ON THE VIBRATIONS GENERATED BY WOODEN CHURCH BELLS

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Abstract. In *Maramureş* County, Romania, there is Budeşti Josani wooden church inscribed on the World Heritage List. The major deterioration of its 18th and 19th paintins represent a big problem in restoration program. The vibrations generated by the bells produced cracks and detachments in the paintings layer. In order to calculate the oscillations of a bell, the bell and its clapper were simulated as a double pendulum.

Key words: wooden church, vibrations, bell, clapper, double pendulum.

1. **INTRODUCTION**

Budeşti Josani Church (Figs. 1 and 2), with Saint Nicolas titular saint, has been erected in 1643 and is representative to the characteristic Maramureş County wooden churches covered with double eaves, because it preserves the original shape, even their dimensions are larger than usual (18 m × 8 m). The edifice is composed by one narthex, a nave with a raised semi-cylindrical dome and a polygonal altar. The church is made of oak beams, mounted in Blockbau system. The steep bell-tower, pillared on four lateral beams, has a bell-room with an open gallery, four corner pinnacles and a slender roof.

The roofs covered with wood shingles were popular in that region of Romania in that period of time. The size and shape of the shingles as well as the detailing of the shingle roof differed according to regional craft practices. People within particular regions developed preferences for the local species of wood that most suited their purposes. The oak was frequently used in *Maramureş* County [1].

The narthex, the nave, the iconostasis, and a lot of icons of the church were painted by Alexandru Ponehalschi, in 1762, on textile pieces, applied on the walls covered with a preparation layer based on lime, and the altar was painted directly on the wooden walls by Ioan Opriş, in 1832.

The vibrations produced by the bells generate the deterioration of the murals, developing cracks and fractures, as well as the detachment of the painting layer (Fig. 3). The bells are considered to have an imperfect shape, because of the irregularities of their shape, the material's properties and the local defects during the casting process. The applications of ornaments and reliefs on the surface of the bell produce the asymmetry of the bell [2]. The forces induced by the swinging of

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the bells can interact dynamically with the base of the bells, according to their angular frequencies and to their imbalance [3].

Fig. 1 – Budeşti Josani Church, Maramureş County.

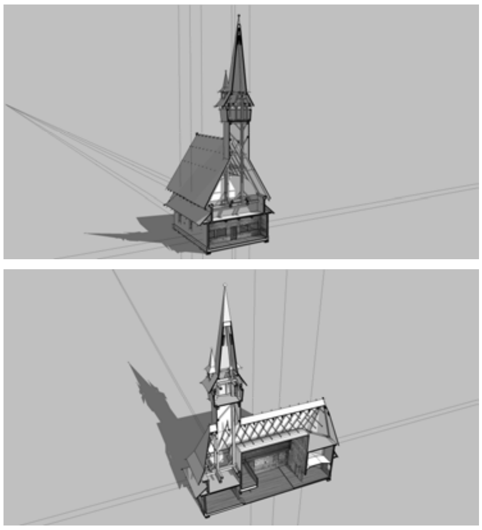

Fig. 2 – Budeşti Josani Church, Maramures County – perspective geometries realized by restorer Dinu Săvescu.

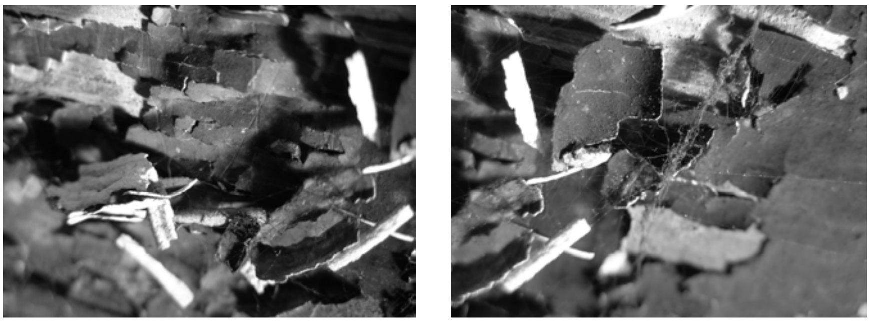

Fig. 3 – Detachment of the painting layer (photo: restorer Dinu Săvescu).

