CALCULUS OF JOINT FORCES IN DYNAMICS OF A 3-PRR PLANAR PARALLEL ROBOT

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Abstract. Recursive matrix relations for the joint forces in dynamics of a planar 3-PRR parallel robot are established in this paper. Knowing the kinematics of the platform, we develop the inverse dynamic problem, using an approach based on the principle of virtual work. Some graphs of simulation for the internal joint forces are obtained.

Key words: planar parallel robot, dynamics, joint forces, principle of virtual work, recursive matrix relations.

1. INTRODUCTION

Equipped with revolute or prismatic actuators, parallel manipulators have a robust construction and can move bodies of large dimensions with high velocities and accelerations. That is reason why the devices, which produce translation or spherical motion to a platform, technologically are based on the concept of parallel manipulators [1].

Parallel manipulators have received more and more attention from researches and industries. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2]; Merlet [3]). The prototype of Delta parallel robot (Clavel [4]; Tsai and Stamper [5]) and the Star parallel manipulator (Hervé and Sparacino [6]) are both equipped with three motors, which train on the mobile platform in a general translation motion.

A mechanism is said to be a *planar robot* if all the moving links in the mechanism perform the planar motions that are parallel to one another. In a planar linkage, the axes of all revolute joints must be normal to the plane of motion, while the direction of translation of a prismatic joint must be parallel to the plane of motion.

Bonev, Zlatanov and Gosselin [7] describe several types of singular configurations by studying the direct kinematics model of a 3-RPR planar parallel robot with actuated base joints. Pennock and Kassner [8] present a kinematical study of a planar parallel robot, where a moving platform is connected to a fixed

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base by three links, each leg consisting of two binary links and three parallel revolute joints.

2. KINEMATICS ANALYSIS

A recursive method is introduced in the present paper, to reduce significantly the number of equations and computation operations by using a set of matrices for kinematics and inverse dynamics of the 3-PRR planar parallel robot.

The planar parallel robot of three degrees of freedom is a symmetrical mechanism composed of three planar kinematical chains $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$, having variable length and identical topology, all connecting the fixed base $A_0B_0C_0$ to the moving platform $A_3B_3C_3$. The parallel mechanism with seven links consists of three prismatic joints and six revolute joints (Fig. 1).

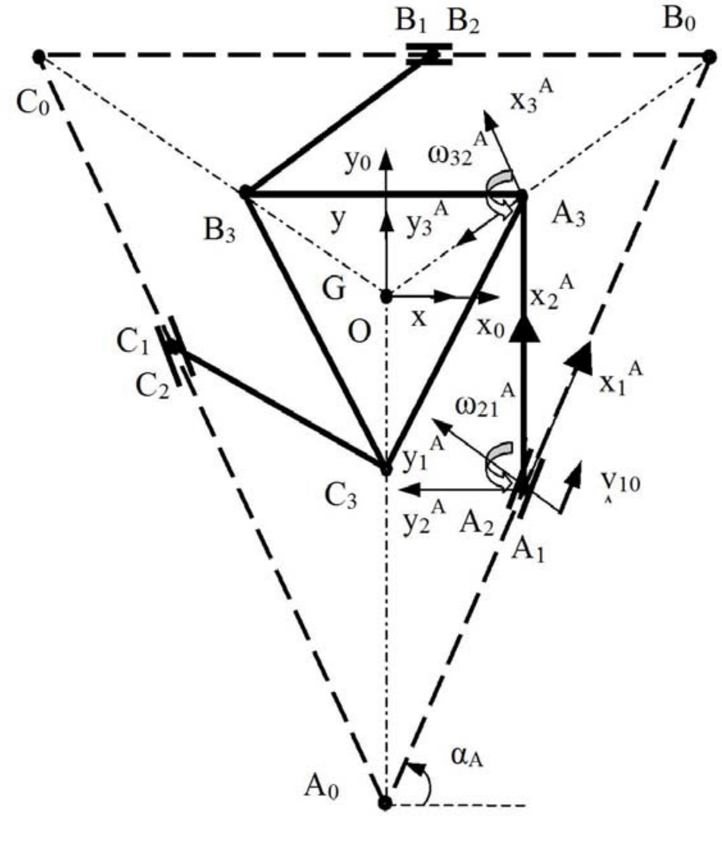

Fig. 1 – Planar 3-PRR parallel robot.

