PROBABILISTIC ASSESSMENT OF THE IMPACT OF STRAIGHTNESS TOLERANCES IN EN 1090-2 ON THE STABILITY DESIGN OF STEEL COLUMNS

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Abstract. This paper addresses a current, practical topic which has interesting implications for design practices and research in the steel construction industry: the impact of new, relaxed fabrication tolerances on the strength of compression members. The current debate on this topic – held at a code committee level – was triggered by the introduction of increased tolerances for the out-of-straightness of steel members in the new European standard for the execution of steel structures for constructional steelwork, EN 1090-2 [1]. The extent to which these changes are still "covered" by the present design rules is currently unknown. To date, a consistent, logically coherent justification for the acceptance of these changes is still missing. The study presented in this paper is therefore intended as a contribution to this debate. In the broader scientific context, it is intended to serve as an example for the possibility of answering questions regarding the impact of changes to production tolerances and manufacturing habits on design rules by probabilistic approaches.

Key words: buckling, imperfections, tolerances, structural reliability, eurocode 3, probabilistic assessment, steel columns.

1. INTRODUCTION

The recent introduction of the harmonized European standard EN 1090-2 [1] for the fabrication and erection of steel structures brought about a number of innovations to the allowable shape deviations (or tolerances) of members, plates etc. In the case of compression members, the most significant – and controversial – change, when compared to most previous national standards, was the introduction of an out-of-straightness limit of $\Delta = L/750$. This value is noticeably larger than the value of $L/1000$ known to form the basis of the stability design rules for column and beam-column buckling in Eurocode 3 – EN 1993-1-1 [2] (EC3). Therefore, the pertinent ECCS and CEN committees are currently confronted with the question of what impact must be expected to the safety level of slender steel structures designed in accordance with EC3 and fabricated following EN 1090-2.

This paper presents the results of a study that addressed this question for the most relevant case of steel compression members ("columns"). Thereby, the case of weak-axis buckling of one exemplary section was considered; this allowed for

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an exemplary representation of the possible changes to the implicit safety level, in terms of relative differences between different scenarios and tolerance prescriptions. The study makes use of plausible scenarios for future column production habits and of an advanced probabilistic and mechanical-deterministic methodology. This methodology includes "Monte Carlo" simulations based on statistical distributions of the basic variables of the stability problem, coupled with geometrically and materially non-linear FEM analyses (GMNIA, see section 4.2) of imperfect members with random distributions of these same variables. A large number (exceeding 2000) of single simulation runs was carried out for this study. The results of these FEM calculation runs were then evaluated, as results of "numerical tests", in accordance with Annex D of EN 1990 [3].

2. TOLERANCES FOR COMPRESSION MEMBERS IN INTERNATIONAL CODES

Most codes regulating the execution of steel structures distinguish between *manufacturing* and *erection tolerances.* The former, also called *shop fabrication tolerances*, are concerned with deviations from the nominal dimensions and are measured in the workshop. They mainly serve as control quantity for the production quality in the shop or factory. The latter are measured on site, after erection. They are a control quantity for the quality of the erection works on site.

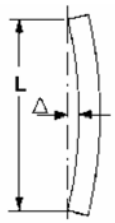
Table 1 summarizes the regulations of the out-of-straightness tolerances for compression members according to different international codes. The following comments can be made:

i. The *manufacturing* (shop) *tolerances* for out-of-straightness of compression members were fixed at a value of $\Delta \leq L/1000$ in all codes of practice preceding the publication of the new European standard EN 1090-2. As a side note, it should be mentioned that hollow sections – produced according to the standard EN 10210-1 (2006) – had and still have a larger tolerance limit of $\Delta \leq L/500$ [4].

ii. The value of $\Delta \leq L/1000$ is identical to the imperfection assumptions on which the European column buckling curves (now found in the Eurocode) are based, $\overline{e}_0 = L/1000$ [5, 6]. Historically, this tolerance value was actually the *initial motivation* for the adoption of $\overline{e}_0 = L/1000$ in the non-linear numerical calculations that were carried out during the development of the European column buckling curves. With $\Delta \leq L/750$, EN 1090-2 increases the tolerance limit by 1/3.

iii. In previous national European and current international codes, the sections devoted to e*rection tolerances* did not generally include provisions regarding the out-of-straightness of compression members. The AISC code only recently added a clarifying drawing where columns are shown to have $\Delta \leq L/1000$ also in the erected configuration, but no comment is given on this – and on how this should be measured. The same can be stated about the former Austrian Standard ÖN B 4300-7 (1994). Other codes only mention positioning and inclination restrictions for the erection of columns.

Table 1
Out-of-straightness of compression members (except hollow sections)
according to different international standards

Measurement Definition	Standard	Country	Manufacturing Tolerance	Erection Tolerance
 (from EN 1090-2)	AISC Code of Standard Practice (2005)	USA	Δ∗L/1000	None . Δ ∗L/1000)
	BS 5920-2 (2001) & NSSS (2007)	UK	Δ ∗max(3mm ; L/1000)	None
	DIN 18800-7 (2002)	DE	Product standards hot-rolled: EN 10034 welded: ISO 13920	None
	ÖN B4300-7 (1994)	AUT	Δ∗*L*/1000	Δ ∗*L*/1000
	ENV 1993-1-1:1992 / ENV 1090-1:1996	Some EU countries	Δ ∗*L*/1000 Δ ∗max (3 mm; *L*/1000)	None
	ECCS Rec. 1978	-	Δ ∗*L*/1000	Δ ∗*L*/667 & statistics
	EN 1090-2:2008	CEN members	Δ ∗*L*/750	Δ ∗*L*/750

iv. The lack of a specification of erection tolerances is explicable by the difficulty and cumbersomeness associated with measuring a column's curvature once it is built in. Furthermore, if the curvature is specified as an erection tolerance it must also be specified how, and with what frequency, this quantity ought to be measured on site – this is not straight-forward and potentially controversial.

v. The only provision known to specify clear and uncontroversial acceptance criteria for column out-of-straightness on site is contained in the ECCS "European Recommendations for Steel Construction" (1978), see [7]. While this code also uses the value of $\Delta \leq L/1000$ for workshop fabrication, it loosens this value to $\Delta \leq L/667 = 0.0015\ L$ on site. According to this recommendation, if individual measured values exceed this value, the construction contract's parties are expected to agree upon the subsequent procedure by judging the fitness-for-purpose of the compression element.

vi. Additionally to this criterion, the emphasis is placed on the statistical distribution of the measured curvature. This is schematically illustrated in Fig. 1. A series of measurements is accepted as conforming to the requirements when the mean value plus α times the estimated standard deviation of measurements lies within the tolerance limit. The value of α decreases with a rising number of measurements n, spanning a range of 3.7 for $n = 6$ to 2.1 for $n = 100$. They are meant to lead to a 90% confidence level regarding the non-exceedance of the tolerance value.