2. **FORMULATION OF THE PROBLEM**

The periodic forces induced by the bell's swinging, as well as the wind loadings determine the bell-tower to vibrate, mainly on the horizontal direction. If the endurance limit of the bell-tower's masonry and the church's nave are exceeded, it results cracks and fractures, which represent the major problem of this type of monuments. It is hard to determine which of these forces is more responsible with the initial micro-cracks, but all the forces mentioned above, especially the horizontal component of the bell's swinging force contribute to the developing of already existing cracks. The dynamic problem is complex and difficult to solve, because the dynamic behaviour of the whole structure must be considered, including the parameters of the different materials used and the soil characteristics [4, 5].

In the case of wooden churches, where the bell-tower is erected upper to the dome, the momentums of the tower are transmitted to the nave, producing not only the developing of the existing cracks, but fractures in the connection zone. When the driving force is close to the natural frequency of the tower there is produced the most important degradation of the structure.

Table 1

The inclination of the bell typical for different countries

System	Country	Inclination*
Central European	Germany Other European countries USA	54 – 80° 80 – 110° 50 – 160°
Spanish	All the countries	Complete circuit
English	All the countries	Complete circuit

* The inclination regarding to the vertical axis

Ivorra *et al*. [6] analyzed three bell-tower systems, typically for Central Europe (which is called in Italy *alla Romana*), for England and Spain. In the first system, the bell swings on both sides of the symmetry axis. In the English system, it accomplishes complete circles, in which the sense of rotation interchanges after a complete cycle. The English and the Spanish systems are in imbalance, and the bells are mounted in the interior of a tower [6, 7].

Table 1 indicates the inclination of bells for all the three systems; their values are important to find the horizontal and vertical forces generated by the balance and their effect over the entire structure [8]. We have to add that the Budeşti Josani church belongs to the Central European category.

Bell-towers are under axial charges bigger than the torsion and bending, because of the geometry, type of the material used and the fixing mode of the bell [4]. In this paper, the bell is modeled in all three systems, in the spirit of the Zlatescu paper [9] where the Lie transformation theory [10] is extended through the study of a double pendulum, subjected to the non-conservative periodic loads. Fig. 4 presents a section of a bell with clapper pendulant regarding to the vertical axis. The model is consisted from two straight rods $\mathrm{O_1O_2}$ and $\mathrm{O_2O_3}$ of masses M_1, M_2, lengths $2l_1$, $2l_2$, and mass centres $\mathrm{C_1, C_2}$. The rods are articulated in

O_2 and suspended in O_1, so that they can move in the vertical plane xO_1y without friction.

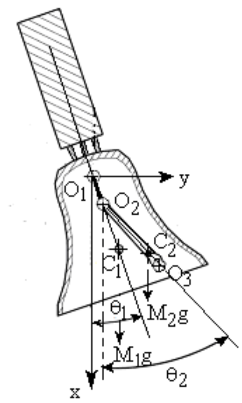

Fig. 4 – The simplified model of the bell.

Other notations from Fig. 4 are: $l = O_1O_2$, $l_1 = O_1C_1$, $l_2 = O_2C_2$. We note by θ_1 and θ_2 the displacement angles with respect to the vertical O_1x, I_1 the mass moment of inertia of O_1O_2 with respect to C_1, I_2 the mass moment of inertia of O_2O_3 with respect to C_2, and g the gravitational constant [9,13]. The forces acting upon the pendulum are, firstly, the weights of bars. The generalized forces are

$$G_1 = -M_1 l_1 g \sin\theta_1 - M_2 g l \sin\theta_1, \quad G_2 = -M_2 g l_2 \sin\theta_2. \tag{1}$$

Let us to consider the case of non-conservative force $A\cos\omega t$ acting into the point O_3. The motion equations are obtained from the Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot\theta_1}\right) - \frac{\partial T}{\partial \theta_1} = G_1, \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot\theta_2}\right) - \frac{\partial T}{\partial \theta_2} = G_2 + A\cos\omega t. \tag{2}$$

