Au_x = A(e_t + Ae_x) , \quad (u + w)_t - Au_x = -A(e_t - Ae_x) \text{ in } Q ,$$

$$w(0) = w_0 , \quad u(0) = u_0 , \tag{11}$$

where $u = y_t$ and $e = y_x$ and $u_0, w_0 \in L^2(\Omega)$, $|w_0| \le 1$, $w_0 = -1$ if $u_0 < \rho_1$, $w_0 = 1$ if $u_0 > \rho_2$ in Ω.

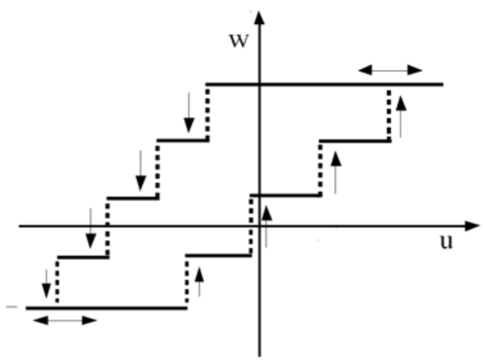

Fig. 2 – The Preisach model.

The operator (9) can be represented in a weak formulation. The confinement condition to the pair (u, w) is written as (Fig. 3):

$$|w| \leq 1, \ \ (w-1)(u-\rho_2) \geq 0, \ \ (w+1)(u-\rho_1) \geq 0 \ \ \text{in } [0, T]. \tag{12}$$

The dissipation condition is added also [14].

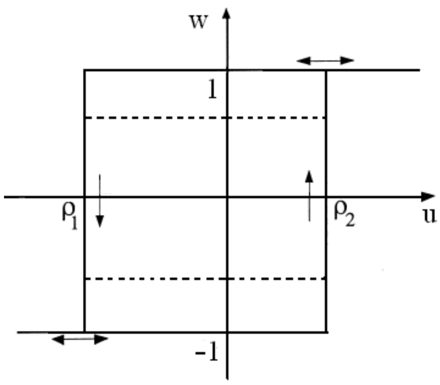

Fig. 3 – The weak formulation of the relay operator.

To see the effect of hysteresis on the wave propagation in DRIP media, we consider a simple wave profile $f(x) = \text{sech}\, x$. This is the case of interaction of two pulses having a soliton profile, travelling in a DRIP medium. The calculations are performed numerically.

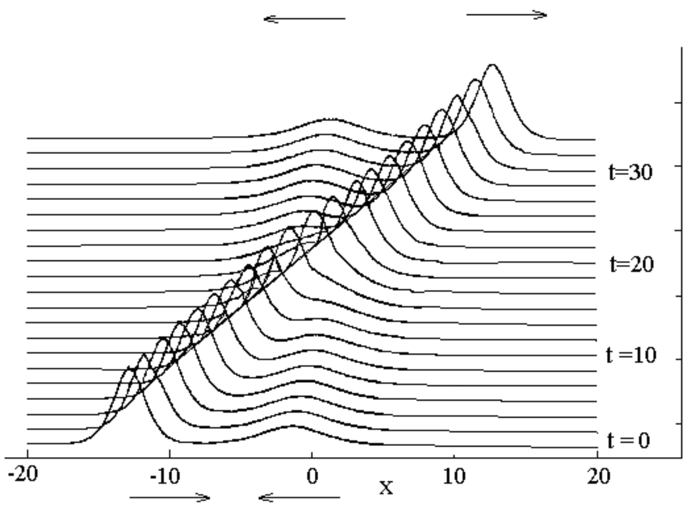

Fig. 4 – Profiles of waves without hysteresis against x for several t [1].

Fig. 4 illustrates the case of interactions without hysteresis [4]. From this figure we see that, in contrast to known theory of solitons interaction, these pulses travel in opposite directions, interact and emerge unaffected by the interaction. In the interaction region no coupling between waves is visible. This suggests that the waves may be regarded individually. Speaking from a physical viewpoint, this interaction requires that the energy of each field is carried individually without any transfer of energy between fields.

The central result of the Preisach model is presented in Fig. 5. In this figure, the interaction region exhibits the coupling between waves. Therefore, the waves may not be regarded individually. The coupling of waves can be explained by the energy transfer between waves. This property may be of the transmitting medium rather than of the particular wave profiles.

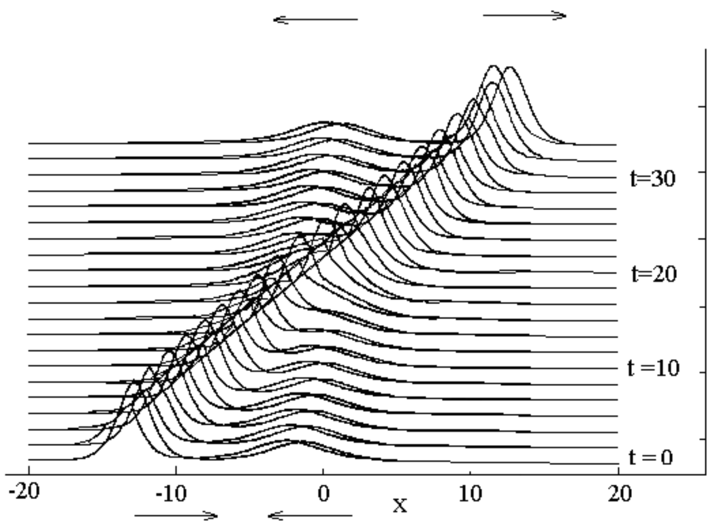

Fig. 5 – Profiles of waves with hysteresis against x for several t .

4. CONCLUSIONS

In this paper, a Preisach model is proposed to describe the hysteretic interaction phenomenon for waves of arbitrary shape and amplitude in DRIP media. In such media the profile of the interacting waves is affected by hysteresis. The waves admit the solitonic features, but in contrast to solitons, they distort as they propagate by an amount that is altered by the interaction. The hysteretic effect of the interaction is to alter the arrival time of their fronts at any point.

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