For the purpose of analysis, we attach a Cartesian frame $Ox_0y_0z_0(T_0)$ to the fixed base with its origin located at triangle centre O, the z_0 axis perpendicular to the base and the x_0 axis pointing along the direction C_0B_0. A mobile reference frame $Gx_Gy_Gz_G$ is attached to the moving platform, with the origin located just at the centre G of the triangle.

One of three active legs (for example leg A) consists of a prismatic joint, which is as well as a piston of mass m_1 linked at the $A_1x_1^Ay_1^Az_1^A$ frame, having a rectilinear translation of displacement λ_{10}^A. Second element is a rigid rod linked at the $A_2x_2^Ay_2^Az_2^A$ frame, having a relative rotation about z_2^A axis with the angle φ_{21}^A. It has the length l_2, mass m_2 and tensor of inertia $\hat{J}_2$ with respect to T_2^A frame. Finally, a revolute joint is introduced at a planar moving platform, which is schematised as an equilateral triangle with edge $l = r\sqrt{3}$, mass m_3 and inertia tensor $\hat{J}_3$ with respect to A_3, which rotates with the angle φ_{32}^A about z_3^A.

At the central configuration, we consider that all legs are symmetrically extended with the angles $\alpha_A = \pi/3,\ \alpha_B = 3\alpha_A,\ \alpha_C = -\alpha_A$ of orientation of three edges of fixed platform.

We call the matrix $a_{k,k-1}^{\varphi}$, for example, the orthogonal transformation 3×3 matrix of relative rotation with the angle $\varphi_{k,k-1}^A$ of link T_k^A around z_k^A axis. Starting from the reference origin O and pursuing three independent legs $OA_0A_1A_2A_3$, $OB_0B_1B_2B_3$, $OC_0C_1C_2C_3$, we obtain the following transformation matrices

$$q_{10} = \theta_{\alpha}^{i},\ \ q_{21} = q_{21}^{\varphi}\theta,\ \ q_{32} = q_{32}^{\varphi}\theta \quad (q = a, b, c) \quad (i = A, B, C)\,, \tag{1}$$

where

$$q_{k,k-1}^{\varphi} = \mathrm{rot}(z, \varphi_{k,k-1}^{i})\,,\ \ \theta_{\alpha}^{i} = \mathrm{rot}(z, \alpha_i)\,,\ \ \theta = \mathrm{rot}(z, \frac{\pi}{6})\,. \tag{2}$$

It can be considered that the position of the mechanism, in the inverse geometrical problem, is completely given through the coordinates x_0^G, y_0^G of the mass centre G of the moving platform and the orientation angle ϕ of the mobile central frame $Gx_Gy_Gz_G$. The orthogonal known rotation matrix of the platform from $Ox_0y_0z_0$ to $Gx_Gy_Gz_G$ reference system is $R = \mathrm{rot}(z, \phi)$.

We suppose that the position vector of G centre $\vec{r}_0^{\,G} = [x_0^G \ \ y_0^G \ \ 0]^T$ and the orientation angle ϕ, which are expressed by following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{\phi}{\phi^*} = 1 - \cos\frac{\pi}{3}t\,, \tag{3}$$

can describe the general absolute motion of the moving platform in its *vertical plane*. From the conditions concerning the orientation of the platform

$$\overset{\circ}{q}{}_{30}^{T} q_{30} = R\,,\;\; q_{30} = q_{32}q_{21}q_{10},\;\; \overset{\circ}{q}_{30} = \theta\theta\theta_{\alpha}^{i},\;\; (q = a,b,c) \tag{4}$$

we obtain the following relations between angles $\varphi_{21}^i + \varphi_{32}^i = \phi\ (i = A,B,C)$.