Introducing the notations

$$\begin{aligned}
\alpha &= \frac{M_2 l l_2}{I_1 + l_1^2 M_1 + l^2 M_2}, \\
\beta &= \frac{M_1 l_1 + M_2 l}{I_1 + l_1^2 M_1 + l^2 M_2} \frac{I_2 + l_2^2 M_2}{M_2 l_2}, \\
\gamma &= \frac{M_2 l l_2}{I_2 + l_2^2 M_2}, \quad \delta = \frac{A}{g M_2 l_2},
\end{aligned} \tag{3}$$

equations (2) become

$$\begin{cases} \ddot{\theta}_1 + \alpha[\ddot{\theta}_2 \cos(\theta_2 - \theta_1) - \dot{\theta}_2^2 \sin(\theta_2 - \theta_1)] + \beta \sin\theta_1 = 0, \\ \ddot{\theta}_2 + \gamma[\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + \dot{\theta}_1^2 \sin(\theta_2 - \theta_1)] + \sin\theta_2 = \delta \cos\omega t. \end{cases} \tag{4}$$

Without reducing the generality of the problem we consider $I_1 = M_1 \dfrac{l_1^2}{3}$, $I_2 = M_2 \dfrac{l_2^2}{3}$. By setting [13]

$$m = \frac{M_1}{M_2}, \quad r = \frac{l}{l_1}, \quad s = \frac{l_2}{l_1}, \tag{5}$$

it is simple to show that α, β, γ become

$$\alpha = \frac{3rs}{3r^2 + 4m}, \quad \beta = \frac{4s(r+m)}{3r^2 + 4m}, \quad \gamma = \frac{3r}{4s}. \tag{6}$$

Finally, we introduce the new variables

$$\begin{aligned} &\theta_1 = z_1, \quad \theta_2 = z_2, \quad \dot{\theta}_1 = z_3, \quad \dot{\theta}_2 = z_4, \\ &\cos\omega t = z_5, \quad -\omega \sin\omega t = z_6. \end{aligned} \tag{7}$$

For $t > 0$ we have the relation $\omega^2 z_5^2 + z_6^2 = \omega^2$. The motion of the double pendulum depends on the control parameters r, s, m, δ and ω. Equations (4) are rewritten in the state-space form as

$$\begin{aligned} &\dot{z}_1 = z_3, \\ &\dot{z}_2 = z_4, \\ &\dot{z}_3 = \Delta[-\beta \sin z_1 + \alpha \sin z_2 \cos(z_2 - z_1) + \alpha z_4^2 \sin(z_2 - z_1) + \\ &\qquad + \alpha\gamma z_3^2 \sin(z_2 - z_1)\cos(z_2 - z_1) - \alpha\delta z_5 \cos(z_2 - z_1)], \\ &\dot{z}_4 = \Delta[-\sin z_2 + \beta\gamma \sin z_1 \cos(z_2 - z_1) - \\ &\qquad - \gamma z_3^2 \sin(z_2 - z_1) - \alpha\gamma z_4^2 \sin(z_2 - z_1)\cos(z_2 - z_1) - \delta z_5], \\ &\dot{z}_5 = z_6, \\ &\dot{z}_6 = -\omega^2 z_5, \end{aligned} \tag{8}$$

where $\Delta = \dfrac{1}{1 - \alpha\gamma \cos^2(z_2 - z_1)}$ and $1 - \alpha\gamma\cos^2(z_2 - z_1) \neq 0$. The initial conditions are

$$z_1(0)=z_{10}\,,\quad z_2(0)=z_{20}\,,\quad z_3(0)=z_{30}\,,$$
$$z_4(0)=z_{40}\,,\quad z_5(0)=1\,,\quad z_6(0)=0\,. \tag{9}$$

After suitable simplifications, the system (8) and (9) can be rewritten under the form [13]

$$\dot z_n = A_{np}z_p + \frac{g_n(z)}{h_n(z)}\,,\quad n=1,2,...,6\,, \tag{10}$$

with

$$\begin{aligned} g_n(z,t) &= B_{np}z_p + C_{np}\sin z_p + D_{npq}\cos(z_p - z_q)\sin z_p + E_{npqr}z_p\,\cos(z_q - z_r) \\ &\quad + F_{npqr}z_p^2\sin(z_q - z_r) + G_{npqr}z_p^2\sin(z_q - z_r)\cos(z_q - z_r) - A_n(z) + \\ &\quad + L_{nnpqm}A_m(z)\cos(z_p - z_q)\,, \end{aligned} \tag{11}$$

$$h_n(z) = 1 - H_{npq}\cos^2(z_p - z_q)\,,$$

and $h_n \neq 0$. It is assumed to be valid the summation law with respect to repeated indices $(n,p,q,r=1,2,3,...,6)$. The constants are

