PART II: DESIGN CODIFICATION ORIENTED STUDIES

LATERAL TORSIONAL BUCKLING RESISTANCE OF CASTELLATED BEAMS

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Abstract. Castellated beams are likely used structural elements in building structures especially in deck systems and in steel frames. The castellated beams are usually produced by cutting a traditional I-girder and welded by creating large openings in the web. The web openings can reach 70–80% of the whole web depth, therefore the effect of the holes on the structural behavior and in the design of castellated beams is not negligible. The openings have influence on the shear and on the lateral torsional buckling resistances. Most of the previous investigations in this topic were focusing on the shear buckling resistance determination and there is only a few which dealing with the lateral torsional buckling resistance. Due to the reduced out-of-plane stiffness of the web, however, different failure modes can be relevant for castellated beams comparing to traditional I-girders. In the paper the lateral torsional buckling resistance of castellated beams is studied by advanced numerical model and analysis. The lateral torsional buckling resistances are determined for different specimen geometries covering a wide parameter range. Based on the numerical investigations enhanced design methods are proposed to determine the lateral torsional buckling resistances.

Key words: castellated beam, lateral torsional buckling resistance, web distortion.

1. INTRODUCTION

Castellated beams are likely used in floor and roof systems of building structures and as main girders of steel frames, as illustrated by the examples of Fig. 1. The holes in the web are usually produced by cutting of a hot-rolled I-girder and welded again by increasing the web depth of the original girder. The bending stiffness can be significantly increased by this manufacturing process without any increase in the self-weight of the girder. This fact makes the application of castellated beams an efficient and economical solution compared to traditional welded or hot-rolled I-girders. The commonly used web openings in the practice can reach 70–80% of the whole web depth, what should be considered in the design. It is known, that the lateral stiffness of the web is smaller for the castellated beams than for traditional I-girders due to the web openings, therefore the importance of the distortion in the structural behavior has a larger importance, which can results in decrease in the lateral torsional buckling (LTB) resistance.

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There are numerous previous investigations dealing with the shear buckling and bending resistance determination of the castellated beams, but there are only a smaller number of previous investigations available in the international literature dealing with the LTB resistance. The majority of the previous investigations in this topic are focusing on the critical bending moment for castellated beams and the LTB ultimate behavior is rarely studied in details. The current study has the aim to analyze the complex nonlinear phenomena and to work out proposal for the lateral torsional buckling resistance of castellated beams.

Fig. 1 – Application examples for castellated beams [1, 2].

To investigate the special structural behavior of castellated beams under bending and to study the effect of the web distortion on the LTB resistance a numerical research program is executed. The numerical model is developed and verified based on the results of previous investigations and it is used to determine the critical bending moment and the ultimate resistance for various girder geometries with different openings. The applicability of the general LTB design method of the EN1993-1-1 [3] is studied in details. The previously developed design equations for the determination of the critical bending moment are evaluated and modifications are proposed. The cross-section modulus and the relevant buckling curves to be applied are also determined to ensure safe and economic design. The applicability of the simplified LTB design method of the EN1993-1-1 [3] is also studied and modification factors are developed to consider the special structural behavior.

2. LITERATURE REVIEW

The structural behavior of the castellated beams is a widely investigated research field by different researchers in the last decades. In the first generation of the castellated beams the openings were mainly polygonal, and nowadays most of the openings are circular. Therefore the main part of the current investigations are focusing on castellated beams with circular openings and smaller number of studies are focusing on girders with polygonal openings. The previous research investigations can be separated on three different topics: (i) shear and bending cross-sectional resistance, (ii) shear buckling and (iii) lateral torsional buckling resistance. Large research program was executed on the shear and bending resistance by Chung *et al*. [4] in 2000 and by Liu *et al*. [5] in 2003. Based on their investigations design models were developed for the determination of the bending and shear resistance of the castellated beams, which were based on a Vierendeel mechanism between the openings. Hagen *et al*. [6], [7] investigated the bending and shear capacity in 2009 and design methods were proposed to determine the bending and shear resistances and for the M+V interaction.

Shear buckling behavior was investigated by Shanmugan *et al*. [8] in 2002 and by Soltani *et al*. [9] in 2012. Numerical research program was performed to investigate the shear buckling resistance by various opening geometries and sizes. Tsavdaridis [10] highlighted the importance of the opening geometry on the shear buckling behavior in 2012 and demonstrated different Vierendeel mechanisms using various opening geometries.