Pursuing the kinematical modeling developed in [9], six independent variables $\lambda_{10}^A,\ \varphi_{21}^A,\lambda_{10}^B,\ \varphi_{21}^B,\lambda_{10}^C,\ \varphi_{21}^C$ will be determined by the analytical equations

$$(l_1 + \lambda_{10}^i)\sin\alpha_i + l_2\sin(\varphi_{21}^i + \frac{\pi}{6} + \alpha_i) = y_0^G - y_{00}^i - r\cos(\phi + \frac{\pi}{3} + \alpha_i)$$

$$(l_1 + \lambda_{10}^i)\cos\alpha_i + l_2\cos(\varphi_{21}^i + \frac{\pi}{6} + \alpha_i) = x_0^G - x_{00}^i + r\sin(\phi + \frac{\pi}{3} + \alpha_i) \tag{5}$$

$$(i = A,\ B,\ C)\,.$$

Now, we compute in terms of the angular velocity of the platform and velocity of mass centre G the relative velocities $v_{10}^A, \omega_{21}^A, \omega_{32}^A$, starting from following *matrix conditions of connectivity*

$$v_{10}^A \vec{u}_j^T a_{10}^T \vec{u}_1 + \omega_{21}^A \vec{u}_j^T a_{20}^T \tilde{u}_3 \{\vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA}\} + \omega_{32}^A \vec{u}_j^T a_{30}^T \tilde{u}_3 \vec{r}_3^{GA} = \vec{u}_j^T \dot{\vec{r}}_0^G \;\; (j = 1,2)\,,$$

$$\omega_{21}^A + \omega_{32}^A = \dot{\phi}\,. \tag{6}$$

Concerning the first leg A, the characteristic *virtual velocities* are expressed as functions of the pose of the mechanism by the general kinematical equations (6), where we add the *contributions* of successive virtual translations or rotations during some fictitious displacements of the prismatic joint A_1 and of the revolute joints A_2 and A_3, as follows:

$$v_{10}^{Av} \vec{u}_j^T a_{10}^T \vec{u}_1 + v_{10}^{Ayv} \vec{u}_j^T a_{10}^T \vec{u}_2 + \omega_{10}^{Av} \vec{u}_j^T a_{10}^T \tilde{u}_3 a_{21}^T \{\vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA}\} +$$

$$+ v_{21}^{Axv} \vec{u}_j^T a_{10}^T \vec{u}_1 + v_{21}^{Ayv} \vec{u}_j^T a_{10}^T \vec{u}_2 + \omega_{21}^{Av} \vec{u}_j^T a_{20}^T \tilde{u}_3 \{\vec{r}_{32}^A + a_{32}^T \vec{r}_3^{GA}\} +$$

$$+ v_{32}^{Axv} \vec{u}_j^T a_{20}^T \vec{u}_1 + v_{32}^{Ayv} \vec{u}_j^T a_{20}^T \vec{u}_2 + \omega_{32}^{Av} \vec{u}_j^T a_{30}^T \tilde{u}_3 \vec{r}_3^{GA} = \vec{u}_j^T \vec{v}_0^{Gv} \;\;\; (j = 1,2), \tag{7}$$

$$\omega_{10}^{Av} + \omega_{21}^{Av} + \omega_{32}^{Av} = \omega_0^v\,.$$

Now, let us assume that the robot has successively some virtual motions determined by following sets of velocities:

$$
\begin{aligned}
& v_{10a}^{iv}=0\,,\, v_{10a}^{Ayv}=1\,,\, v_{10a}^{Byv}=0\,,\, v_{10a}^{Cyv}=0\,,\, \omega_{10a}^{iv}=0\,,\, v_{21a}^{ixv}=0\,,\, v_{21a}^{iyv}=0\,,\, v_{32a}^{ixv}=0\,,\, v_{32a}^{iyv}=0\,;\\
& v_{10a}^{iv}=0\,,\, v_{10a}^{iyv}=0\,,\, \omega_{10a}^{Av}=1\,,\, \omega_{10a}^{Bv}=0\,,\, \omega_{10a}^{Cv}=0\,,\, v_{21a}^{ixv}=0\,,\, v_{21a}^{iyv}=0\,,\, v_{32a}^{ixv}=0\,,\, v_{32a}^{iyv}=0\,;\\
& v_{10a}^{iv}=0\,,\, v_{10a}^{iyv}=0\,,\, \omega_{10a}^{iv}=0\,,\, v_{21a}^{Axv}=1\,,\, v_{21a}^{Bxv}=0\,,\, v_{21a}^{Cxv}=0\,,\, v_{21a}^{iyv}=0\,,\, v_{32a}^{ixv}=0\,,\, v_{32a}^{iyv}=0\,;\\
& v_{10a}^{iv}=0\,,\, v_{10a}^{iyv}=0\,,\, \omega_{10a}^{iv}=0\,,\, v_{21a}^{ixv}=0\,,\, v_{21a}^{Ayv}=1\,,\, v_{21a}^{Byv}=0\,,\, v_{21a}^{Cyv}=0\,,\, v_{32a}^{ixv}=0\,,\, v_{32a}^{iyv}=0\,; \qquad (8)\\
& v_{10a}^{iv}=0\,,\, v_{10a}^{iyv}=0\,,\, \omega_{10a}^{iv}=0\,,\, v_{21a}^{ixv}=0\,,\, v_{21a}^{iyv}=0\,,\, v_{32a}^{Axv}=1\,,\, v_{32a}^{Bxv}=0\,,\, v_{32a}^{Cxv}=0\,,\, v_{32a}^{iyv}=0\,;\\
& v_{10a}^{iv}=0\,,\, v_{10a}^{iyv}=0\,,\, \omega_{10a}^{iv}=0\,,\, v_{21a}^{ixv}=0\,,\, v_{21a}^{iyv}=0\,,\, v_{32a}^{ixv}=0\,,\, v_{32a}^{Ayv}=1\,,\, v_{32a}^{Byv}=0\,,\, v_{32a}^{Cyv}=0\,.
\end{aligned}
$$

These virtual velocities are required into the computation of virtual power and virtual work of all forces applied to the component elements of the manipulator.

As for the relative accelerations $\gamma_{10}^{A}, \varepsilon_{21}^{A}, \varepsilon_{32}^{A}$ of the robot, new conditions of connectivity are obtained by the derivative of above equations (6):

$$
\begin{aligned}
& \gamma_{10}^{A}\vec{u}_j^{T} a_{10}^{T}\vec{u}_3 + \varepsilon_{21}^{A}\vec{u}_j^{T} a_{20}^{T}\tilde{u}_3\{\vec{r}_{32}^{\,A} + a_{32}^{T}\vec{r}_3^{\,GA}\} + \varepsilon_{32}^{A}\vec{u}_j^{T} a_{30}^{T}\tilde{u}_3\vec{r}_3^{\,GA} = \\
& = \vec{u}_j^{T}\ddot{\vec{r}}_0^{\,G} - \omega_{21}^{A}\omega_{21}^{A}\vec{u}_j^{T}\, a_{20}^{T}\tilde{u}_3\tilde{u}_3\{\vec{r}_{32}^{\,A} + a_{32}^{T}\vec{r}_3^{\,GA}\} - \omega_{32}^{A}\omega_{32}^{A}\vec{u}_j^{T} a_{30}^{T}\tilde{u}_3\tilde{u}_3\vec{r}_3^{\,GA} - \qquad (9)\\
& -2\omega_{21}^{A}\omega_{32}^{A}\vec{u}_j^{T} a_{20}^{T}\tilde{u}_3 a_{32}^{T}\tilde{u}_3\vec{r}_3^{\,GA}, \quad \varepsilon_{21}^{A} + \varepsilon_{32}^{A} = \dot{\phi} \quad (j = 1, 2)\,.
\end{aligned}
$$

