ON THE ACOUSTICS OF THE STICK-SLIP PHENOMENON

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Abstract. In this paper, the acoustics and friction of the stick-slip phenomenon is discussed. The interaction friction-vibration gives rise to oscillations within solids which frequently lead to radiation of sound to the surrounding media. The analysis of a friction contact model between two vibrating grains provides an example of the *mode lock-in* and the sound.

Key words: acoustics, friction sounds, stick-slip phenomenon, contact interface model, hysterezis, mode lock-in.

1. THE FRICTION SOUNDS

A relevant paper in the frame of the acoustics of friction is due to Akay (2002) [1]. Akay presents a remarkable overview of the acoustics of friction by covering friction sounds, friction-induced vibrations and waves in solids, and descriptions of other frictional phenomena related to acoustics. Starting from [1], in this paper we analyze several aspects of the acoustics of friction inside the stick-slip phenomenon.

Examples of sounds that result from friction-excited vibrations and waves appear frequently in our life. Starting by the violin music, the string instruments, the insect sounds and ending by the brake noise in automobiles and the squeal of the hips prosthesis, the acoustics of friction extends beyond noise and music.

As an example, the extracting of a nail from a piece of hard wood can produce friction forces and sounds. The nail diameter determines the pressure, and thus the friction force on the nail surface. The speed of the nail extraction affects the response because of the temperature.

Another illustrative example of a friction system in which friction excitation takes place away from the radiation source is the *cuica*, the Brazilian friction drum. The *cuica* consists of a cylindrical metal shell covered as in a dum at one end with a membrane. Inside the cylinder, a bamboo stick suspends from the center of the membrane where one end of the stick is tied with a knot. The player strokes the stick with a piece of moist cloth, thereby exciting the membrane at its center with the resulting stick-slip motion.

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A historical example demonstrated by Tyndall suggests that rubbing a glass tube along its length can break off rings at its free end [2]. A bowed string presents a useful example because of its simple configuration. A dimensionless coupling constant was shown to characterize the classical stick-slip phenomenon that consists of a mass, *m*, restrained by a spring of constant k on a conveyor belt that moves at a speed V_0, *i.e.* $\Gamma = F_0\omega_0 / kV$, where F_0 is the maximum static friction force and $\omega_0 = \sqrt{k/m}$ [3]. The larger the value of Γ, the longer the sticking period of the mass and the longer the period of its oscillation. The string and other friction-excited systems should be considered not as having a single degree of freedom, but as continuum systems with many degrees of freedom [4]. An example of how sound quality depends on the interaction of friction and vibration appears in the phenomenon called *double slip*. *Double slip* occurs when the actual bowing force falls below the minimum bowing force necessary for *playability.* Double slip manifests itself by splitting the fundamental frequency and producing the *wolf tone* [5–7]. One animal that produces sound using stick-slip friction is the spiny lobster which rubs its antennae over smooth surfaces on its head [8]. Other examples which produce sound using stick-slip friction can be found in [9-12]. In this paper a friction contact model between two vibrating grains is considered starting from the results provided in [13-15]. A peculiar aspect of the hysteresis loop is obtained, due to the instabilities which lead to the *mode lock-in*, where the system responds at one of its fundamental frequencies and its harmonics.

2. THE STICK-SLIP PHENOMENON

This model represents a contact interface between two grains with an initial gap n_0, modeled as a massless elastic element characterized by two linear springs of lengths l_u and respectively l_v, and moduli k_u and respectively k_v, and a friction contact point which obeys the Coulomb friction law with the friction coefficient μ (Fig. 1). Here u is the tangential relative displacement, v is the normal relative displacement and w is the tangential displacement of the contact point relative to grain 2 considered being the ground. When the grains are in contact it is possible to appear a tangential stick-slip motion and when the normal relative motion v becomes large we can have an intermittent separation of grains. The variable normal load n and the induced friction f are given by:

$$n = \begin{cases} n_0 + k_v v, & \text{when } v \geq -n_0 / k_v \\ 0, & \text{when } v < -n_0 / k_v \end{cases}, \tag{1}$$

$$f = k_u(u - w). \tag{2}$$

When the vibratory displacements are small, the contact interface sticks and the friction force is proportional to u with reference to w, which is zero. The slip load always limits its magnitude $\pm\mu n$. When the friction force exceeds the positive value μn, the contact interface starts to slip towards the positive u direction. The friction force remains equal to the varying slip load until the contact interface sticks again. The friction coefficient μ is a function on time $\mu(t)$ if the load $P(t)$ and the friction force $f(t)$ at a contact vary with time:

$$\mu(t) = \frac{f(t)}{P(t)}. \tag{3}$$

The average friction can be interpreted as the time average of $\mu(t)$, denoted by $<\mu(t)>$, or as an average friction force divided by the average normal load, so that [16]:

$$\mu_{av} = \frac{<f(t)>}{<P(t)>}. \tag{4}$$

If the normal load remains constant or the friction coefficient does not change with time, the two interpretations are equivalent. Otherwise they are not.