Investigations dealing with the LTB behavior of castellated beams was started by Nethercot and Kerdal [11] in 1982. The structural behavior of the castellated beams were compared to the conventional I-girders and it was concluded, that the LTB behavior of the castellated beams are similar to the conventional I-girders. If the cross-section properties are calculated based on the smallest net section geometry, the application of the standard design methods leads to safe design. Mohebkhan in 2004 [12] made numerical investigations dealing with the effect of the bending moment diagram on the critical bending moment. The conclusion was that the modification factor due to the bending moment diagram is not constant, it depends on the slenderness of the analyzed beam. Zhirakian *et al*. [13–15] made extensive research on this research field between 2006 and 2012. The structural behavior of the castellated beams was studied and the effect of web distortion on the load carrying capacity was described. The lateral displacements and the deformed shape of the web was studied and characterized based on the web distortion. The conclusion was that the distortion of the web has influence on the LTB failure mode of the castellated beams, but the resistance can be approximated by the design method used for conventional I-girders by several modifications. Sweeden [16] executed a large numerical research program in 2010 to determine the critical bending moment for castellated beams under different loading conditions and girder geometries. Based on the study a modification factor (χ_{LB}) is proposed to consider the effect of the loading condition, the bending moment diagram, the opening sizes and the distance between the openings. Ellobody [17] performed a numerical investigation in 2011 and determined the LTB resistance for more than one hundred castellated beams with different geometries and opening sizes. Ellobody also studied the application possibilities of high strength steel for castellated beams, and he found that it is an efficient solution. Furthermore the calculated LTB resistance was also compared to standard design models and it was found that if the observed failure mode is the combination of LTB and web distortion the standard design method developed for conventional I-girders may lead to unsafe design. If the web distortion has negligible effect on the failure mode, the LTB resistance can be determined on the same way as for conventional I-girders. Lakušić *et al*. [18] executed an experimental research program on 5 test specimens in 2008 and the test results were extended by numerical simulations. Based on these studies it was concluded that the buckling curves according to the EN1993-1-1 [3] can be applied also for castellated beams with minor changes. Since the castellated beams are produced mainly form hot-rolled sections by cutting and welding, the buckling behavior can be classified in between the traditional welded and hot-rolled girders.

Based on the literature review two major conclusions can be drawn, as follows. Most of the previous investigations were executed in the last 10 years, what shows the actuality of the problem. The majority of the previous studies describe the special structural behavior of the castellated beams, classify the observed failure modes. But there are only a limited number of previous studies dealing with design method development. Beside that all the previous studies dealing with the design of castellated beams are focusing on the determination of the critical bending moment and on the applicability of the general LTB design method of the EN1993-1-1 [3]. No investigations are found by the authors dealing with the simplified LTB method for the application of castellated beams.

The aims of the current investigations are (i) to study the applicability of the available research results in a larger parameter range, (ii) to improve the previously developed general LTB resistance models for castellated beams and (iii) to study the applicability of the simplified LTB design method of the EN1993-1-1 [3] for castellated beams.

3. NUMERICAL MODELING

In the first phase of the research a numerical model is developed and verified on the basis of previous results. Using the models the critical bending moment is determined by bifurcation analysis (GNB) and the LTB resistance is calculated by nonlinear FE simulation (GMNI). The numerical model is developed in Ansys [19] environment by 4-node-thin shell elements. Linear elastic material model is used in the GNB and linear elastic-hardening plastic material model using isotropic hardening rule with von-Mises yield criterion is applied in the GMNI analysis with the characteristics as follows: Young's modulus 210000 MPa; the yield plateau is modeled to 1% strains and from the yield stress it follows linear hardening with a reduced modulus until reaching the ultimate stress by 15% strain level. At the ends of the analyzed girders simply supported conditions are used, allowing warping and rotation at both ends; no internal lateral supports are applied.