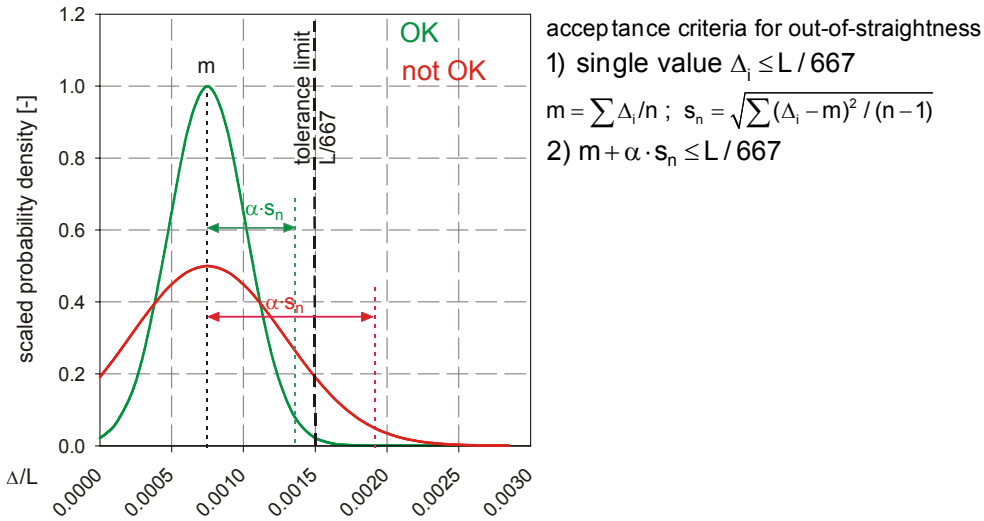


Fig. 1 – Visualization of the acceptance criteria for column curvature according to [7].

vii. EN 1090-2 does specify erection tolerances for the out-of-straightness of compression members. However, no indication is given about how, and how often, these ought to be measured. One must assume that, in theory, all columns must be measured and checked against the tolerance criterion if EN 1090-2 applies as part of a construction contract.

3. COLUMN OUT-OF-STRAIGHTNESS "COVERED" BY THE EUROCODE 3 BUCKLING RULES

3.1. BUCKLING TESTS AND NUMERICAL CALCULATIONS

The code survey in the preceding section confirmed that the tolerance definition for column straightness in EN 1090-2 differs from – and is more "generous" than – all other important international codes, including those that were valid (with earlier editions) in Europe when the current EC3 rules were created. In this section, a closer look is taken at the imperfections that can be considered to be "covered" by these rules. As is discussed in more detail in [5] and [6] and was more recently summarized in [8], the current EC3 column buckling rules are based on an extremely thorough, international experimental campaign, combined with an equally thorough theoretical/numerical study, both carried out under the auspices of ECCS. The experimental campaign consisted of both large scale buckling tests and small-scale "auxiliary tests" that included the measurement of yield stress, cross-sectional geometry and column out-of-straightness. While the buckling tests

formed the direct experimental basis of the current EC3 rules, the auxiliary tests were originally mainly used for the purpose of documentation and test quality control. Figure 2 shows the measurements of the out-of-straightness amplitude Δ for some of these tests; as can be seen, the limit of $\Delta/L = 1/1000 = 0.001$, valid at the time of the tests, was exceeded by a non-negligible amount of tested columns, especially for smaller, slender cross-sections like the IPE 160 shown in the figure. However, the statistical distribution of the out-of-straightness had a fairly narrow scatter-band, with a mean value of Δ/L of $m = 0.00085 = 1/1176$ and a standard deviation of $s = 0.0002 = 1/5000$. This would have fulfilled the above-mentioned procedure in the 1978 ECCS recommendation for field measurements, which also required $\Delta \leq L/1000$ in the shop. The probabilistic/statistical evaluation of the full-scale tests corresponding to the above measurements for IPE 160 sections is illustrated in Fig. 3, leading to a maximum value of the necessary safety factor γ_M^* of approximately 1,13. In the main diagram, the test results are plotted over the nominal value of the slenderness $\bar{\lambda}$, evaluated statistically and shown as vertical lines with markers at the values m and $m \pm 2s$. This evaluation was found to be reproducible by numerical simulations of a model beam with nominal member geometry, bow-imperfections of amplitude $\bar{e}_0 = L/1000$ and standardized patterns of residual stresses [5]. This is also shown in Fig. 3, where the line marked with $f_{y,k}$ represents the "characteristic curve" (evaluated with the measured value $m - 2s$ of the yield stress), while $f_{y,\mathrm{nom}}$ identifies the curve evaluated with nominal values of yield stress. As can be seen, the "characteristic" GMNIA line describes the $m - 2s$ points of the test result evaluation very well. This knowledge was then used to *numerically* determine the shape and position of the 5 European column buckling curves in [5].

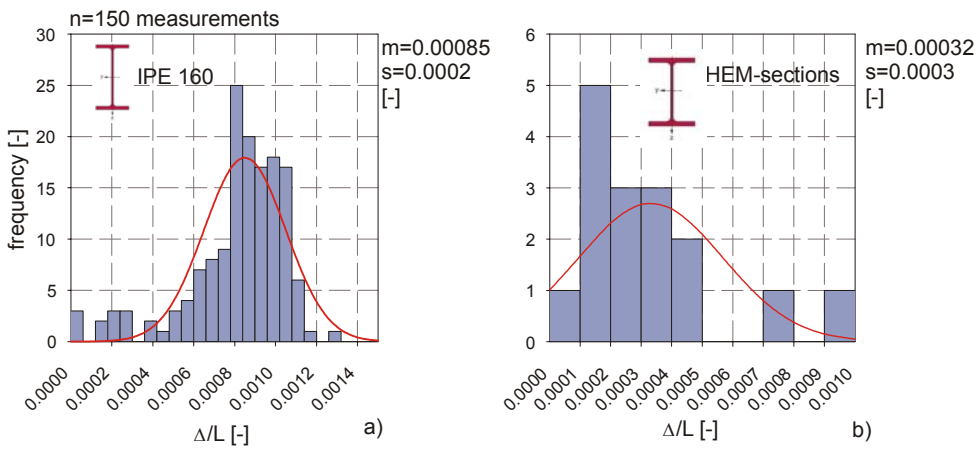


Fig. 2 – Measured initial curvature Δ as fraction of the column length for some of the ECCS column tests.

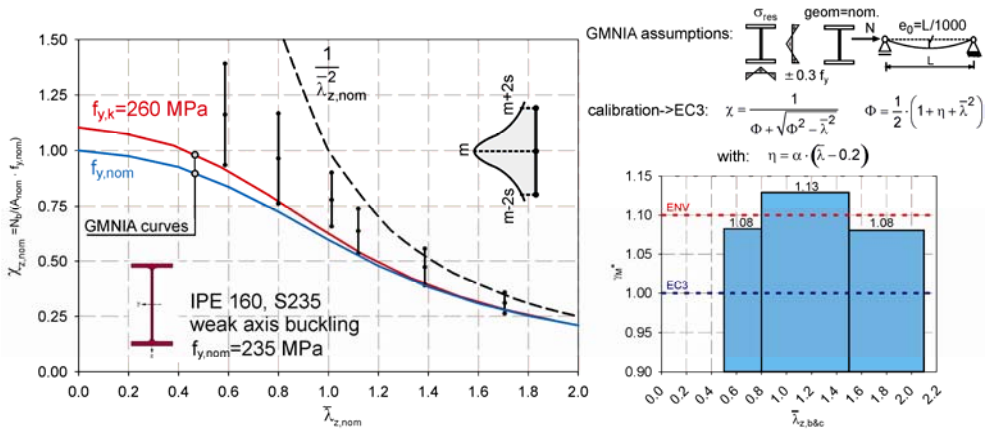


Fig. 3 – Original ECCS test results [6] for IPE 160 sections compared with GMNIA analyses and safety-factor evaluation of the current, calibrated EC3 design formula.

3.2. IMPLICIT EQUIVALENT IMPERFECTIONS FOR SECOND-ORDER ANALYSIS AND THEIR RELATIONSHIP TO STRAIGHTNESS TOLERANCES

In order to obtain mechanically consistent and meaningful design equations for column buckling, the ECCS column bucking curves were transformed into Ayrton-Perry type design formulae by Rondal and Maquoi [9]. These design expressions, which represent a combination of second-order internal forces with a linear cross-sectional failure criterion and equivalent, generalized and *calibrated* imperfections, are the familiar expressions currently found in EC3, Eqs. 1 and 2.

