ON THE RESPONSE OF THE CAR FROM ROAD DISTURBANCES

RĂZVAN-VLAD VASIU[1], OCTAVIAN MELINTE[2], VICTOR VLĂDĂREANU[2], DAN DUMITRIU[2]

Abstract. The tire/road contact dynamics is investigated in this paper by assuming a rocky road. The vehicle model to be used in simulation is a seven degree of freedom full-car model. The contact domain between the tire and the road has a shape defined by the Lamé curve, and it is identified by checking the minimum distance between bodies in contact. The nonlinear response from road disturbances is analyzed by assuming explicit relationships for contact and friction forces, respectively. Since the road is rocky, a bristle model was chosen to take into account the effect of the road irregularities on the tire's action.

Key words: full-car vehicle model, contact force, friction force, rocky road, contact dynamics.

1. INTRODUCTION

The impact and the frictional slip are met together when they simultaneously develop in a contact interface, such as the interface between the tire and the road. The distribution of forces can be manifested in many different forms and the friction model changes the characteristics of the vibro-impact phenomenon in terms of duration, dissipation of energy, accelerations and decelerations [1–4].

The irregularities of the road (bumps, corners, peaks, shallows, etc.) can not be taken into account without introducing the contact and friction forces in the contact interfaces. We adopt in this paper a continuum approach for the vibro-contact dynamics by admitting explicit relationship between contact force and deformation. In this continuous approach, no difference is made between impact and contact, therefore the methods of non-impact dynamics can be used to solve the problem [4]. The modeling of the contact friction in the interfaces has been performed in a series of relevant papers [5–10]. The contact force depends on the deformation and it is defined by the interference distance or penetration. Impacts between bodies are generally defined by the condition of impenetrability [11].

The continuum modeling of tire/road vibro-contact dynamics is developed in this paper for a rocky road and the contact domain is modeled by checking the minimum distance between bodies. The model takes as inputs the function of

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bristle displacement and the vertical tire force and produces, as outputs, the distribution of contact pressure in the interfaces between the tire and the road in a short interval of time.

The novelty of the paper in relation to mentioned publications from the literature is referring to the modeling of the tire/road contact for a rocky road. Modelings for such roads we do not found in the literature. For such roads only the bristle model is able to take into account the effect of the irregularities on the tire's action. Our modeling is based on a seven degree of freedom full-car model. The idea of taking the Lamé curve for modeling the contact domain between the tire and the road is new. This idea together with the explicit relationships for contact and friction forces in the contact interfaces, form a new strategy introduced in this paper for investigation the tire behavior on rocky roads.

2. CONTACT BETWEEN THE TIRE AND THE ROAD

The contact between the tire and the road can be identified by checking the minimum distance between the tire and the road [5]

$$\min\left(\frac{1}{2}(r_1-r_2)^{\mathrm{T}}(r_1-r_2)\right),\quad f_1(r_1)\le 0,\ \ f_2(r_2)\le 0, \tag{1}$$

where r_1 and r_2 are the position vectors of two points belonging to the tire and the road, respectively, and f_1 and f_2 are bounding surface constraints. The interference distance is defined as

$$\min(-d),\quad f_1(r_1)\le -\frac{d}{2}e_1,\quad f_2(r_2)\le -\frac{d}{2}e_2 \tag{2}$$

The tire load and velocity generate forces at the interface between the tire and the road. These forces act in three directions, namely the contact force $F_c = F_{cz}$ acting in the z direction, the longitudinal component of the friction force F_{tx} acting in the x direction, and the lateral component of the friction force F_{ty} acting in the y direction, respectively (Fig. 1).

Figure 2 shows the force and moments vectors acting on an arbitrary point P of the tire located at the central intersection point of the tire and road surface plane. The undeformed tire has the radius R and rotates at constant angular speed Ω about the wheel axis. In the tire plane x and y axes are parallel to the longitudinal axis of the tire and the road surface normal, respectively. F_x is the longitudinal force acting in the direction of the forward motion of the tire in the plane of the road, F_y is the lateral force acting in the direction perpendicular to F_x in the plane of the road, and F_z is the vertical force acting in the direction of the vector normal to the road plane. M_x is the overturning moment acting about the axis parallel to F_x vector, M_y is the

rolling resistance moment acting about the axis parallel to the F_y vector, and M_z is the aligning moment acting about the axis parallel to F_z vector.