In the study three loading conditions are analyzed: (i) uniformly distributed load along the whole length, (ii) concentrated force at the mid-span, and (iii) concentrated bending moments in the ends. Three load positions within the cross-section are investigated, namely the load is placed in the (i) upper flange, (ii) center of gravity and (iii) lower flange. The numerical model is verified based on the investigations of Sweeden [16] by the comparison of the calculated critical bending moments. The results of the numerical simulations are also compared to the test results of Showkati *et al*. [15]. Both comparisons showed good agreement with the published results. The typical failure mode of the analyzed girders can be seen in Fig. 2. It can be observed that the deflected shape of the cross-section shows significant distortion of the web, what should be considered in the LTB resistance calculation.

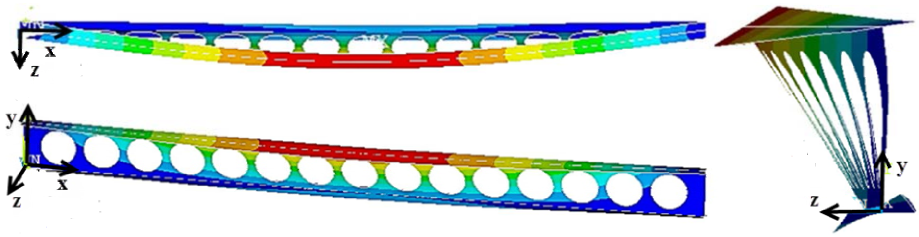


Fig. 2 – Observed failure mode.

4. NUMERICAL RESEARCH PROGRAM

The numerical research program is completed with two aims. The first one is to determine the critical bending moment of the analyzed girders and the second one is to determine the LTB resistance of the castellated beams with different geometries. The analyzed parameters are the followings: (i) diameter of the holes (*d*), (ii) distance between the openings (*s*), (iii) flange width and thickness (b_f; t_f); web depth and thickness (h_w; t_w), (iv) span of the girder (*L*), (v) shape of the holes (diagonal or polygonal), (vi) load type (uniformly distributed, concentrated, end-moment), (vii) load position (upper flange, center of gravity, lower flange). The notations are shown in Fig. 3 and the investigated parameter range can be seen in Table 1.

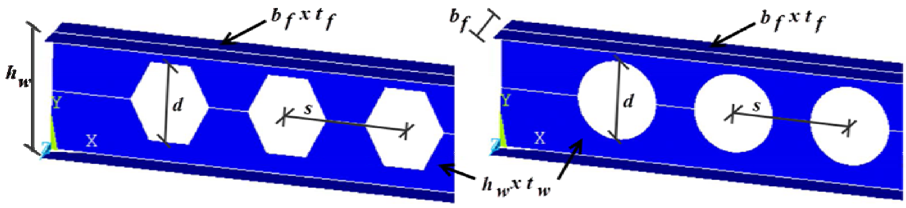


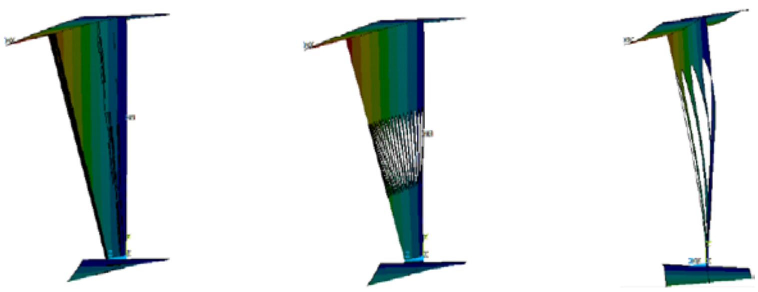
Fig. 3 – Notations.

The parameters are determined on the bases of typical girder geometries used in the practice and by the product description of manufacturers producing castellated beams. In the cases of the diameter of the holes (*d*) and the distance between the openings (*s*) the parameter range is extended in order to analyze the distortional phenomenon in details. The total number of studied girder geometries is 240. For each girders the critical bending moments and the LTB resistances are determined with and without openings, in order to investigate the reduction due to the openings.