$$\begin{aligned} &A_{13}=1,\quad A_{24}=1,\quad A_{56}=1,\quad A_{65}=-\omega^2,\quad B_{45}=1, \\ &C_{31}=-\beta,\quad C_{42}=-1,\quad D_{412}=\beta\gamma,\quad D_{321}=\alpha,\quad E_{3521}=-\alpha, \\ &F_{3421}=\alpha,\quad F_{4321}=-\gamma,\quad G_{3321}=\alpha\gamma,\quad G_{4421}=-\alpha\gamma, \\ &H_{321}=\alpha\gamma,\quad H_{421}=\alpha\gamma,\quad L_{3214}=\alpha,\quad L_{4214}=\gamma, \end{aligned} \tag{12}$$

and the rest are null. The initial conditions are

$$z_1(0)=z_{10}\,,\quad z_2(0)=z_{20}\,,\quad z_3(0)=z_{30}\,,$$
$$z_4(0)=z_{40}\,,\quad z_5(0)=1,\quad z_6(0)=0. \tag{13}$$

The problem (10)–(13) is solved by using the Linear Equivalence Method (LEM), introduced and developed by Toma [11, 12]. The values of the initial angular velocities [rad/s] are depending on the chosen system bell.

3. LEM INVESTIGATIONS AND CONCLUSIONS

Let us apply LEM to the system (10) and (11), *i.e.* Theorem 1 in order to get the corresponding LEM solution. In this case, the LEM exponential mapping will depend on 5 parameters. The linearized form of equations (12) is

$$\dot z = Az\,, \tag{14}$$

where

$$A=\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\beta\varsigma & \alpha\varsigma & 0 & 0 & -\alpha\varsigma\delta & 0 \\ \beta\gamma\varsigma & -\varsigma & 0 & 0 & \varsigma\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega^2 & 0 \end{pmatrix}, \tag{15}$$

where $\varsigma = 1+\alpha\gamma$. The characteristic equation is $\left|A-\lambda I\right| = 0$. The equivalent LEM system is written under the form [13]

$$\frac{\partial v}{\partial t} = \sum_{n=1}^{6}\left(\sum_{p=1}^{6}\sigma_n a_{np}\frac{\partial v}{\partial \sigma_p} + \sum_{p,q=1}^{6}\sigma_n b_{npq}\frac{\partial^2 v}{\partial\sigma_p\partial\sigma_q} + \sum_{p,q,r=1}^{6}\sigma_n c_{npqr}\frac{\partial^3 v}{\partial\sigma_p\partial\sigma_q\partial\sigma_r} + \right.$$
$$\left. + \sum_{p,q,r,l=1}^{6}\sigma_n d_{npqrl}\frac{\partial^4 v}{\partial\sigma_p\partial\sigma_q\partial\sigma_r\partial\sigma_l} + \sum_{p,q,r,l,m=1}^{6}\sigma_n e_{npqrlm}\frac{\partial^5 v}{\partial\sigma_p\partial\sigma_q\partial\sigma_r\partial\sigma_l\partial\sigma_m}\right) \tag{16}$$

and the initial conditions

$$v(0,\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_5,\sigma_6) = \exp(\sigma_1 z_1^0 + \sigma_2 z_2^0 + \sigma_3 z_3^0 + \sigma_4 z_4^0 + \sigma_5 z_5^0 + \sigma_6 z_6^0). \tag{17}$$

In order to present and compare the performance of the aforementioned method, we consider an example for each systems of bell illustrated in Table 1. The weight and initial angular velocities, respectively, are presented in Table 2.

Table 2

Examples

System	Weight [N]	Initial angular velocities $\dot{\theta}_1 / \dot{\theta}_2$ [rad/s]
Central European (1)	4300	2.77 / 3.14
Spanish (2)	4300	2.20 / 3.14
English (3)	4318	0.96 / 1.30

Initial conditions are chosen in the interval [–1.5, 1.5]. Results show that the solutions θ_1 and θ_2 are bounded and stable for $\omega = \delta = 0$. For certain values of ω and δ we have also bounded motions, but we can depict other regions of these parameters for which the solutions may sudden change to irregular, chaotic type motions.