3. DYNAMICS EQUATIONS

The dynamics of parallel mechanisms is complicated by existence of multiple closed-loop chains. In the context of the real-time control, neglecting the friction forces and considering the gravitational effects, an important objective of the dynamics is first to determine the input torques or forces which must be exerted by the actuators in order to produce a given trajectory of the end-effector, but also to calculate all *internal joint forces or torques*.

Upon to now, several methods have been applied to formulate the dynamics of parallel mechanisms, which could provide the same results concerning these actuating torques or forces. First method applied to formulate the dynamics modelling is using the Newton-Euler procedure [10], the second one applies the Lagrange's equations and multipliers formalism [11] and the third approach is based on the principle of virtual work [12].

Knowing the position and kinematics state of each link as well as the external forces acting on the planar 3-PRR parallel manipulator, in the present paper we apply the principle of virtual work for the inverse dynamic problem in order to establish some definitive recursive matrix relations for the calculus of internal forces in the joints.

Three independent mechanical systems acting along the planar directions $A_1x_1^A$, $B_1x_1^B$ and $C_1x_1^C$ with the forces $\vec{f}_{10}^A = f_{10}^A\vec{u}_1$, $\vec{f}_{10}^B = f_{10}^B\vec{u}_1$, $\vec{f}_{10}^C = f_{10}^C\vec{u}_1$ can control the general motion of the moving platform. The force of inertia $\vec{f}_{k0}^{inA} = -m_k^A\left[\vec{\gamma}_{k0}^A + \left(\tilde{\omega}_{k0}^A\tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A\right)\vec{r}_k^{CA}\right]$ and the resulting moment of inertia forces $\vec{m}_{k0}^{inA} = -[m_k^A\tilde{r}_k^{CA}\vec{\gamma}_{k0}^A + \hat{J}_k^A\vec{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A\hat{J}_k^A\vec{\omega}_{k0}^A]$ of an arbitrary rigid body T_k^A, for example, are determined with respect to the centre of joint A_k. On the other hand, the wrench of two vectors $\vec{f}_k^{*A}$ and $\vec{m}_k^{*A}$ evaluates the influence of the action of the weight $m_k^A\vec{g}$ and of other external and internal forces applied to the same element T_k^A of the manipulator.

Two significant recursive relations generate the vectors

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^T\vec{F}_{k+1}^A,$$
$$\vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^T\vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T\vec{F}_{k+1}^A, \tag{10}$$

with the notations $\vec{F}_{k0}^A = -\vec{f}_{k0}^{inA} - \vec{f}_k^{*A}$, $\vec{M}_{k0}^A = -\vec{m}_{k0}^{inA} - \vec{m}_k^{*A}$.

As example, starting from (10), we develop a set of six recursive matrix relations for the leg A:

$$\vec{F}_3^A = \vec{F}_{30}^A,\quad \vec{F}_2^A = \vec{F}_{20}^A + a_{32}^T\vec{F}_3^A,\quad \vec{F}_1^A = \vec{F}_{10}^A + a_{21}^T\vec{F}_2^A,$$
$$\vec{M}_3^A = \vec{M}_{30}^A,\quad \vec{M}_2^A = \vec{M}_{20}^A + a_{32}^T\vec{M}_3^A + \tilde{r}_{32}^A a_{32}^T\vec{F}_3^A, \tag{11}$$
$$\vec{M}_1^A = \vec{M}_{10}^A + a_{21}^T\vec{M}_2^A + \tilde{r}_{21}^A a_{21}^T\vec{F}_2^A.$$

The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Assuming that frictional forces at the joints are negligible, the virtual work produced by all remaining forces of constraint at the joints is zero.