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \leq 1.0, \tag{1}$$

with

$$\Phi = \frac{1}{2}\left[1 + \eta + \bar{\lambda}^2\right]; \quad \eta = \alpha\left(\bar{\lambda} - 0.2\right). \tag{2}$$

The starting point for the derivation of these equations is represented by a first-yield buckling condition using second-order internal forces resulting from a sinusoidal pre-deformation.

$$\frac{N}{A \times f_y} + \frac{N \cdot \bar{e}_0}{W \times f_y} \cdot \frac{1}{1 - (N/N_{cr})} \leq 1.0, \tag{3}$$

with: N – axial force ; N_{cr} – elastic ("Euler's") critical buckling load;
A – cross-sectional area; W – section modulus (plastic or elastic);
$\bar{e}_0$ – amplitude of a sinusoidal imperfection shape ("out-of-straightness");
f_y – yield strength.

Equation 3 can be rewritten in a normalized form by introducing the EC3 variables $\chi = N/(A \cdot f_y)$, $\bar{\lambda} = \sqrt{(A \times f_y)/N_{cr}}$, and $\eta = A \times \bar{e}_0 / W$, leading to Eq. 4:

$$\chi + \eta \cdot \frac{\chi}{1 - \chi \cdot \bar{\lambda}^2} = 1.0. \tag{4}$$

Equation 4 is a quadratic equation, the lowest solution of which is given by Eq. 1 and 2. Thus, in the Eurocode column buckling curves, the original elastic term for η (= $A \cdot \bar{e}_0$ / W) was substituted by a calibration term containing the buckling curve coefficient α. This substitution can be reversed to obtain the "implicit" *equivalent imperfection amplitudes* $\bar{e}_0$ contained in the Eurocode column buckling curves. In the following Eq. 5, these are represented as a fraction of the column length L:

$$\left(\bar{e}_0/L\right) = \frac{W}{A}\eta \cdot \frac{1}{L} = \frac{W}{A} \cdot \frac{\alpha}{L} \cdot \left(\bar{\lambda} - 0.2\right). \tag{5}$$

In some reviews (e.g. [10]) of the current Eurocode design formulae for column buckling, it is argued that the loosening of the curvature tolerances Δ/L may be justifiable on the basis of the above equivalent imperfections for second-order calculations. Acknowledging the fact that the values of the generalized imperfection coefficient α also have to cover the influence of residual stresses in the Ayrton-Perry formulation, it is argued (*assumption* 1) that, at $\bar{\lambda} = \infty$, the influence of the residual stresses is practically zero. Furthermore, it is – implicitly – assumed (*assumption* 2) that the buckling curve a_0 ($\alpha = \alpha_{a0} = 0.13$) is representative for a column with practically no locked-in stresses. The following formula can then be derived for strong-axis flexural column buckling $FB_{y\text{-}y}$:

$$\left(\bar{e}_0 / L\right)_{\text{lim,GEOM,FB}_{y-y}} = \frac{2 \cdot I_y}{h \times A} \cdot \frac{\alpha_{a0}}{L} \cdot \bar{\lambda} = \frac{2 \cdot i_y}{h} \cdot \frac{\alpha_{a0}}{\pi} \cdot \sqrt{\frac{f_y}{E}}. \tag{6}$$

In a third assumption (*assumption* 3), the ratio i_y/h is set to 0.5. With this value, the following theoretical limit value of the purely geometrical imperfection that is already covered by the current buckling rules can be calculated, for the most inconvenient case of steel grade S235:

$$\left(\bar{e}_0 / L\right)_{\text{lim,GEOM,FB}_{y-y}} = 2 \cdot 0.5 \cdot \frac{0.13}{\pi} \cdot \sqrt{\frac{235}{210000}} = 1/722. \tag{7}$$

Since $\bar{e}_0/L = 1/722$ is larger than $\Delta/L = 1/750$, one may believe that the new tolerance limit is covered by the current design rules. In fact, this line of argumentation appears to represent the main background for the loosening of the straightness tolerances for columns.

While at first glance this derivation is appealing, a number of problems become evident when a more in-depth verification of its basic assumptions is performed. The following points can be brought forward:

i. First of all, the assumption regarding i_y/h is not realistic for most commercial I– & H-sections, where this value moves in the range of 0.35–0.42. This simple corrective remark, regarding only one of the three assumptions made above, would disprove the above argumentation, since e.g. with i_y/h = 0.4, $\overline{e}_0/L_{\text{lim,GEOM,FBy-y}}$ = = 1/903, which is smaller than Δ/L = 1/750.

ii. If assumption 2 were true, applying Equ. 7 in the case of weak-axis flexural buckling $FB_{z\text{-}z}$ of typical I- and H-sections (which have ratios of i_z/b~0.25) would correspondingly lead to the conclusion that a tolerance of only Δ/L = 1/1444 is covered by the design rules – an even smaller value than the one that was previously valid at Δ/L = 1/1000.

iii. If assumption 2 is dropped, or said not to apply for weak-axis buckling of a member made of S235 steel, the next most plausible buckling curve for this assumption is line b, the "highest" curve for weak-axis buckling of S235 I-sections. In this case, the "covered" imperfection is calculated as follows:

$$\left(\overline{e}_0 / L\right)_{\text{lim,GEOM,FB}_{Z-Z}} \cong 2 \cdot 0.25 \cdot \frac{0.34}{\pi} \cdot \sqrt{\frac{235}{210\,000}} = 1/552. \tag{8}$$

This is larger than both Δ/L = 1/1000 and Δ/L = 1/750, and (importantly) larger than any value ever recorded in the ECCS tests (Fig. 2), it is hard to imagine that this could be covered by the present rules.

These two points confirm that it is not purposeful to attempt to draw any conclusions regarding the permissibility of loosening geometrical shape tolerances *by inference* from considerations stemming from the second-order beam theory and the generalized imperfection amplitude factor α. The current column buckling formulae are essentially based on curve-fitting of a second-order equation onto previously established buckling curves, derived from tests and numerical analyses (**g**eometrically and **m**aterially **n**on-linear analyses, including **i**mperfections: GMNIA, see section 4.2). Thereby, these "manipulated" second-order equivalent imperfections lost much of their physical meaning, particularly in the *quantitative* sense that is of interest here. Finally, it must be remembered that, from a safety point of view, the values of α must "cover" more than just residual stresses and column curvature: since the structural designer eventually calculates a column's strength with nominal values of cross-sectional geometry and material strength, but in reality these values scatter, the values of α must also account for the scatter of these quantities. This is not acknowledged in the above considerations.

4. PROBABILISTIC STUDY – METHODOLOGY

4.1. GENERAL PRINCIPLES

In this paper, a probabilistic approach was used in order to assess the impact of changes to the straightness tolerances on the safety of the stability design rules for steel compression members. Thereby, First Order Reliability Methods were considered, following the general philosophy employed in the Eurocodes (EN 1990 [3]). Possible scenarios for the future impact of "loosened" straightness tolerances were considered in a numerical parametric study with randomly generated input parameters, whereby physical tests were simulated/replaced by non-linear numerical tests with scattering input parameters. In the steel research community, this type of "test data" generation is generally referred to as a type of "Monte Carlo" assessment procedure, in reference to related, fully probabilistic approaches using random input data generation. These test results were then analyzed statistically, using the FORM procedures of EN 1990, in order to obtain representative values for the necessary safety factors of the design rules.

The study was performed – exemplarily – for *flexural buckling about the weak z-z axis* ($FB_{z\text{-}z\text{-}}$) and for *one section*. This apparent limitation is not particularly relevant for the aims of this paper, which are to highlight potential *relative* differences in the safety level prompted by the chance in tolerances, rather than absolute values. Thus, a representative I-section geometry had to be chosen for the Monte Carlo simulations of this paper. The *IPE 160 section made of S235 steel*, already shown in Fig. 3, was selected because it was very thoroughly studied during the development of the European column buckling curves and thus comparisons with physical tests are possible.