Table 1
Parameter range

parameter	*investigated parameter range*
d / h_w	0,3 – 0,4 – 0,5 – 0,6 – 0,7 – 0,8
s / h_w	1,2 – 1,4 – 1,6 – 1,8 – 2,0 – 2,5 – 3,0 – 4,0
b_f / t_f	10 – 12 – 14 – 16 – 20 – 30
h_w / t_w	40 – 50 – 60 – 70 – 80 – 90
L / h_w	7,5 – 10 – 12,5 – 15 – 20 – 25
opening shape	circular, polygonal
load position	upper flange, center of gravity, lower flange
load type	uniformly distributed, concentrated, end-moment

In the study three typical failure modes are observed, as shown in Fig. 4. In the absence of holes the failure mode is LTB with quasi-straight deflected webs (Fig. 4a). By increasing the diameter of the openings the web distortion becomes more critical and it results in decrease in the LTB resistance, as shown in Fig. 4b, c.



a, no holes b, d/h_w=0,2 d, d/h_w=0,8

Fig. 4 – Typical failure modes.

Based on the parametric study the following conclusions can be drawn:

1. The opening size has significant influence on the failure mode of the castellated beams since the web distortion becomes more critical by increasing the opening size. Figure 5 shows the relationship between the opening size and the resistance reduction factor (factor 1,0 refers to the girder without openings). It is

concluded that the critical bending moment and the LTB resistance decreases quasi-linearly by increasing the opening size. The resistance decrease for girders with larger web depth is larger, as shown in Fig. 5, what can be explained by the effect of web distortion. Webs can give more efficient lateral support in the case of girders with relatively smaller web depth.

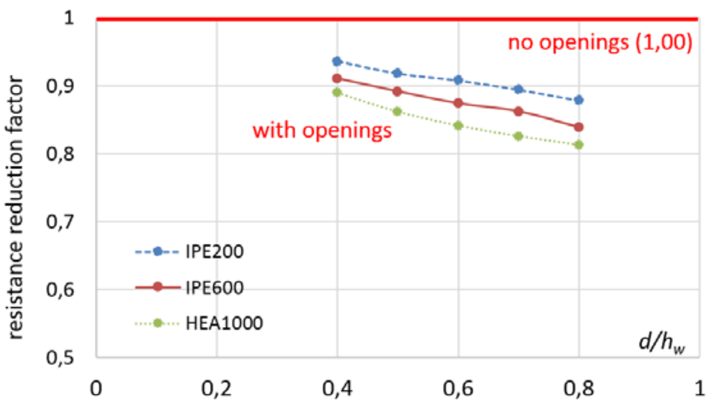


Fig. 5 – Effect of d/h_w on the resistance reduction due to openings.

2. It is observed that the distance between the holes has also a quasi-linear effect on the critical bending moment and on the LTB resistance, as shown in Fig. 6. The resistance reduction for three girders with two opening sizes are presented in Fig. 6. The dashed lines show the resistance decreases of the girders with smaller openings and the continuous lines represent the results of larger openings. The resistance reduction tendencies are the same, the values depend on the d/h_w; h_w/t_w; b_f/t_f ratios.

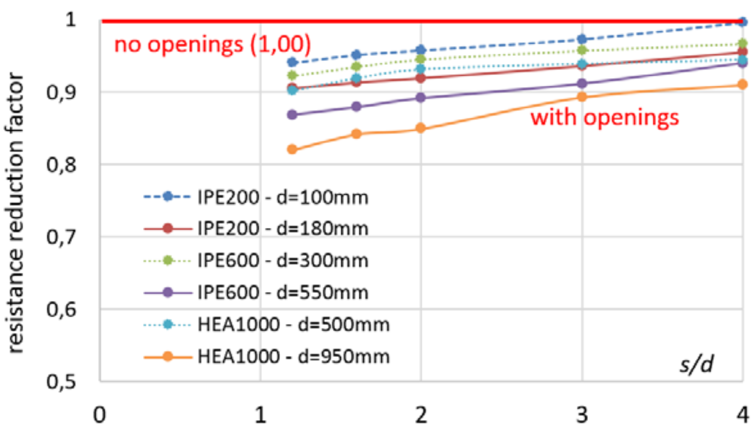


Fig. 6 – Effect of s/d on the resistance reduction due to openings.

3. The analysis of the cross-section geometry (b_f/t_f; h_w/t_w) showed that the tendencies are the same as for the conventional I-girders without web openings but on a smaller resistance level. This observation proves that the general design method of the Eurocode standard can be applied also for castellated beams with changes taken the effect of the holes into account.