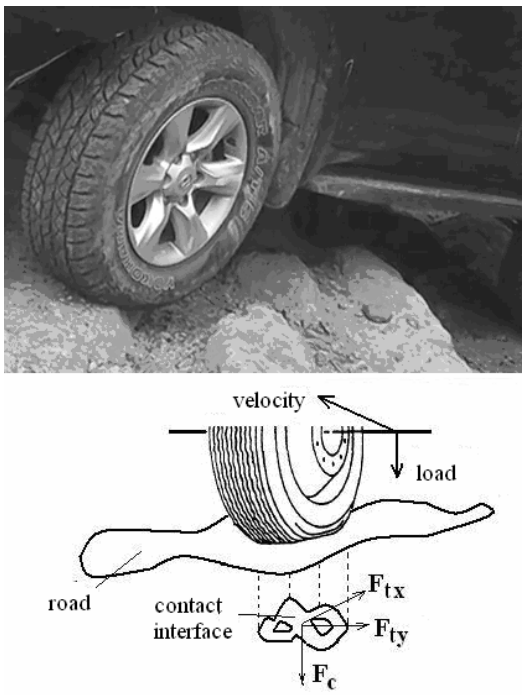

Fig. 1 – The contact between the tire and the road.

The τ, $|\tau| < 90°$ is the slip angle between the travel direction and the direction in which the tire is oriented, γ is the camber angle, that means the inclination angle between the *z*-axis and the tire plane and ω is the angular velocity of the wheel measured at the centre of the wheel-tire assembly (the linear speed of the center of the wheel in the longitudinal direction is V), and T is the axle torque.

If we denote with p_x, p_y and p_z, the longitudinal tire contact stress, the lateral tire contact stress and vertical tire contact pressure, the forces and moments, respectively in the point $P(x, y)$ belonging to the contact area A are given by

$$F_x(P) = -\int_A p_x \mathrm{d}A, \quad F_y(P) = -\int_A p_y \mathrm{d}A, \quad F_z(P) = -\int_A p_z \mathrm{d}A, \tag{3}$$

$$M_x(P) = -\int_A p_z \cdot y \mathrm{d}A, \quad M_y(P) = \int_A p_z \cdot x \mathrm{d}A, \quad M_z(P) = \int_A (p_x \cdot y - p_y \cdot x) \mathrm{d}A. \tag{4}$$

From (1.3) and (1.4) we can determine the forces F_x, F_y, F_z and moments M_x, M_y, M_z acting at the center of the tire in the tire reference frame, by applying a rotation γ about x [12]

$$\begin{pmatrix} F_x(P) \\ F_y(P) \\ F_z(P) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix},$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} M_x(P) - F_y(P)R\cos\gamma - F_z(P)R\sin\gamma \\ M_y(P) + F_x(P)R\cos\gamma \\ M_z(P) + F_x(P)R\sin\gamma \end{pmatrix}. \quad (5)$$

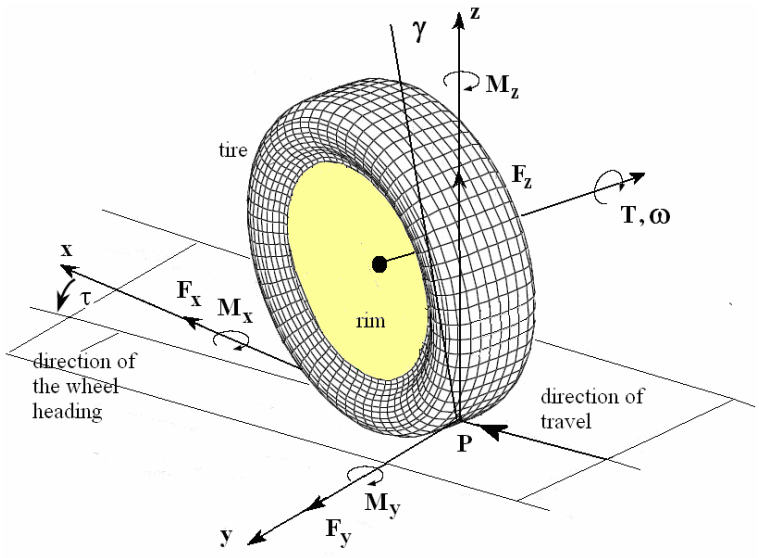

Fig. 2 – Forces and moments acting on the tire.