Total virtual work contributed by the inertia forces and moments of inertia forces, by the wrench of known external forces and by some internal joint forces, for example, can be written in a compact form, based on the relative virtual velocities.

Applying *the fundamental equations of the parallel robots dynamics* [13, 14, 15] the following compact matrix relations results:

$$f_{10y}^{A} = \vec{u}_2^T \vec{F}_1^A + \vec{u}_3^T \{\omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21a}^{Bv} \vec{M}_2^B + \omega_{21a}^{Cv} \vec{M}_2^C\} \quad (12)$$

for the *external joint force* acting in the prismatic joint A_1,

$$m_{10}^{A} = \vec{u}_3^T \vec{M}_1^A + \vec{u}_3^T \{\omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21a}^{Bv} \vec{M}_2^B + \omega_{21a}^{Cv} \vec{M}_2^C\} \quad (13)$$

for the *joint torque* acting in the prismatic joint A_1,

$$f_{21x}^{A} = \vec{u}_1^T a_{21}^T \vec{F}_2^A + \vec{u}_3^T \{\omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21}^{Bv} \vec{M}_2^B + \omega_{21}^{Cv} \vec{M}_2^C\} \quad (14)$$

for the first *joint force* and

$$f_{21y}^{A} = \vec{u}_2^T a_{21}^T \vec{F}_2^A + \vec{u}_3^T \{\omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21}^{Bv} \vec{M}_2^B + \omega_{21}^{Cv} \vec{M}_2^C\} \quad (15)$$

for the second *joint force* acting in the joint A_2,

$$f_{32x}^{A} = \vec{u}_1^T a_{32}^T \vec{F}_3^A + \vec{u}_3^T \{\omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21}^{Bv} \vec{M}_2^B + \omega_{21}^{Cv} \vec{M}_2^C\} \quad (16)$$

for the first *joint force* and

$$f_{32y}^{A} = \vec{u}_2^T a_{32}^T \vec{F}_3^A + \vec{u}_3^T \{\omega_{21a}^{Av} \vec{M}_2^A + \omega_{32a}^{Av} \vec{M}_3^A + \omega_{21}^{Bv} \vec{M}_2^B + \omega_{21}^{Cv} \vec{M}_2^C\} \quad (17)$$

for the second *joint force* acting in the joint A_3.

In what follows we can apply the Newton-Euler procedure to establish the set of analytical equations for each compounding rigid body of a prototype robot in a real application. These equations give all connecting forces in the external and internal joints. Several relations from the general system of equations could eventually constitute verification for the input forces already obtained by the method based on the principle of virtual work.

As application let us consider same planar mechanism 3-PRR analysed in [9], which has the following geometrical and architectural characteristics:

$$x_0^{G*} = -0.025\,\text{m}, \; y_0^{G*} = 0.025\,\text{m}, \; \phi^* = \frac{\pi}{12}, m_1 = 1\,\text{kg}, \; m_2 = 1.5\,\text{kg}, \; m_2 = 3\,\text{kg}$$

$$r = 0.1\,\text{m}, \; l_1 = l = r\sqrt{3}, \; l_0 = 0.3\,\text{m}, \; l_2 = 0.2\,\text{m}, \; \Delta t = 3\,\text{s}.$$

Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the planar parallel manipulator. To illustrate the algorithm, it is assumed that for a period of three second the platform starts at rest from a central configuration and rotates or moves along rectilinear directions.

Assuming that there are no external forces and moments acting on the moving platform, a dynamic simulation is based on the computation of the *joint forces* $f_{10y}^i, f_{21x}^i, f_{21y}^i, f_{32x}^i, f_{32y}^i$ and of the *external joint torques* m_{10}^i $(i = A, B, C)$ during the platform's evolution.