4.2. NUMERICAL METHODOLOGY

The modelling techniques sketched in Fig. 4 were used for the numerical analyses with scattering input parameters, using the software package ABAQUS (Dassault Systems). The boundary conditions of the member, as well as the chosen shapes of imperfection and the used finite element mesh, are shown in the figure. Single-span compression members with in-plane, out-of-plane and torsional restraints at the supports were considered ("end fork" conditions). An end load eccentricity in the same direction as the assumed column out-of-straightness was considered as one of the possible scattering quantities.

Four-node linear shell elements (S4) with six degrees of freedom per node and finite strain formulation were chosen to model the flange and web plates of the studied sections. The contribution of shear stresses in plasticity is accounted for by this modelling technique. The mesh density was left constant, with 16 elements per flange and web plate generally found to be a sufficient number. The number of nodes in longitudinal direction was varied between 100 and 200, depending on

total member length. Rigid coupling beams were used to connect the single plates with each other. At the supports, the stiffness terms of these elements were manipulated in order to obtain a stiff load introduction mechanism (allowing one to define concentrated loads to simulate moments and axial loads) that nevertheless allowed the cross-section to warp and rotate freely.

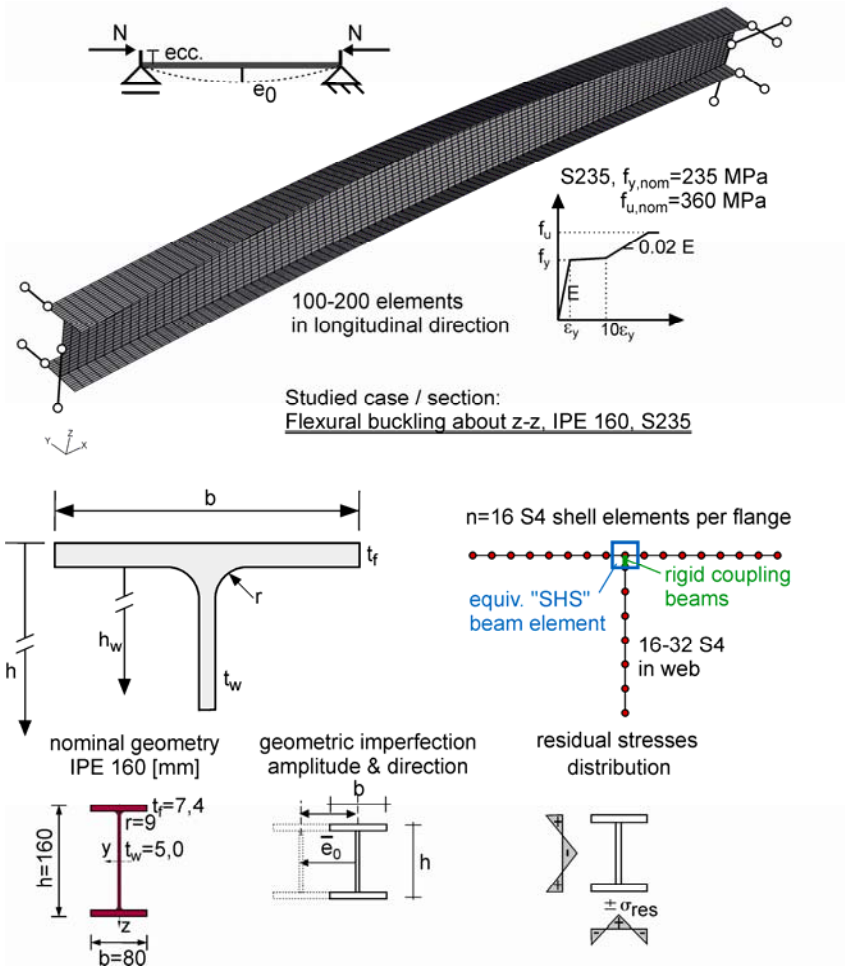


Fig. 4 – Overview of the FEM modelling techniques and assumptions.

Since the studied section is a hot-rolled IPE 160, the fillets were included in the calculations by adding equivalent beam elements that were placed in the centroids of the flanges. By defining the cross-sections of these beam elements as equivalent box or square hollow sections (SHS) of variable depth and wall thickness, the total area and torsional stiffness of the modelled member could be calibrated to precisely match the tabulated values given by the production standards for rolled

sections. The exact bending capacity, as well as the weak- and strong-axis sectional inertia (moments of area) were thereby also approximated with negligible error (in the range of less than 2% error).

In the calculation of all relative quantities (such as slenderness $\bar{\lambda}$, the buckling reduction factor χ, etc.), the properties of the actually modelled section (with the equivalent SHS section) were considered, rather than the original rolled section, thereby further reducing the error of the calculation results, when expressed again in these relative variables.

In order to obtain realistic numerical values of the ultimate buckling strength, **G**eometrically and **M**aterially **N**on-linear **A**nalyses with **I**mperfections (GMNIA) were carried out – these were already mentioned above and were also used for the development of the column buckling curves themselves. As the name implies, imperfections are included in these calculations, whereby both geometric and structural imperfections (residual stresses) were considered:

i. The initial geometric imperfections were assumed to follow a sinusoidal shape. Since a double-symmetric section failing in weak-axis flexural buckling was studied, no twist of the section needed to be considered.

ii. The residual stresses were assumed to vary linearly over the single cross-section components, following the provisions given by ECCS in [11].

In addition to the imperfections, the material non-linearity (stress-strain curve), as well as geometric non-linearities (equilibrium in the deformed state), were considered. Strain hardening was included in the calculations, again following the long-established recommendations of ECCS in [11].

4.2. PROBABILISTIC EVALUATION ACCORDING TO EN 1990 [3]

EN 1990 – Annex D contains a standardized procedure – based on First Order Reliability Methods – that allows for the determination of appropriate values (in a semi-probabilistic design concept) of partial safety factors γ_M on the basis of test results. A summary of the procedure, as applicable to the problem studied in this paper, is given in the following.

EN 1990 recognizes the relevance of the accuracy of a design formulation and of its ability to react to the variability of the input parameters that govern a design problem. The design formulation therefore takes on an immediate, central part in the statistical evaluation procedure of EN 1990. In the notation of the code, the theoretical value of strength r_t is expressed as a function of its individual input parameters

$$r_t = g_n\left(\underline{X}\right). \tag{9}$$

As an example for a function g_{rt}, the buckling formula leading to a column strength $r_t = \chi\ A\ f_y$ may be considered, with the vector of the basic variables $\underline{X}$ containing such parameters as the cross-sectional geometry, the yield stress, the column length, etc.

The theoretical strength r_t is compared with the experimental strength r_e in the methodology of Annex D. Thereby, the "actual" values of the basic parameters should be determined for each individual test result $r_{e,i}$. This information about the basic variables is used to calculate the specific strength prediction $r_{t,i}$ for a single test result, which can then be plotted as shown in Fig. 5.

For a number of n pairs $r_{e,i}/r_{t,i}$, plotted in the r_t/r_e plane, a regression line through the origin can then be calculated through least-square approximation, using the following formula:

$$b = \sum_{i=1}^{n} r_{e,i} \cdot r_{t,i} \Big/ \sum_{i=1}^{n} \left(r_{t,i}\right)^2 . \tag{10}$$

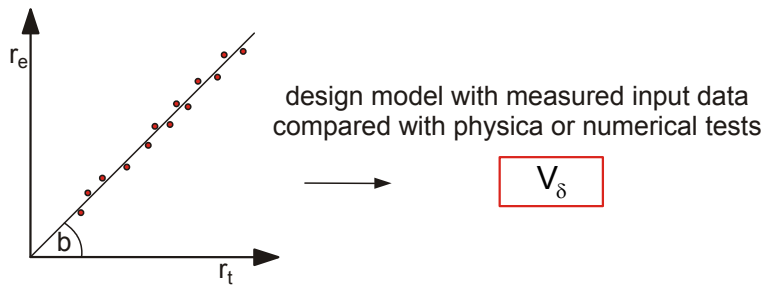


Fig. 5 – Schematical representation of the interpretation of V_δ as variance of the design model.