To shape of the unknown contact domain between the tire and the road is taken as a superellipse shape defined by the Lamé curve [8],[9]

$$\left(\frac{x}{a(t)}\right)^n + \left(\frac{y}{b(t)}\right)^n = 1, \quad n > 0, \quad (6)$$

where x and y define the envelope of the contact area, a is half of the contact length, b is half of the contact width (radii of the oval shape are depending of time), and n the power of the ellipsoid.

For the contact area A area of (6), we have

$$A(t)=4b\int_0^a\left(1-\left(\frac{x}{a}\right)^n\right)^{1/n}\mathrm{d}x=\frac{4^{1-1/n}a(t)b(t)\sqrt{\pi}\Gamma\left(1+\frac{1}{n}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{n}\right)}, \tag{7}$$

where Γ is the Gamma function

$$\Gamma(z)=\lim_{n\to\infty}\frac{n!n^z}{z(z+1)...(z+n)}\quad(z\neq 0,-1,-2,...).$$

3. MODEL DESCRIPTION

The description of the car is carried out based on the papers [13–15]. An alternative approach can be found in [16] concerning only the vertical dynamics by using a 7 DOF car model. The dynamic contact between the wheels and the road cannot be studied by alone, but only considering the car as a unique dynamical system. The sprung mass m of the vehicle is defined as a rigid body that can translate and rotate, exhibiting angular oscillations around the mass centre G. The tires B_j, $j=1,2,3,4$ are represented as the tips of the tetrahedron Q. The trajectories of these tips are arbitrary in the space, but subjected to given geometric requirements. Let us consider in the following that the projections of the trajectories of the contact points tyre-road are parallel lines situated in the plane Q. In Fig. 3, H is the horizontal plane, O_1A a segment on the intersection of planes Q and H, α is the angle between Q and H, n the unit vector defining the direction with the highest inclination of the plane Q, and P the projection of G in the plane Q, which describes the line O_1P. A mobile reference frame T_1 of an orthogonal set of vectors i, is attached to P ($i_1, i_2 \subset Q$, the sense of i_1 is along the vehicle motion direction). A fixed reference frame T_0 of an orthogonal set of vectors k, is attached to O_1. The relation between i and k is given by

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}=\begin{bmatrix} -\sin\beta & \cos\alpha\cos\beta & \sin\alpha\cos\beta \\ -\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha\sin\beta \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}, \tag{8}$$

where α, $0\le\alpha\le\pi/2$, and β, $-\pi\le\beta\le\pi$, are the relative angular displacements of g having the directions i_1 and i_2.

The roll and pinch angles are $n=k_2\cos\alpha+k_3\sin\alpha$ and $i_1=n\cos\beta-k_1\sin\beta$, where n is the unit vector defining the direction with the highest inclination of the faces of tetrahedron Q with respect to H.

A new mobile frame T of base vectors u of the principal inertial directions of the sprung mass, is assigned fixed on the sprung mass (Fig. 4)

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\theta_2 \\ 0 & 1 & \theta \\ \theta_2 & -\theta_1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}, \tag{9}$$

with $i_1 \times u_1 = \tilde{\gamma} \sin\theta$, $i_2 \times \tilde{\gamma} = i_1 \sin\psi$, $\tilde{\gamma} \times u_2 = u_1 \sin\varphi$, and θ, φ and ψ are the Euler angles. The frame T rotates with respect to T_1 to angular velocity ω defined as

$$\omega = \omega_1 u_1 + \omega_2 u_2 + \omega_3 u_3, \tag{10}$$

$$\omega_1 = \dot{\psi} + \dot{\varphi} = \dot{\theta}_1, \quad \omega_2 = \dot{\theta}_2, \quad \omega_3 = 0. \tag{11}$$