5. LTB RESISTANCE DETERMINATION METHOD

In the LTB design method of the castellated beams the calculation of the cross-section resistance and the critical bending moment have the main interest. The basic principle of the slenderness calculation suggest to use the cross-section modulus which results in the largest normal stress in the flanges. If the holes are equally spaced along the girder length and the slope of the bending moment diagram between the holes is not significant, it can be assumed, that the maximum stress can be approximated by using the net section properties, as proposed by Sweeden [16]. In this proposal the modification factors taking the effect of the holes in the critical bending moment into account. The results of the Sweeden's design method are compared to the current numerical results. It is observed, that the proposed modification factors give good approximation in a specific parameter range. But the calculated critical bending moments according to Sweeden's [16] design method does not follow all the tendencies observed in the numerical calculations (e.g. for the d/h_w ratio). Therefore the calculation method of the critical bending moment (M_{cr}) is investigated in details and refined according to the current results. The basis of the method is the equation of the M_{cr} developed by Sweeden and the proposed enhanced design method is the following: the LTB design resistance of the castellated beam can be determined by Eq. (1).

$$M_{b,Rd} = \chi_{LT} \cdot \frac{W_y \cdot f_y}{\gamma_{M1}}, \tag{1}$$

where: W_y is the cross-section modulus of the net section according to the classification made on the original cross-section without openings, and χ_{LT} is the reduction factor using the buckling curve "*c*" according to EN1993-1-1 [3].

The slenderness of the castellated beam can be calculated by Eqs. (2)–(9).

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}}, \tag{2}$$

$$M_{cr} = \kappa_{CB,p} \cdot \kappa_{CB,\beta} \cdot \kappa_{CB,L} \cdot \kappa_{LB} \cdot M_{0,cr}, \tag{3}$$

where: $K_{CB,p}$ depends on the load position: upper flange: 0,7; center of gravity: 1,00; lower flange: 1,4;

$K_{CB,p}$ depends on the load type: uniformly distributed load: 1,0; concentrated force: 1,21; end-moments: 0,95.

$$\kappa_{CB,L} = \left(\frac{L}{15 \cdot h_w}\right)^{0.25}, \tag{4}$$

$$\kappa_{LB} = \frac{-0{,}03}{\psi \cdot \frac{b_f}{t_f}} \cdot \frac{h_w}{t_w} + \left[1{,}21 - 0{,}002 \cdot \left(\frac{b_f}{t_f} - 10\right)\right] \cdot \sqrt{\psi} \tag{5}$$

$$\psi = \left[\kappa_{CB,Hb} + 0{,}25 \cdot \frac{s}{h_w} - 0{,}07 \cdot \left(\frac{s}{h_w}\right)^{1,5}\right] \cdot \kappa_{CB,wD}, \tag{6}$$

$$\kappa_{CB,Hb} = 0{,}172 \cdot \frac{h_w}{b_f} + 0{,}248, \tag{7}$$

$$\kappa_{CB,wD} = 3 \cdot \left(\frac{h_w}{t_w}\right)^{-0{,}08 \cdot \frac{d}{h_w} - 0{,}24}, \tag{8}$$

$$M_{0,cr} = \frac{\pi^2 \cdot E \cdot I_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}}. \tag{9}$$

The notations in Eqs. (4–8) are shown in Fig. 3; the notations used in Eq. (9) are the same as given in the EN1993-1-1 [3] with the only difference, that the value of I_t and I_z should be calculated based on the net cross-section geometry. Using the results of Eq. (3) and the numerical simulations the reduction factors for the analyzed girders are presented in Fig. 7.

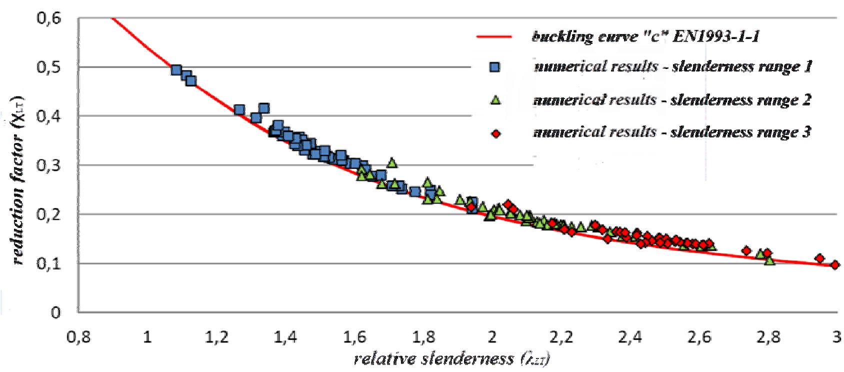


Fig. 7 – Calculated reduction factors.