We start with illustrating the time history of θ_1 and θ_2, respectively, by Figs. 5–7 for each system, and $\omega = \sqrt{\beta}$, $\delta = 1.1$. The LEM procedure is convergent and the behavior of the system is quasi periodic. For $\delta \geq 1.9$ the solutions show a typical transition between the regular and non-regular motions, when increasing the time. The cascade of period-doubling solutions suggests easily a route to chaos. The amplitude of oscillations increases and the system will lose its stability.

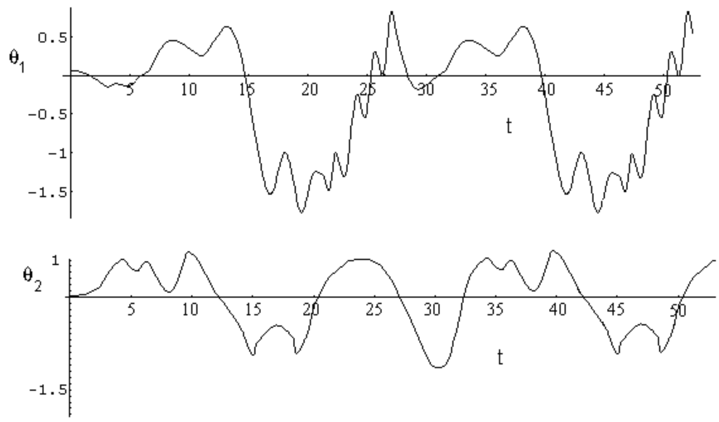

Fig. 5 – Time history of θ_1 and θ_2 ($\delta = 1.1$) in the case of Central European system.

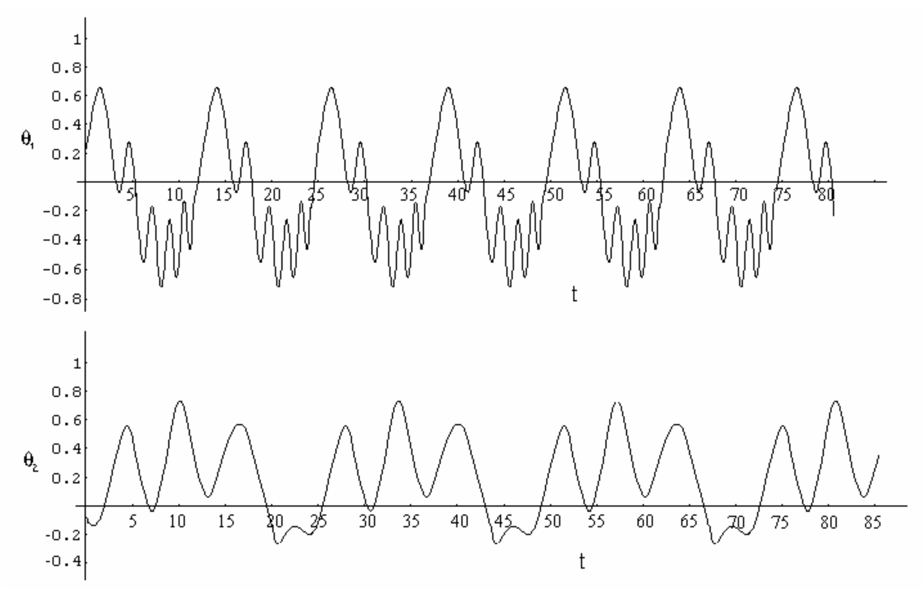

Fig. 6 – Time history of θ_1 and θ_2 ($\delta = 1.1$) in the case of the Spanish system.

Furthermore, a comparison between the LEM and Ivorra *et al.* [6] methods for the time history of the horizontal force (δ=1.1), is illustrated in Fig. 8 for the Spanish system. From the figure it can be seen that the LEM results show a small distorsional aspect in the decreasing regions of the horizontal force with respect to time. We explain this by the fact that the motion is very sensitive with respect to the variation of the parameter m .