The motion equations of the sprung mass m with respect to T_0, with neglecting of the term $\omega \times \tau\omega$, are written under the form

$$m \frac{\mathrm{d}^2}{\mathrm{d}t^2}(OG) = R, \quad \tau_c \dot{\omega} = M_G, \tag{12}$$

where τ_c is the moment of central principal inertia tensor, R the force acting on m, M_G the moment with respect to G. The force R acting on the sprung mass is defined as

$$R = F + F_a + F_w + \sum_j P_j. \tag{13}$$

Here $F = -mgk_3$ is the weight, $F_a = F_a^1 i_1 + F_a^3 i_3$, $F_a^2 = 0$ the aerodynamic force applied in the pressure center P_1 linked on the frame T_1, on the position vector $\rho_{P_1} = l_a^1 i_1 + l_a^3 i_3$, $l_a^2 = 0$, depending only on the speed v of the vehicle, $F_w = \sum_k F_w^k i_k$ the wind force applied in the point P_2 linked on the frame T_1, on the position vector $\rho_{P_2} = \sum_k l_w^k i_k$, and supposed to be a constant. The quantities P_j are the constraint forces acting in points A_j, $j = 1,2,3,4$, that link the sprung mass m to four unsprung masses m_j, $j = 1,2,3,4$ of the tires B_j, $j = 1,2,3,4$ (Fig. 5).

The variation ΔV_j of the normal reaction forces in points A_j is given by $\Delta V_j = -\sum_j c_j \Delta_j - \sum_j k_j \dot{\Delta}_j$, where c_j and k_j, $j = 1,2,3,4$, are stiffness and damping coefficients of the suspensions associated to the static equilibrium point

$\Delta_j = 0$ of A_j. The quantities Δ_j represent the variation of the distance between the mass centers of the tires B_j and the points A_j

$$\Delta_j = z - z_j + \alpha b_j - \beta a_j, \quad j = 1,2,3,4. \tag{14}$$

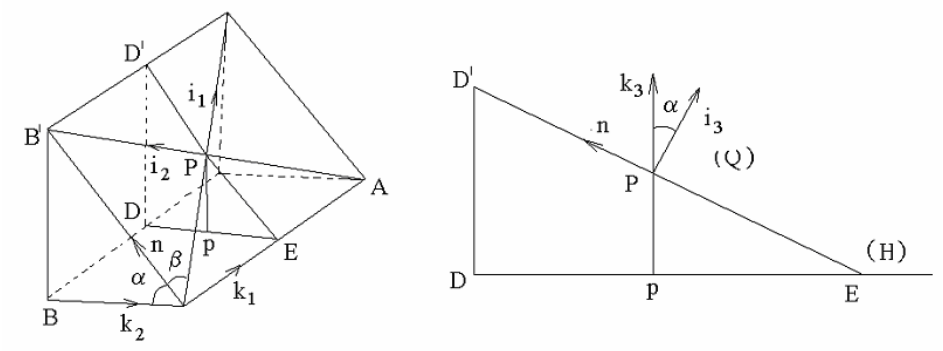

Fig. 3 – Geometry of the problem.

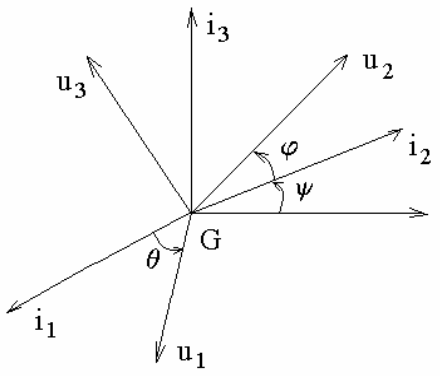

Fig. 4 – The Euler angles.

The initial conditions of the system motion are

$$t = 0,\ v = 0,\ F_a = 0,\ P_j = P_j^0,\ \varphi = \varphi_0,\ \psi = \psi_0,$$

$$\theta = \theta_0,\ \ \varphi_0 + \psi_0 = \alpha_0,\ \ \theta_0 = \beta_0. \tag{15}$$