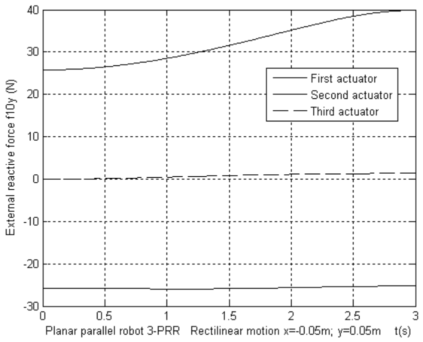

Fig. 2 – Joint forces f_{10y}^{A}, f_{10y}^{B}, f_{10y}^{C}.

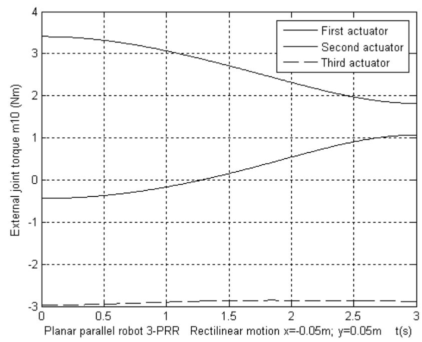

Fig. 3 – Joint torques m_{10}^{A}, m_{10}^{B}, m_{10}^{C}.

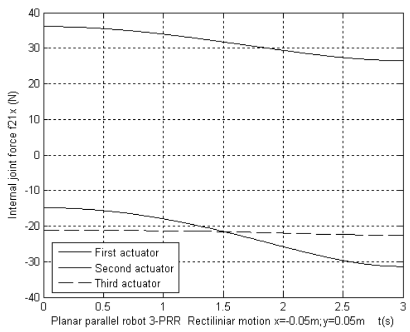

Fig. 4 – Joint forces f_{21x}^{A}, f_{21x}^{B}, f_{21x}^{C}.

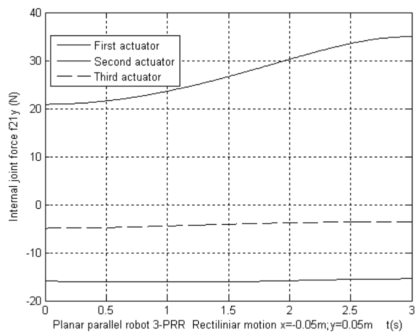

Fig. 5 – Joint forces f_{21y}^{A}, f_{21y}^{B}, f_{21y}^{C}.

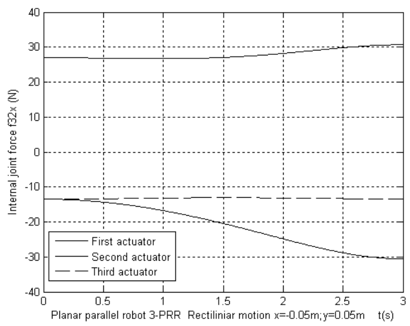

Fig. 6 – Joint forces f_{32x}^{A}, f_{32x}^{B}, f_{32x}^{C}.

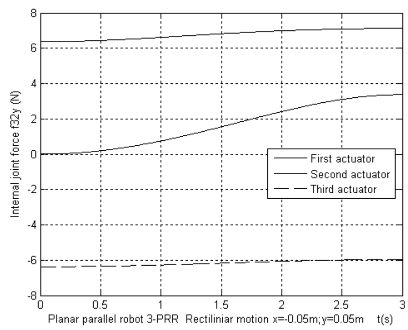

Fig. 7 – Joint forces f_{32y}^{A}, f_{32y}^{B}, f_{32y}^{C}.

If the platform's centre G moves along a *rectilinear planar trajectory* without rotation of platform, the intensities of internal joint forces or torques are calculated by the program and plotted versus time as follows: Fig. 2–Fig. 7. For the second example we consider the *rotation motion* of the moving platform about z_0

horizontal axis with variable angular acceleration while all the other positional parameters are held equal to zero (Fig. 8–Fig. 13).