The results show that the buckling curve "*c*" of the EN1993-1-1 [3] gives a good approximation of the numerical results for castellated beams in the analyzed parameter range. These results prove the conclusions of Lakušić *et al*. [18], that for castellated beams with $h/b_f > 2$ the buckling curve "*c*" and with $h/b_f < 2$ the buckling curve "*b*" could be used. Note that in the case of the current investigations the h/b_f ratio is larger than 2 for all the analyzed specimens. Accordingly they belongs to the buckling curve "*c*" as proposed by Lakušić *et al*. [18]. It can be concluded that the current numerical study proves the applicability of this proposal. Figure 8 shows the comparison of the critical bending moment and the LTB resistance between the enhanced design method and the numerical results. It can be seen on the diagrams, that the calculated values with the proposed enhanced design method gives a good approximation to the numerical results. The average ratio between the results of the developed design method and the numerical calculations is 0,927 for the critical bending moment and 0,909 for the lateral torsional buckling resistance.

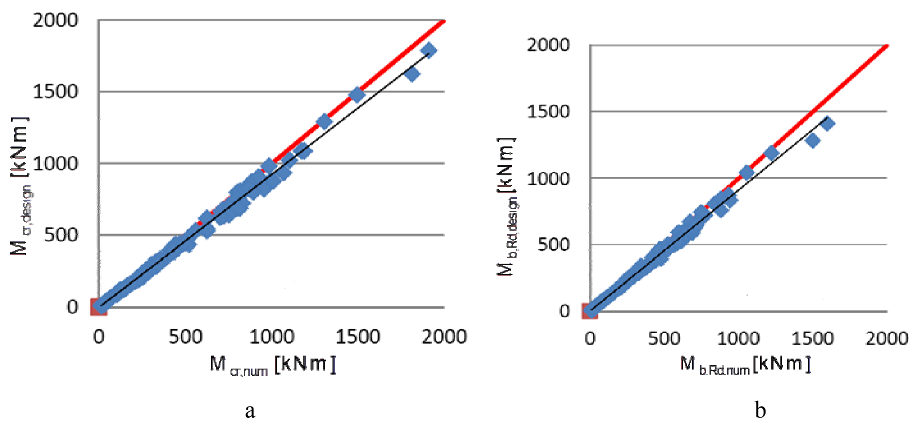


Fig. 8 – Comparison of the developed design method and numerical results: a) critical bending moment; b) LTB resistance.

6. SIMPLIFIED METHOD FOR LTB RESISTANCE CALCULATION

In the simplified LTB design method the equivalent compression flange is created from the whole flange and from a certain part of the compressed web. The simplified LTB design method is an efficient and accurate tool in the case of lateral distortional buckling failure modes, too, and therefore the applicability for castellated beams is studied. The holes slightly reduce the area of the equivalent compression flange, therefore the consideration of the holes is implemented in the cross-section modulus. For this purpose an equivalent cross-section is introduced having the same web depth as the original castellated beam with an average hole size calculated by Eq. (10).

$$h_{eq} = \left(1 - \frac{n \cdot \frac{d^2 \cdot \pi}{4}}{L \cdot h_w} \right) \cdot \frac{h_w}{2} \tag{10}$$

where: n – total number of the holes along the girder length (L); d – hole diameter.

The geometrical presentation of the equivalent girder geometry is shown in Fig. 9.

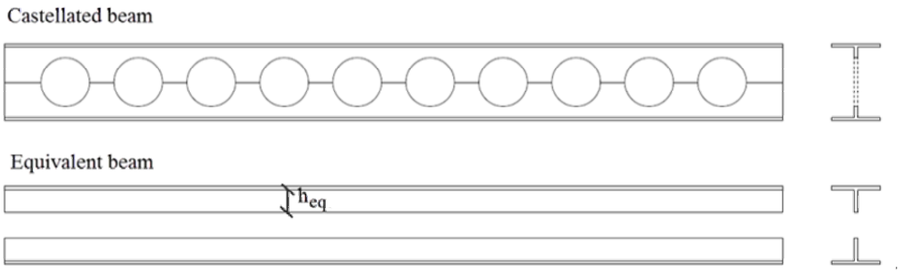


Fig. 9 – Original and equivalent beam geometries.