From (3.6) we have at $t = 0$

$$R_0 = F + F_w + \sum_j P_j^0 = 0. \tag{16}$$

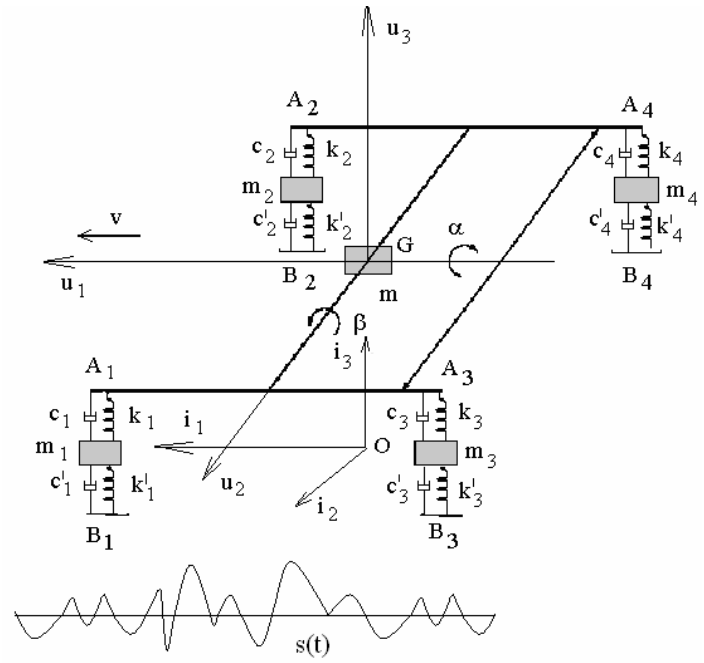

Fig. 5 – The full car model with seven degrees of freedom.

The suspension of the vehicle with two symmetric axes are modeled by the following seven equations of motion for m and m_j, $j=1,2,3,4$, on the direction i_3, and the second equation (12)

$$m\ddot{z}+\sum_j c_j\Delta_j+\sum_j k_j\dot{\Delta}_j=F_a^3, \tag{17}$$

$$m_j\ddot{z}_j+c'_jz_j+k'_j\dot{z}_j-c_j\Delta_j-k_j\dot{\Delta}_j=g_j,\ j=1,2,3,4, \tag{18}$$

$$A\ddot{\alpha}+\sum_j b_jc_j\Delta_j+\sum_j b_jk_j\dot{\Delta}_j=M_1, \tag{19}$$

$$B\ddot{\beta}-\sum_j a_jc_j\Delta_j-\sum_j a_jk_j\dot{\Delta}_j=M_2, \tag{20}$$

where c'_j and k'_j, $j=1,2,3,4$, are the stiffness and damping coefficients of the tires B_j, M_1 and M_2 are the moments acting on the mass m with respect to G, and A,B the principal central inertial moments corresponding to the directions u_1 and u_2. The forces g_j, $j=1,2,3,4$, are given by

$$g_j = c'_j s_j + k'_j \dot{s}_j, \ j = 1,2,3,4, \tag{21}$$

with $s_j(t)$, $j = 1,2,3,4$, the bristle displacement functions.

The boundary conditions attached to the tires can be written as

$$M_j = z_0 F_{fj}, \ F_{shj} = F_{cj} - F_z, \ j = 1,2,3,4, \tag{22}$$

where M_j are the bending moment of B_j corresponding to the contact domain, F_z is the vertical load acting on B_j, and F_{sh} is the shearing force in B_j, respectively. The moments M_j are equal to the couple moments created by the friction force F_{fj}, $j = 1,2,3,4$ in B_j. The friction force F_{fj} is given by [18, 19]

$$F_{tj} = \begin{cases} k_{fj} s_j(t_0) + \int_{t_0}^{t} v_{tj} \mathrm{d}t, & |s_j| < s_{j\max}, \\ \mu \dfrac{|F_{cj}|}{k_{fj}} \dfrac{v_{tj}}{|v_{tj}|}, & \text{otherwise}, \end{cases} \tag{23}$$

where k_{fj} is the bristle stiffness, $s_j(t)$ the bristle displacement functions, $j = 1,2,3,4$, t_0 is the start time of the last sticking at that contact point, v_{tj} the relative tangential velocity in B_j and parameter $s_{j\max}$ is the maximum allowable deflection of the bristle. The contact force F_{cj} are the contact forces in B_j, given by [17]

$$F_{cj} = k\delta^{n_j} + b\delta^{p_j}\dot{\delta}^{q_j}, \ j = 1,2,3,4, \tag{24}$$

with n_j, p_j, q_j, $j = 1,2,3,4$, are constants, the coefficient *k* depends on the tire's material and the geometric properties of the tire, and *b* is defined with respect to the coefficient of restitution $0 \le e \le 1$, and the F_{tj} are friction forces in B_j respectively, given by (23).