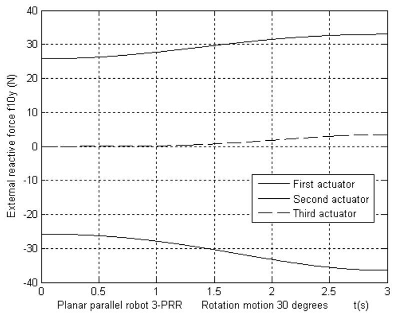

Fig. 8 – Joint forces $f_{10y}^{A}, f_{10y}^{B}, f_{10y}^{C}$.

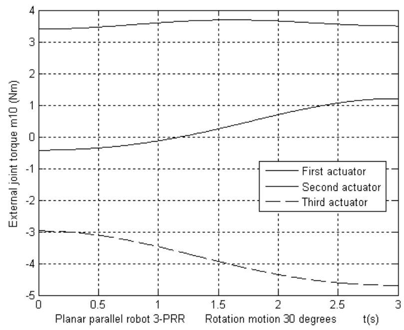

Fig. 9 – Joint torques $m_{10}^{A}, m_{10}^{B}, m_{10}^{C}$.

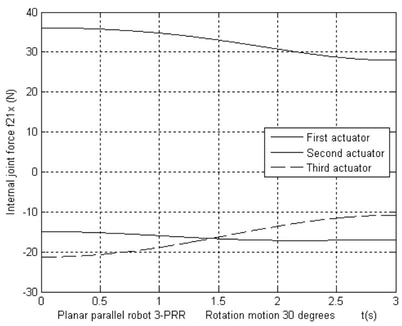

Fig. 10 – Joint forces $f_{21x}^{A}, f_{21x}^{B}, f_{21x}^{C}$.

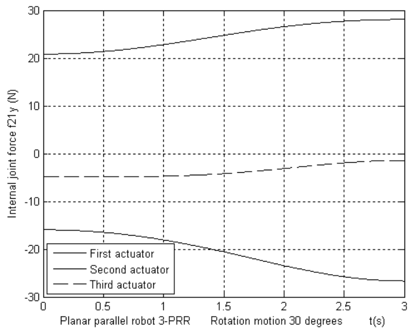

Fig. 11 – Joint forces $f_{21y}^{A}, f_{21y}^{B}, f_{21y}^{C}$.

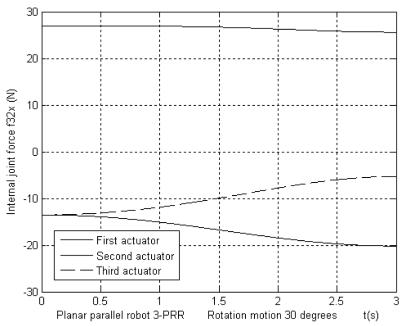

Fig. 12 – Joint forces $f_{32x}^{A}, f_{32x}^{B}, f_{32x}^{C}$.

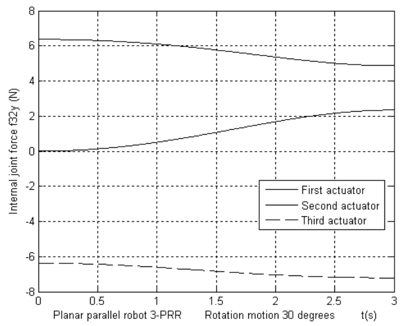

Fig. 13 – Joint forces $f_{32y}^{A}, f_{32y}^{B}, f_{32y}^{C}$.

The simulation through the MATLAB program certify that one of the major advantages of the current matrix recursive formulation is a reduced number of additions or multiplications and consequently a smaller processing time of numerical computation.

4. CONCLUSIONS

The present dynamics model takes into consideration the mass, the tensor of inertia and the action of weight and inertia force introduced by all compounding elements of the parallel mechanism. Based on the principle of virtual work, this approach establishes a direct determination of the time-history evolution for the internal forces or torques in joints. Choosing appropriate serial kinematical circuits connecting many moving platforms, the present method can easily be applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures*, when the number of components of the mechanisms is increased.

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