The coefficient of variation V_δ of the error terms δ_i (with $\delta_I = r_{e,i}/(b \cdot r_{t,i})$) of the design function is calculated as follows:

$$V_\delta = \sqrt{\exp(s_\Delta^2) - 1}, \tag{11}$$

with

$$s_\Delta^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(\Delta_i - \overline{\Delta}\right) \tag{12}$$

$$\overline{\Delta} = \frac{1}{n} \sum_{i=1}^{n} \Delta_i \tag{13}$$

$$\Delta_i = \ln\left(\frac{r_{e,i}}{b \cdot r_{t,i}}\right) = \ln \delta_i . \tag{14}$$

Up to now, the methodology has only accounted for the differences between a certain pool of test data results (usually in terms of strength) and the prediction of

the same strength according to the design function. In the next step, the sensitivity of the design function itself to the variability of the basic input variables must be accounted for, by calculating the error propagation term $V_{r,t}$. In the usual case of a complex, multi-variable design function, $V_{r,t}$ is calculated using the following formulae:

$$V_{r,t}{}^2 = \frac{\mathrm{VAR}\left[g_{r,t}(\underline{X})\right]}{g_{r,t}(\underline{X}_m)^2} = \frac{1}{r_{m,t}{}^2} \cdot \sum_{i=1}^{j} \left(\frac{\partial g_{r,t}}{\partial X_i} \cdot \sigma_i \right)^2, \tag{15}$$

with $\frac{\partial g_{r,t}}{\partial X_i} \cdot \sigma_i$ ….. partial derivative for the variable X_i times its standard deviation.

The log-normal variation coefficients can now be calculated as follows:

$$Q_{r,t} = \sqrt{\ln\left(V_{r,t}{}^2 + 1\right)}, \tag{16}$$

$$Q_{\delta} = \sqrt{\ln\left(V_{\delta}{}^2 + 1\right)}, \tag{17}$$

$$Q = \sqrt{\ln\left(V_{r}{}^2 + 1\right)}, \tag{18}$$

with

$$V_{r}{}^2 = V_{r,t}{}^2 + V_{\delta}{}^2. \tag{19}$$

In the next step, the design value of the resistance r_d is calculated:

$$\text{for } n \le 100\text{: } r_d = b \cdot g_{r,t}\left(\underline{X}_m\right) \cdot \exp(-k_{d,\infty} \cdot \frac{Q_{rt}{}^2}{Q} - k_{d,n} \cdot \frac{Q_{\delta}{}^2}{Q} - 0.5 \cdot Q^2), \tag{20}$$

$$\text{for } n > 100\text{: } \quad r_d = b \cdot g_{r,t}\left(\underline{X}_m\right) \cdot \exp(-k_{d,\infty} \cdot Q - 0.5 \cdot Q^2), \tag{21}$$

with $g_{rt}\left(\underline{X}_m\right) \hat{=} r_{m,t}$ representing the value of the design function evaluated with the mean values of all basic input variables, and $k_{d,n}$ and $k_{d,\infty}$ being the design fractile factors for n and infinite single test results (see [3], Annex D, Table D.2).

Finally, the required partial safety factor $\gamma_M{}^*$, applicable for designs based on nominal input data, can be calculated as follows

$$\gamma_M{}^* = \frac{r_{tk}}{r_d} \underbrace{\hat{=}}_{\text{in EC3}} \frac{r_{nom}}{r_d}, \tag{22}$$

with $r_{nom} = g_{rt}\left(\underline{X}_{NOM}\right)$, i.e. the design function evaluated with nominal values of the input parameters. This is the quantity usually determined by designers in EC3.

4.3. RANDOM INPUT DATA GENERATION

As stated previously, simulated tests obtained from GMNIA calculations with random input variables ("Monte Carlo simulations") are nowadays increasingly being used instead of expensive and time-consuming physical tests. This procedure is fully legitimate, provided that the real scatter bands of the single input parameters, as well as the way these parameters correlate, are known [12].

This paper makes use of so-called "Monte Carlo simulations" in order to generate random sets of input parameters for these numerical tests. The numerical values for the single calculations were generated using the standard *inverse transform method* [13] on the basis of cumulative distribution functions (cdf) F for the normal or log-normal distribution, depending on the modelled parameter. A single, randomly generated value X is therefore obtained from F and a randomly generated value $U \sim \mathrm{Unif}[0,1]$, i.e. a random, uniformly distributed real number in the interval [0,1]:

$$X = F^{-1}(U),\ U \sim \mathrm{Unif}\left[0,1\right]. \tag{23}$$

An exemplary illustration of the results of data generation of this kind are shown in Fig. 6, where the frequency plot for randomly generated variable values for column curvatures and load eccentricities is shown to follow the underlying distribution (in form of its probability density function) very well.

An issue for the application of the FORM and Monte Carlo methods is the question of the correlation of the single parameters. This is graphically illustrated in Fig. 7 for the example of the initial curvature and the residual stresses in a column, generated either without any correlation (a) or with strong correlation (b). In the latter case, the maximum of one variable corresponds to the minimum of the other variable, while no such relationship exists in Fig. 7a.

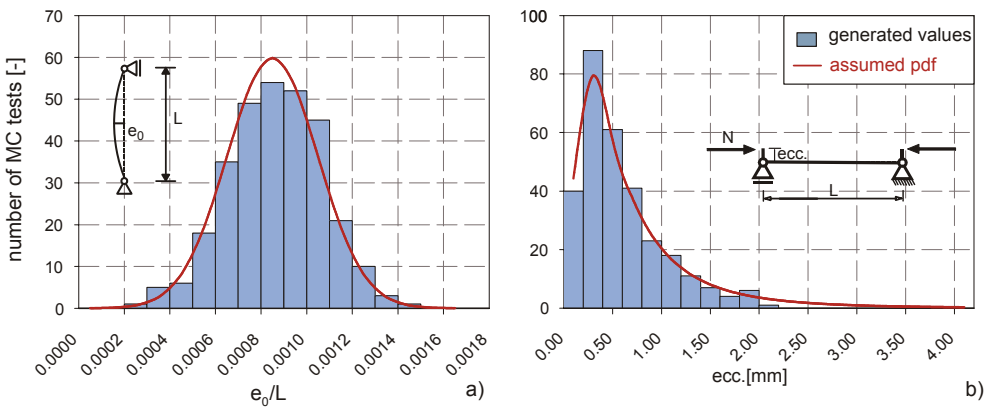


Fig. 6 – Exemplary representation of the generation of random variables on the basis of distribution functions.

As a matter of fact, precisely the two parameters plotted in Fig. 7 are sometimes suspected to be inversely correlated with each other, meaning that smaller residual stresses are systematically present whenever high curvatures are present (see e.g. [14, p. 387]). However, for the purposes of this paper, no systematic correlation was considered. This is motivated by the following considerations:

i. No statistical evaluations exist that would definitively prove such correlations between parameters that are relevant for buckling.

ii. More importantly, the statistical evaluation procedure integrated in the Eurocode – EN 1990 and discussed in section 4.2 is a First Order Reliability Method, based on the assumption that all variables are independent. Since the results of Monte Carlo simulations are intended to be evaluated using this procedure, any information stemming from the inclusion of the correlation would be "ignored" by the EN 1990 evaluation procedure itself.