In the following we consider that the stiffness and damping coefficients c'_j and k'_j, $j = 1,2,3,4$, of the tires are constants. The suspension force is considered to vary continuously between a soft suspension (to insulate against road disturbances), and a hard suspension (to insulate against load disturbances)

$$c_1 = c_2 = c_1(t), \ c_3 = c_4 = c_2(t), \ k_1 = k_2 = k_1(t), \ k_3 = k_4 = k_2(t). \tag{25}$$

$$c_{1,2} = \begin{cases} c_s^h, & \text{if } c_s^h < c_s \text{ (hard suspension)}, \\ c_s, & \text{if } c_s^s < c_s < c_s^h, \\ c_s^s, & \text{if } c_s < c_s^s \text{ (soft suspension)}. \end{cases} \tag{26}$$

$$k_{1,2} = \begin{cases} k_s^h, & \text{if } k_s^h < k_s \text{ (hard suspension)}, \\ k_s, & \text{if } k_s^s < k_s < k_s^h, \\ k_s^s, & \text{if } k_s < k_s^s \text{ (soft suspension)}. \end{cases} \tag{27}$$

4. RESULTS

We select the following parameters for the model: $m = 1\,500$ kg, $m_1 = m_2 = 40$ kg, $m_3 = m_4 = 40$ kg, $k_i' = 250$ kN/m, $c_i' = 0.6$ kN/m, $i = 1,2,3,4$, $k^h = 20$ kN/m, $k^s = 12$ kN/m, $c^h = 5$ kNs/m, $c^s = 0.5$ kNs/m. The identification of the contact patches in the interval T_0 is performed by checking the minimum distance between bodies according to (1) and (2). As results, for a speed of the vehicle of 10 m/s, a number of more than 500 contact points were detected in the given interval of time in $t \in T_0 = [0; 5\text{s}]$. By defining a contact patch consisting from a minimum nearest 5 contact points, a number of more than 100 contact patches were identified. The functions of bristle-displacement $s_j(t)$, $j = 1,2,3,4$, are represented in Fig. 6.

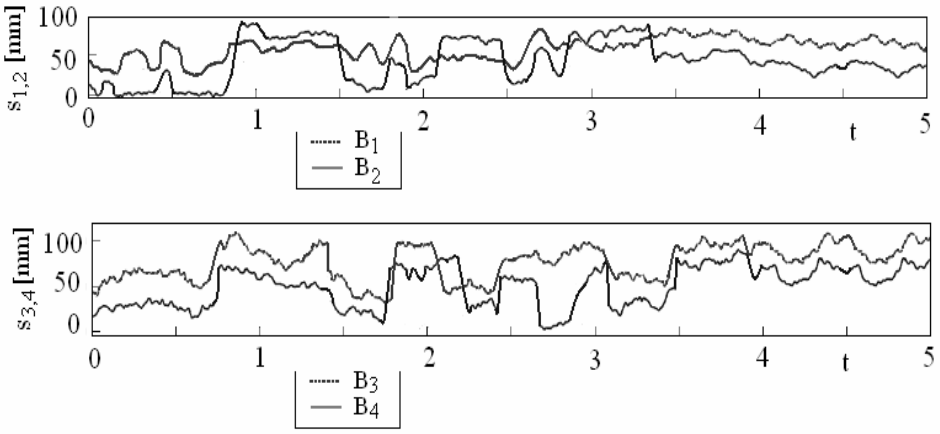

Fig. 6 – Functions of bristle-displacement $s_j(t)$.

Table 1 shows the characteristics of eight contact patches identified in T_0 for four values of the vertical loads, 2 000, 3 000, 5 000 and 8 000 N, respectively.