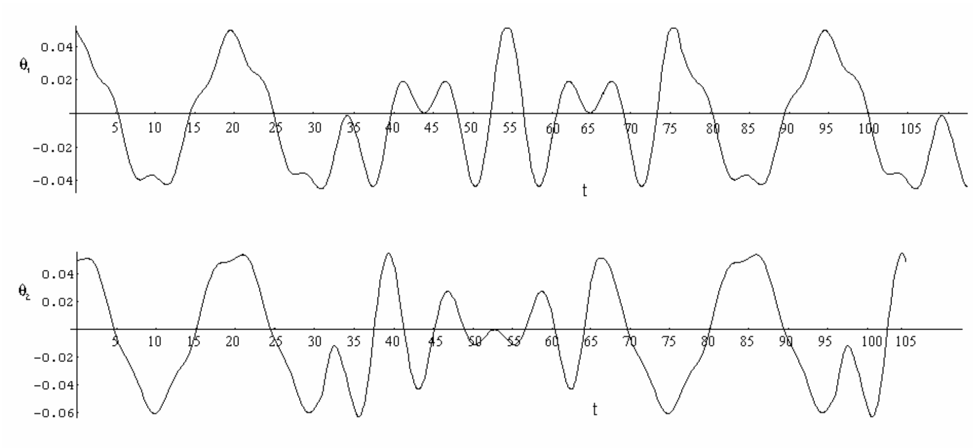

Fig. 7 – Time history of θ_1 and θ_2 (δ = 1.1) in the case of the English system.

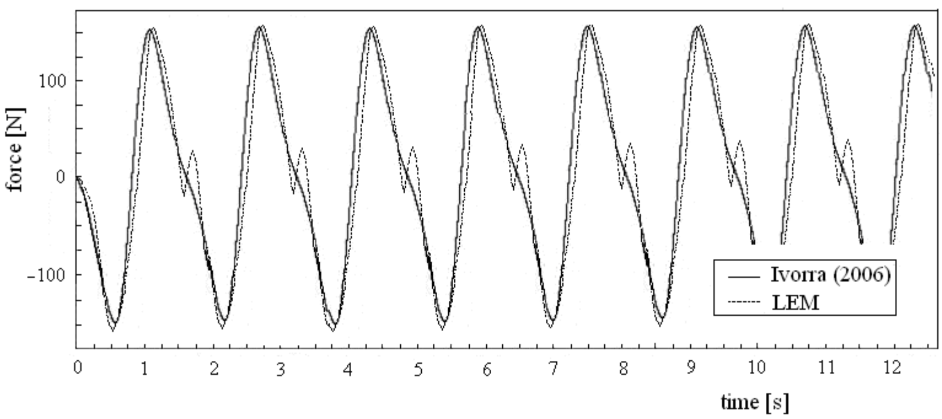

Fig. 8 – Comparison between LEM and Ivorra *et al.* [6] methods for the horizontal force time history (δ = 1.1) for the Spanish system.

It is interesting to consider a crack in the bell in order to study the way in which the crack affects the bell motion. In order to do this, the wave analysis is needed because the crack affects the resonance at the fundamental and subharmonic waves. Different cracks into the bell contribute to the deterioration of the harmonics. Fig. 9 represents the free decay vibrations of the Central European system for $\delta = \omega = 0$ and the perturbed parameter $m + \Delta m$, $\Delta \approx 10^{-3}$. We see that the equilibrium position of the double pendulum is shifting during the motion.

After oscillations died out, the equilibrium position remains shifted. We must remark that a similar behaviour was experimentally observed on different damaged material vibrators [14–16]. Though the noise is not typically treated in an engineering discussion of damping, we believes that there is a connectivity among vibrations, damping, and noise, because the mechanical damping of hysteretic type is closely to the form of noise ($1/f$ = flicker) [17].

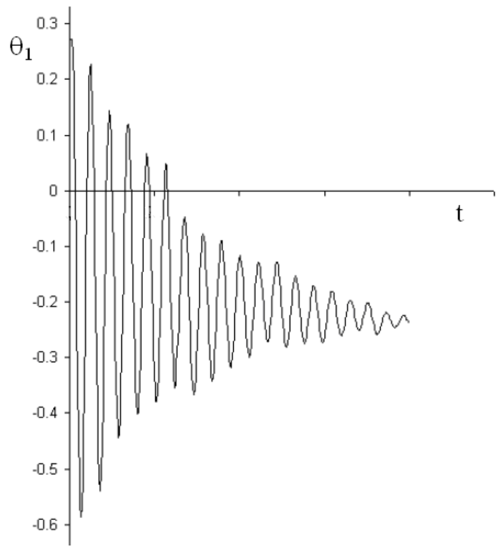

Fig. 9 – Free decay oscillation θ_1 of the Central European system.

In conclusion, the motion of double pendulum can be, especially for damaged materials, very complex, and cannot be explained by classical double pendulum models.

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