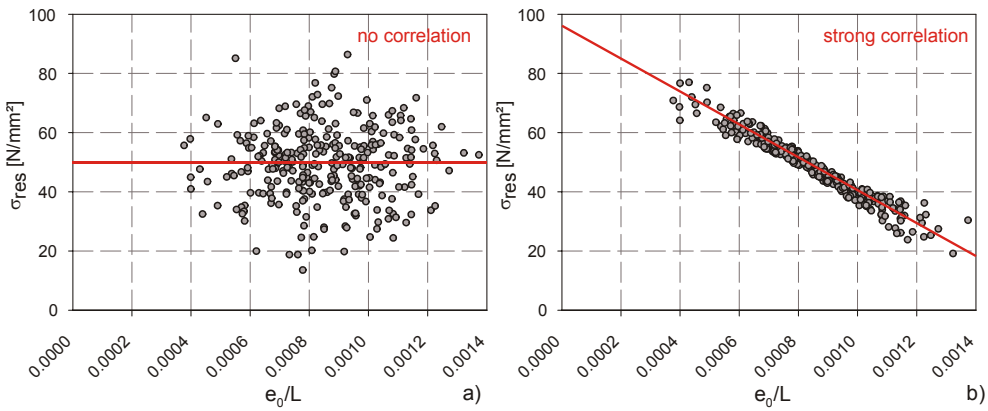


Fig. 7 –Example: two variables that: (a) do not correlate at all; (b) strongly correlate.

5. CONSIDERED INPUT PARAMETERS AND SCENARIOS REGARDING COLUMN STRAIGHTNESS

5.1. CONSIDERED INPUT PARAMETERS

In order to be truly representative of the "real" behavior of physical tests with random input parameters, the numerical calculations in the Monte Carlo simulations need to take into account the scatter of as many parameters as are necessary to represent the scatter of the resulting buckling strengths with sufficient accuracy. In this paper, the case of flexural buckling of a hot-rolled IPE-section about the weak axis is documented; for this case, the main cross-sectional parameters (width, depth, flange and web thickness) as well as the yield strength are of great significance. It is thus important to make use of representative information regarding the scatter band of these – and other related – quantities.

Some very valuable studies [15–17] have already been published that deal with a systematic statistical analysis of the properties of steel products for structural steelwork. Nevertheless, no final consensus seems to have been reached at the present stage as to what set of structural parameters can be regarded as truly representative for the statistical properties of internationally manufactured steel products that are placed onto the European market. Another early application of the general approach used in this paper, which also contained representative input parameters for the buckling of I-sections, is documented in [18].

As was stated in section 4.1, a representative I-section geometry had to be chosen for the Monte Carlo simulations of this paper, and the *IPE 160 section made of S235 steel* was selected for this purpose.

On the basis of the above-mentioned studies, scatter bands of the parameters as shown in Table 2 were considered in this study, in addition to four scenarios regarding the scatter of the member out-of-straightness discussed in section 5.2. The normal or log-normal (ln) distribution was assumed in all cases. These values represent a lower-bound "consensus" for the considered quantities. Sometimes even higher mean values of the yield strength of S235 steel are mentioned in the literature [17]. However, one must consider that the product standard for hot-rolled steel products, EN 10025, only mentions "minimum" values of yield strength, with no mention of a required scatter band or non-exceedance probability, meaning that steel production could in fact comply with this standard without much "over-strength" of the scatter band of the yield stress; thus, it is prudent to use scatter band values for the yield strength that are not exceedingly optimistic.

Table 2
Parameter variation for the Monte Carlo simulations – IPE 160 – S235

Parameter	mean value m	standard dev. s
Initial curvature e_0	See scenarios Fig. 8	
Yield stress f_y	285 N/mm²	17.1 N/mm²
Eccentricity	0.60 mm (ln)	0.45 mm (ln)
Residual stress σ_{res}	0.20–235 N/mm²	0.05–235 N/mm²
Flange thickness t_f	7.4 mm	0.37 mm
Web thickness t_w	5.0 mm	0.25 mm
Depth h	160 mm	1.6 mm
Width b	82 mm	0.82 mm

5.2. SCENARIOS REGARDING OUT-OF-STRAIGHTNESS

In order to assess the possible impact of changes to the production habits for steel compression members that may (plausibly) be caused by the loosening of straightness tolerances in EN 1090-2, four different scenarios were considered in this study. They represent four different assumptions regarding the scatter of the amplitude of the column out-of-straightness. They are represented in Fig. 8 which shows the histograms of the values of the initial column curvature (with index $\overline{e}_0$ – instead of Δ – used to indicate that a sinusoidal "equivalent" imperfection shape

was assumed in the GMNIA calculations) which were randomly generated for the Monte Carlo simulations. All scenarios are referred to the best-documented (by tests and calculations) case of the weak-axis flexural buckling of IPE 160 sections made of steel grade S235, see section 5.1. Scenario 1 (Fig. 8a) is used as reference for the other 3 scenarios, thus the histogram of this scenario is plotted in light grey in Fig. 8b to d. The red continuous lines in the plots shown the assumed, underlying probability density functions, scaled to match the total frequency of produced "numerical tests", while the light-blue histograms show the randomly produced input data. The vertical, dashed red lines show the locations of the tolerance limits of $L/1000$ and $L/750$ in comparison with the produced data.

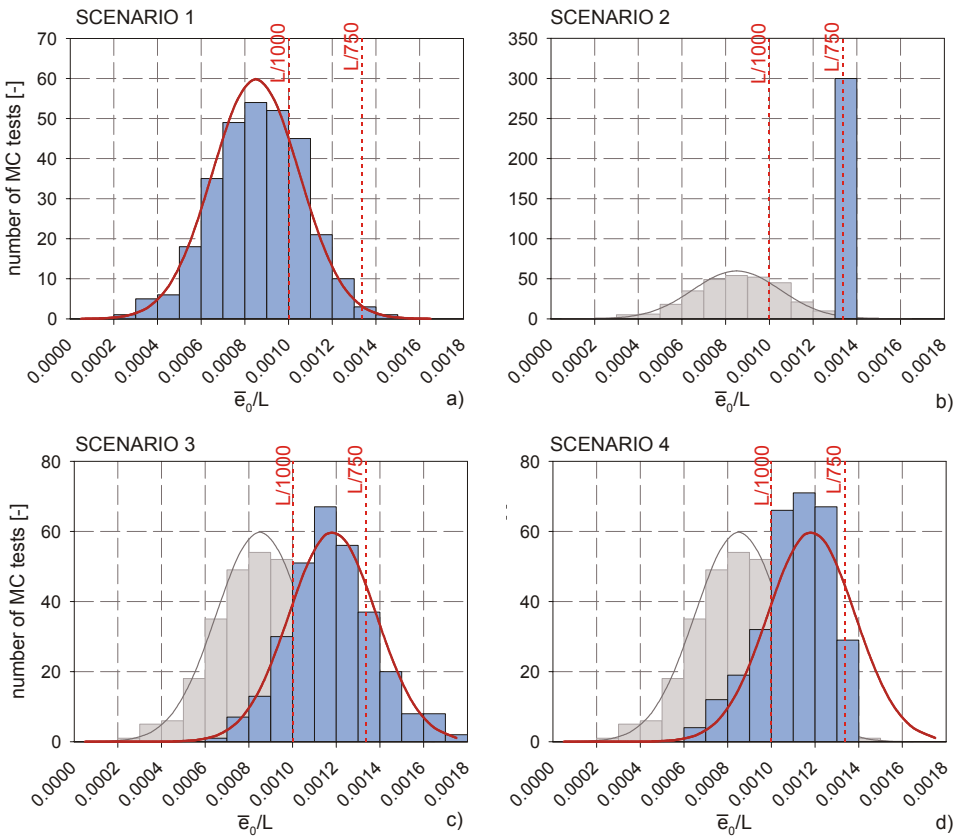


Fig. 8 – Scenarios for the future development of the scatter band of column curvatures.