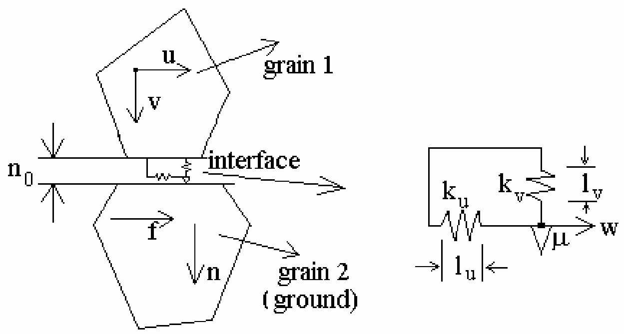

Fig. 1 – The contact interface model.

The stick-slip conditions are characterized by the friction force f and the slip velocity $\dot{w}$, which must be formulated in terms of the input relative displacements. We have from (1)–(4):

$$f = \begin{cases} k_u(u-u_0)+f_0 & \text{for} \quad \dot{w}=0 \quad \text{(stick)}, \\ \mu n = \mu n_0 + \mu k_v v & \text{for} \quad \dot{w}>0 \quad \text{(positive slip)}, \\ -\mu n = -\mu n_0 - \mu k_v v & \text{for} \quad \dot{w}>0 \quad \text{(negative slip)}, \end{cases} \tag{5}$$

where u_0 and f_0 are the initial values of u and f at the beginning of the stick state $t = 0$: $u(0) = u_0$, $f(0) = f_0$. The synthesis of the results is presented in Table 1 [14].

We refer to the rocks, which are fractured rather than intact materials. Rocks, like many ceramics, consist from grains imbedded in a contiguous matrix, or in direct contact with each other. The matrix contains a fine network of micro fractures: cracks, joints, voids of typical size $\approx 1\mu\mathrm{m}$. The interfaces, or the bond systems, are located between the grains and consist of a fabric of defects (cracks, voids) that participate in the elastic response of the material. The strong nonlinearity is associated with the presence of hard and soft phases, where the soft phase occupies a much smaller volume but it is subject to strong deformation and is the origin of the nonlinear response.

Table 1

The transition criteria

Transition	Criteria
stick to positive slip	$f - \mu n = 0$, $\dot{f} - \mu\dot{n} > 0$ or $k_u u - \mu k_v v + (f_0 - \mu n_0 - k_u u_0) = 0$, $k_u\dot{u} - \mu k_v\dot{v} > 0$.
stick to negative slip	$f + \mu n = 0$, $\dot{f} + \mu\dot{n} > 0$ or $k_u u + \mu k_v v + (f_0 + \mu n_0 - k_u u_0) = 0$, $k_u\dot{u} + \mu k_v\dot{v} < 0$.
stick to separation	$n = 0$, $\dot{n} < 0$ or $n_0 + k_v v = 0$, $\dot{v} < 0$.
positive slip to stick	$\dot{w} = 0$, $\ddot{w} < 0$ or $\dot{u} - \frac{\mu k_v}{k_u}\dot{v} = 0$, $\ddot{u} - \frac{\mu k_v}{k_u}\ddot{v} < 0$.
negative slip to stick	$\dot{w} = 0$, $\ddot{w} > 0$ or $\dot{u} + \frac{\mu k_v}{k_u}\dot{v} = 0$, $\ddot{u} + \frac{\mu k_v}{k_u}\ddot{v} > 0$.
positive slip to separation	$n = 0$, $\dot{n} < 0$ or $n_0 + k_v v = 0$, $\dot{v} < 0$.
negative slip to separation	$n = 0$, $\dot{n} > 0$ or $n_0 + k_v v = 0$, $\dot{v} > 0$.
separation (stuck interface) to stick	$-\mu\dot{n} < \dot{f} < \mu\dot{n}$, $\dot{w} = 0$ or $\dot{n} = k_v\dot{v}$, $\dot{f} = k_u(\dot{u} - \dot{w})$ $-\frac{\mu k_v}{k_u}\dot{v} < \dot{u} < \frac{\mu k_v}{k_u}\dot{v}$.

separation (stuck interface) to positive slip	$\dot{u} > \frac{\mu k_v}{k_u}\dot{v}$.
separation (stuck interface) to negative slip	$\dot{u} < -\frac{\mu k_v}{k_u}\dot{v}$.