The cross-section modulus (W_{eq}) of the equivalent beam should be determined and the LTB resistance can be calculated using this value in Eq. (11). On the basis of the numerical results the LTB resistance of the castellated beams can be determined by the same way as for the conventional I-girders with the modifications described in the Eqs. (11) – (14).

$$M_{b.Rd} = \kappa_{CB,p} \cdot \chi \cdot W_{eq} \cdot \frac{f_y}{\gamma_{M1}} . \tag{11}$$

The original LTB calculation method is extended by the factor $\kappa_{CB,p}$, considering the effect of the load position, determined by:

– 0,8 if the upper flange,
– 1,1 if the center of gravity and
– 1,4 if the lower flange is loaded.

The slenderness of the equivalent compressed T-bar can be calculated by Eqs. (12–14). The modification factors of $\kappa_{CB,L}$ and $\kappa_{CB,w}$ takes the effect of the span and the cross-section geometries into account. The value of k_c can be determined according to the EN1993-1-1 [3].

$$\bar{\lambda}_f = \frac{k_c \cdot L_c}{i_{fz} \cdot \lambda_1} \cdot \kappa_{CB,w} \cdot \kappa_{CB,L} , \tag{12}$$

$$\kappa_{CB,L} = \left(\frac{L}{15 \cdot h_w} \right)^{-0,5} \qquad \kappa_{CB,w} = \left(\frac{h_w}{60 \cdot t_w} \right)^{0,1} . \tag{13–14}$$

The calculated resistances according to the enhanced simplified LTB design method and the numerical simulations are compared and good agreement is presented in Fig. 10. The average ratio between the lateral torsional buckling resistances determined by the developed design method and the numerical analyses is 0,902. Note that the differences are slightly larger than for the general LTB design method, but it is considered acceptable from the practically required accuracy point of view.

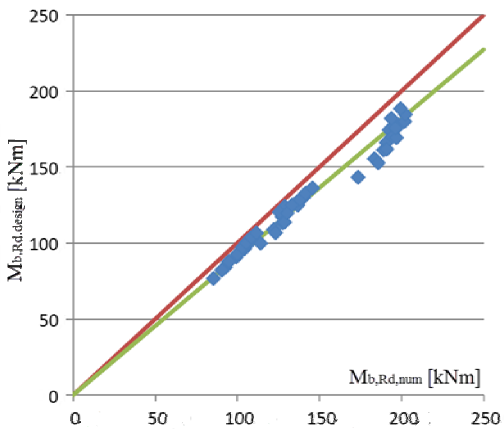


Fig. 10 – Comparison of the enhanced simplified LTB design method and the numerical results.

7. SUMMARY AND CONCLUSIONS

The lateral torsional buckling behavior of the castellated beams are investigated in the current paper. More than 240 girder geometries are studied: girders with and without openings, geometrical and loading conditions are investigated to determine the effect on the LTB resistance. For each analyzed girders the critical bending moment and the LTB resistance are determined. The calculated critical bending moments are compared to the design method of Sweeden [16] and a refinement is proposed to predict M_{cr}. On this basis the applicability of the LTB design method of the EN1993-1-1 [3] for castellated beams is investigated. The investigations proved the applicability of the proposal of Lakušić *et al.* [18] regarding to the applicable buckling curves. The numerical results proved the applicability of the LTB resistance model by the following minor changes for castellated beams.

1. The cross-section properties of the net section should be used in the cross-section modulus determination.
2. The critical bending moment should be calculated according to Eqs. (2)–(9).
3. The buckling curve "*c*" according to the EN1993-1-1 [3] can be applied in the analyzed parameter range ($h/b_f > 2$).

The applicability of the simplified LTB design method for castellated beams is also studied. The results of the numerical simulations proved its applicability by the following changes:

1. The cross-section properties (W_{eq}) of the proposed equivalent section should be used in the LTB resistance calculation.
2. The slenderness of the equivalent compression flange should be determined according to Eqs. (12–14) with minor changes of the slenderness calculation method developed for traditional I-girders.
3. An additional modification factor ($\kappa_{CB,p}$) considering the load position is proposed for castellated beams.

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