The scenarios can be described as follows:

– **SCENARIO 1**. The production habits of steelwork fabricators are not altered by the new manufacturing and erection tolerances. The deviations found during the ECCS tests are thought to be representative of real production; thus, the probability density function (pdf) for $\Delta \sim \bar{e}_0$ found in [18] on the basis of the test results in [6] is assumed to be correct and still valid: $m = 0.00085\ L$, $s = 0.0002\ L$.

– **SCENARIO 2**. All columns are produced with an initial curvature of $\Delta \sim \bar{e}_0 = L/750$. Although very unrealistic, this scenario is the most unfavourable possibility still in full compliance with the new tolerance limits in EN 1090-2.

– **SCENARIO 3**. Steelwork fabricators make use of the new, relaxed tolerances and produce columns that are on average somewhat more curved, i.e. by a shift of $L/750–L/1000 = L/3000$ towards the right in Fig. 8c. The scatter of the production is otherwise left unmodified. No measurements are undertaken, neither in the shop nor on site, therefore columns with curvatures exceeding the erection tolerance of $\Delta = L/750$ are not prevented from being used in the structure.

– **SCENARIO 4**. This final scenario is almost identical to 3, with the addition that all columns are thought to be measured on site. Columns that exceed the erection tolerance value of $\Delta = L/750$ are not allowed; in practical terms, a new random value was generated in this case until $\Delta \sim \bar{e}_0 < L/750$.

6. RESULTS

The results of the statistical analysis are shown in Fig. 9 to Fig. 12 for the four different scenarios discussed in the previous section. In order to give a more complete und understandable picture of the outcome of these simulations, the $\chi–\bar{\lambda}$ form of representation with $m - 2s$ bars representing the statistical distribution (mean value m +/– $2s$ standard deviations s) of the single numerical "test" results is complemented by a plot showing the result of a reliability assessment in accordance with EN 1990 – Annex D, i.e. following section 4.2 of this paper up to Eq. 22. This assessment was performed using the same input parameters of the statistical data as contained in Table 2, and is shown in terms of the required values of the partial safety factor γ_M^* (note: the * is used to differentiate this calculated value from the "normative" value γ_{M1} found in Eurocode 3). This factor gives a clear indication of the impact of the single scenarios on the safety level of the column buckling rules, especially when compared to the current reliability level discussed in section 3.1 and shown in Fig. 3.

The figures can be commented upon as follows:

– **SCENARIO 1**. Figure 9 shows the position of the "$m - 2s$" points to lie very close to the applicable ECCS column buckling curve b. This would not represent any noticeable change with respect to the current reliability level, see Fig. 3. The maximum value of γ_M* that was calculated by following the procedure of EN 1990 is very similar to the one calculated by Müller [19] for the ECCS tests (1.115 *vs*. 1.13). This is not surprising, since Scenario 1 simulates the case where the initial curvatures are left "untouched" by the changes of tolerance.

– **SCENARIO 2**. Figure 10 shows the position of the "$m - 2s$" points to lie noticeably lower than the applicable ECCS column buckling curve b. The difference is most pronounced in the region of intermediate slenderness, around

$\bar{\lambda} = 1.0$, where the "$m - 2s$" points are very close to the line representing curve c. The maximum value of γ_M* has increased to 1.177 in this scenario.

– **SCENARIO 3**. Figure 11 where the "$m - 2s$" points start falling significantly below the applicable buckling curve at a slenderness of $\bar{\lambda} = 0.8$. In terms of γ_M*, a maximum value of 1.161 is calculated for this scenario.

– **SCENARIO 4**. Figure 12 again, the "$m - 2s$" points start falling significantly below the applicable buckling curve at a slenderness of $\bar{\lambda} = 0.8$. In terms of γ_M*, a maximum value of 1.155 is calculated, only slightly lower than for scenario 3.

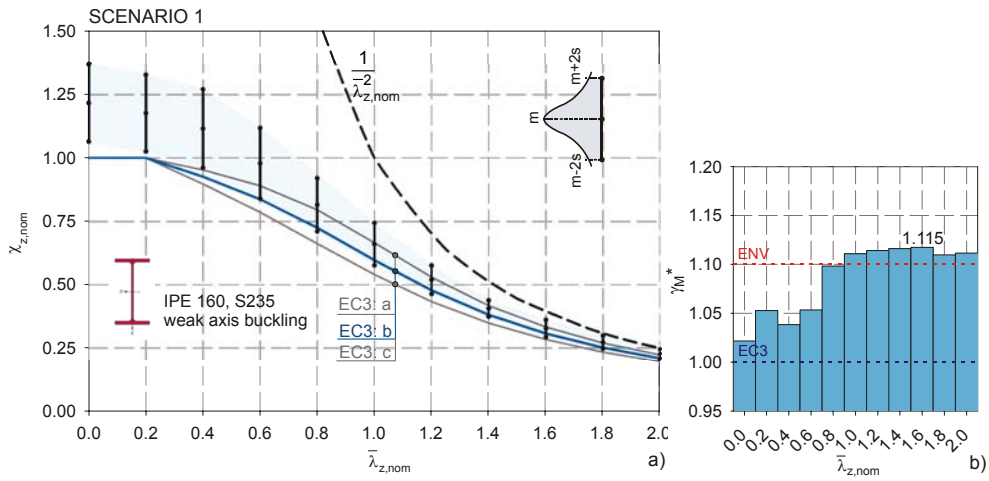


Fig. 9 – Monte Carlo simulation of FBz-z of an IPE 160 section – S235: SCENARIO 1; statistical distribution of the simulated tests in a χ-$\bar{\lambda}$ plot (a); reliability analysis acc. to EN 1990 (b).

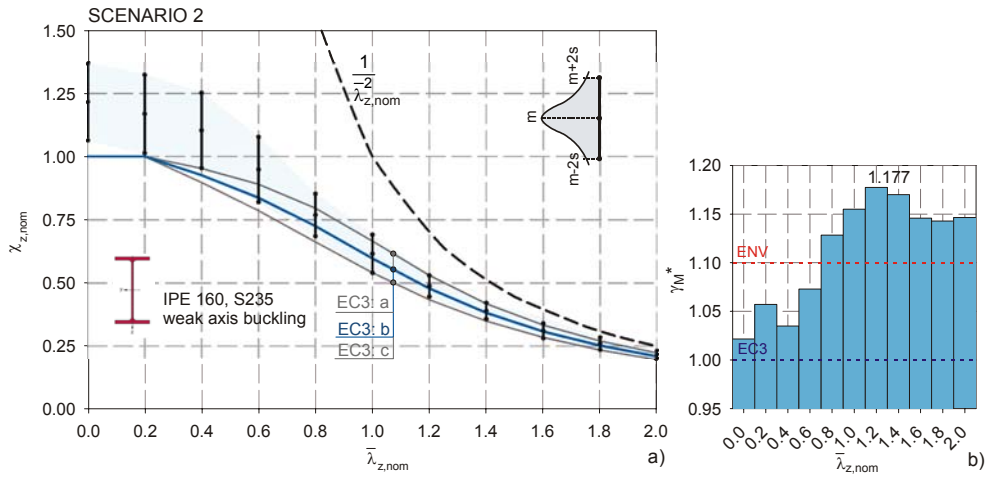


Fig. 10 – Monte Carlo simulation of FBz-z of an IPE 160 section – S235: SCENARIO 2.