The transition between slip and stick depends on the tangential relative displacement u and on the variable normal load, which may decrease to reduce the slip load so that the occurrence of the transition can be postponed to some instant after the reversion of u.

During the cycle of motion the contact normal load may vanish and cause the separation. In the displacement-force plane a hysteresis loop is appearing. Fig. 2 plots the force $f/(n_0\mu_{\min})$ against the displacement u/u_0. The solid, the dash-dotted and the dashed curves indicate the stick, slip and separation regions, respectively. A peculiar aspect of the loop is appearing by the presence of some *teeths allure* in the stick-slip zones.

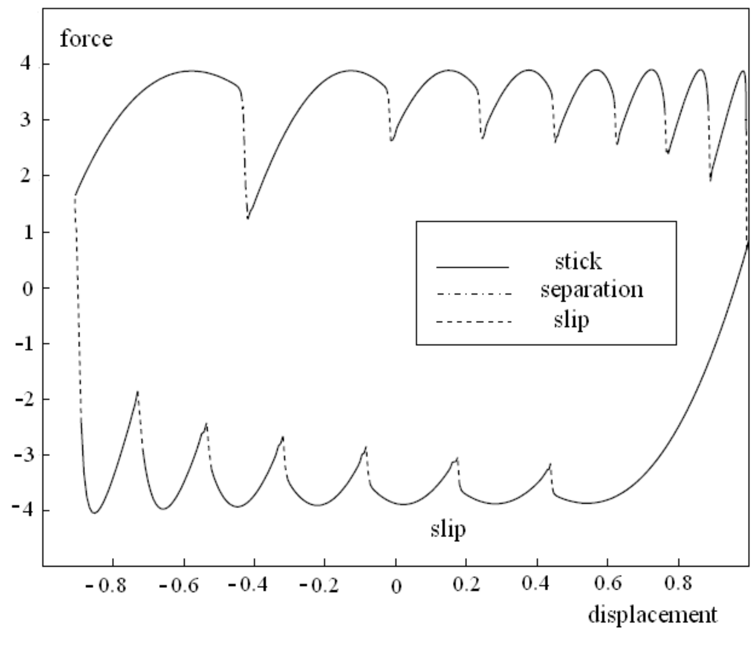

Fig. 2 – Hysteretic behavior of the model.

This peculiar aspect of the hysteresis loop is due to the contact conditions and the transition criteria. The instabilities develop and frequently lead to a condition called *mode lock-in*, where the system responds at one of its fundamental frequencies and its harmonics [1, 17]. Usually, in the mode lock-in, the spectra show one or more fundamental frequencies closely associated with natural frequencies of the system, and their harmonics [18, 19].

A known illustration of the mode lock-in results from applying a violin bow to the free edge of a cantilever beam or a plate [1]. The development of the mode lock-in depends on the normal load, sliding velocity, and the contact geometry. The illustration of the concept of the mode lock-in is given in Fig. 3, for the first mode of vibration. The system locks into this particular mode and oscillates in a stable manner.

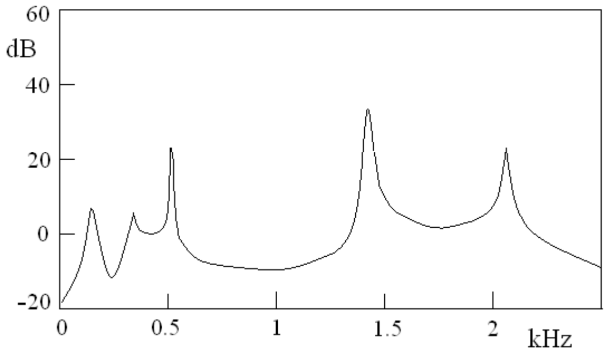

Fig. 3 – Frequency response in the mode lock-in in the first mode of vibration.

3. CONCLUSIONS

The acoustics and friction of the stick-slip phenomenon is discussed in this paper by considering of a friction contact model between two vibrating grains. The interaction friction-vibration gives rise to oscillations within the system which frequently lead to radiation of sound to the surrounding media.

The friction between the grains provides an example of the mode lock-in and the sound. The granular materials provide an effective mechanism of vibration damping by dissipating energy primarily through inelastic stick-slip-separation behavior and friction among the grains.

Acknowledgments. We gratefully acknowledge prof. Ioan Magheţi from "Politehnica" University of Bucharest for fruitful discussions on the acoustics of friction subject.

Received on February 2, 2011

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