Table 1

Dimensions of the contact patches in the interval T_0

Tire	Vertical load F_z [N]	*a* [mm]	*b* [mm]	*n*
B_1	2000	35	46	2.02
		38.3	51.2	2.18
	3000	45.2	53.9	2.41
		50.2	54.8	2.55
	5000	67.8	63.4	2.88
		72.9	65.6	2.92
	8000	90.1	71.8	3.18
		91.8	74.7	3.95
B_2	2000	35.3	45.6	2.02
		38.1	51.5	2.17
	3000	45.0	53.7	2.40
		51.0	54.4	2.53
	5000	68.0	63.2	2.86
		73.6	65.5	2.91
	8000	89.5	72.3	3.15
		92.5	74.9	3.93
B_3	2000	35.8	45.9	2.03
		38.5	51.7	2.16
	3000	49.2	56.8	2.42
		61.4	64.5	2.54
	5000	52,7	57.0	2.67
		63.4	61.2	2.71
	8000	97.8	75.1	4.03
		101.0	76.4	4.13
B_4	2000	35.9	45.8	2.03
		38.5	51.8	2.15
	3000	53,1	57.1	2.66
		63.8	61.4	2.70
	5000	71.9	68.0	2.78
		79.2	67.6	3.00
	8000	98.3	75.0	4.04
		100.1	76.3	4.11

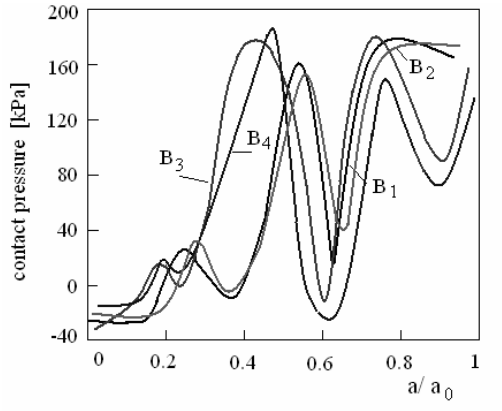

Fig. 7 – The maximum contact pressure in the first contact patch.

The maximum value of the contact pressures for the first contact patch, with respect to a/a_0 and b/b_0, are plotted in Fig. 7. The a_0 and b_0 are the reference radii of the oval shape of the contact domain. The time variation of the roll and pinch angles α and β, are shown in Fig. 8.

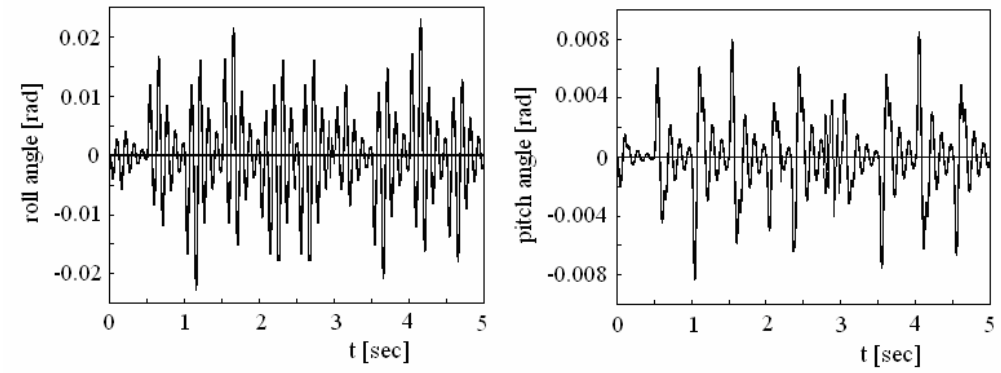

Fig. 8 – The time variation of the roll and pinch angles α and β.

5. CONCLUSIONS

Road profiles are central into investigation of the interaction between the tire and the road. This paper outlines a novel approach for the modeling of the tire/road contact dynamics by assuming a full-car vehicle model in interaction with a rocky road. The vehicle model to be used in simulation is a seven degree of freedom full-car model. The response from road disturbances is analyzed by assuming an explicit relationship between the contact force and the deformation. An important

aspect of this model is that the contact area increases with deformation and a plastic region can appear for larger indentation, i.e. the damping depends on the indentation. Another advantage of this model is that the contact force has no discontinuities at initial contact and separation, and it begins and finishes with the value of zero.

For a rocky road, the bumpiness has to be taken into account. It is not unusual for automotive designers to test virtual models of cars on virtual models of bumpy roads. The bristle model for the friction force is chosen in this paper to take into account the effect of the road irregularities on the tire's action.

The suspension force is considered to vary continuously between a soft suspension (to insulate against road disturbances), and a hard suspension (to insulate against load disturbances).

The contact domain between the tire and the road has a shape defined by the Lamé curve, and it is identified by checking the minimum distance between bodies in contact.

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