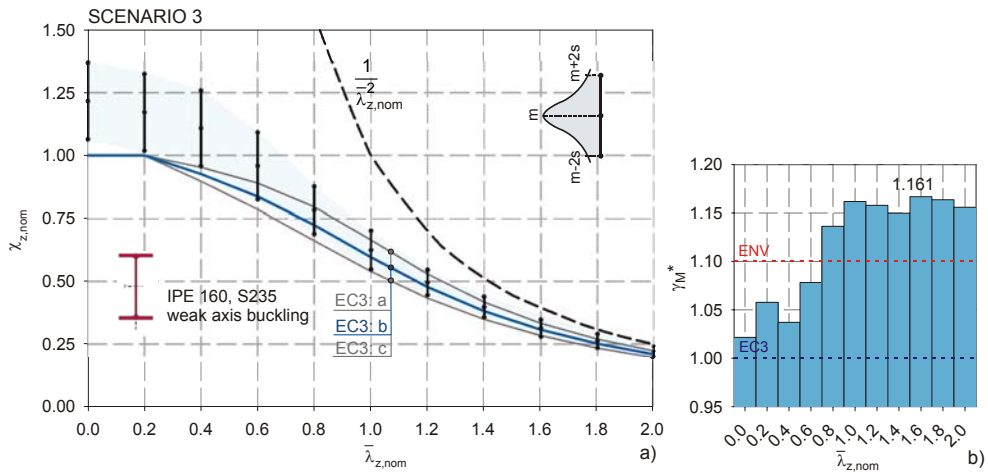


Fig. 11 – Monte Carlo simulation of FBz-z of an IPE 160 section – S235: SCENARIO 3.

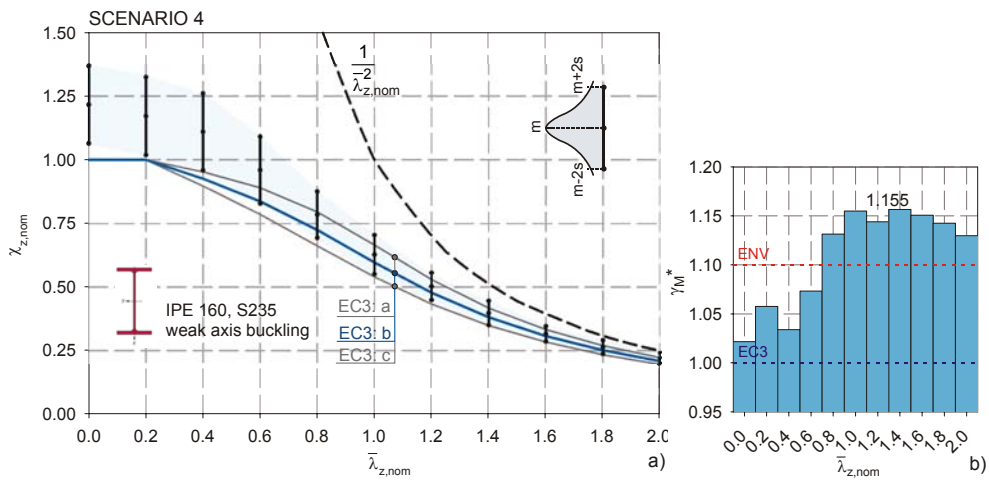


Fig. 12 – Monte Carlo simulation of FBz-z of an IPE 160 section – S235: SCENARIO 4.

7. DISCUSSION OF THE RESULTS

The results of the Monte Carlo simulations shown in Figs. 9 to 12 point out that the reliability level of column buckling *could be affected by the introduction of new tolerance limits*. The scenarios 2 to 4 all treated the possibility that columns produced in the future have – on average – larger initial curvatures than was customary up to now, while all other parameters were left (in terms of scatter band) unmodified.

Thus, it is not surprising that the calculations led to an (average) *loss of column strength*, respectively of margin of safety. The magnitude of this loss is in the order of 4 to 6%.

As far as the likelihood of the single scenarios is concerned, it must certainly be admitted that scenario 2 is not plausible, since it cannot be expected that all steelwork fabricators will produce columns that have a value of out-of-straightness of exactly $\Delta = L/750$. However, this extreme scenario is still interesting when compared to the (much more plausible) scenarios 3 and 4: these scenarios treat the possibility that fabricators "relax" their fabrication habits with respect to column curvatures, an outcome that, even if not *necessary*, must certainly be seen as *desired* by the new tolerance specifications. The calculations in this section have shown the differences between the "extreme" scenario 2 and the "desired" scenarios 3 and 4 to be rather small. Interestingly, scenario 4 is plausible even if every single column is verified for initial curvature on site and "replaced" if curvatures above $\Delta = L/750$ are detected. Compared to the (similar) scenario 3, where no such measurements are undertaken, the advantage stemming from these measurements is negligible.

It is important to understand that it is often *not* the purpose of geometric *erection* (i.e. on-site) tolerance limits to specify the *extreme upper values* of shape deviations in the erected structure, which must never be exceeded in order to design a structure safely with a given set of design rules. On the contrary, as the presentation (in section 2, Fig. 1) of the acceptance criteria in the 1978 ECCS recommendation has shown, the philosophy with respect to tolerances on site was that *single values* above the specified tolerance (in that case, still $\Delta/L = 1/1000$) are acceptable – as long as the *statistical distribution* of these deviations is kept in check, and the production in the shop is confirmed to produce columns within the (shop) limits for Δ. Other international codes implicitly followed a similar philosophy, by assuming that site measurements were entirely unnecessary if $\Delta/L = L/1000$ was checked in the shop, since the statistical distribution of values on site could then be assumed to be acceptable.

Of course, this study only discussed some, possibly "too pessimistic" scenarios. The correlation between tolerance limits and actual deviations is not always quite as clear-cut as has been assumed here. To name one example, Chan & Gardner [4] have found that the tolerance limits of $\Delta = L/500$ for the initial curvature of cylindrical hollow sections "may be unduly lax as evidenced by both the observed structural performance and measured imperfections of real columns". In other words, they observed a poor correlation between actual shape deviations and tolerance limit. They then justified the use of the current buckling rules for such sections, which are also based on GMNIA calculations with $\overline{e}_0/L = 1/1000$ and fixed residual stresses, by the actual statistical distribution of the shape

deviations. In principle, this is not quite unlike Scenario 1 discussed above: fabrication habits don't "acknowledge" actually laxer fabrication tolerances. The conclusion in [4] is coherent: a reassessment of the tolerance limit towards more realistic, lower levels, as opposed to adjusting buckling curves to match current tolerance levels.

Other arguments could be brought up to declare the scenarios discussed above as too pessimistic: for example, residual stress distributions could be lower now than back in the 1960s and '70s, when the ECCS tests were carried out. Then again, actual column imperfections might be smaller than the deviations measured in the ECCS program. Finally, as stated before, a beneficial inverse correlation between Δ and σ_{res} (i.e. σ_{res} is systematically lower when Δ is large) is sometimes thought to exist. These are valid points, but purely speculative at this stage; practically nothing is known about real column curvatures in buildings, and the knowledge and data about residual stresses in mill- or shop-fabricated column sections has also not yet reached a satisfying level in terms of statistical representativeness.

8. CONCLUSIONS

On the basis of the above findings, it can be concluded that the new manufacturing and erection tolerances for compression members, as contained in EN 1090-2, cannot be logically proven to be "covered" by current buckling rules. On the contrary, the *implicit intent* of the new limits, i.e. allowing steelwork fabricators to loosen their fabrication habits with regard to column straightness, has been shown to (plausibly) lead to a drop of the reliability level of column buckling rules by about 5%. Whether this is acceptable or not should be carefully considered by the concerned code committees. In this sense, considering the already present differences between (published, and confirmed in this paper) evaluations of the "appropriate" safety factor for column buckling γ_M* (i.e. values of around 1.1) and the actual code recommendation for compression members in buildings (γ_{M1}=1.0), the implications of further potential loss of 5% of safety must be considered with particular care.

As a more general conclusion, it was shown that the methodology presented in this paper is a suitable tool for the assessment of the influence of parameter scatter bands on the safety level of design rules for steel structures. It can be applied to problems similar to the one studied in this paper, i.e. studies of the impact of changes of production habits, as well as for product and design rule development. Further applications of this method are discussed in [20].

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