DYNAMICS ANALYSIS OF A TWO-MODULE HYBRID PARALLEL MANIPULATOR

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Abstract. Recursive matrix relations for dynamics analysis of a spatial two-module hybrid parallel manipulator are established in this paper. Knowing the relative motions of the moving platforms, the inverse dynamics problem is solved based on a set of recursive explicit equations of parallel robots dynamics. Finally, compact results and graphs of simulation for the input forces and powers of six actuators are obtained.

Keywords: connectivity relations, hybrid parallel robot, dynamics, kinematics.

LIST OF SYMBOLS

$p_{k,k-1}$, $q_{k,k-1}$	– relative orthogonal matrices of transformation
$\vec{u}_1, \vec{u}_2, \vec{u}_3$	– three orthogonal unit vectors
α_j, $j = A, B, C, D, E, F$	– angles giving the position of universal joints
β	– initial inclination of all legs
λ_{32}^j	– relative displacement of T_3^j link
$\varphi_{\tau,\tau-1}^j$	– relative rotation angle of T_τ^j rigid body
$\vec{\omega}_{\tau,\tau-1}^j = \omega_{\tau,\tau-1}^j \vec{u}_3$	– relative angular velocity of T_τ^j
$\vec{\omega}_{\tau 0}^j$	– absolute angular velocity of T_τ^j
$\tilde{\omega}_{\tau,\tau-1}^j$	– skew symmetric matrix associated to vector $\vec{\omega}_{\tau,\tau-1}^j$
$\vec{\varepsilon}_{\tau,\tau-1}^j = \varepsilon_{\tau,\tau-1}^j \vec{u}_3$	– relative angular acceleration of T_τ^j
$\vec{\varepsilon}_{\tau 0}^j$	– absolute angular acceleration of T_τ^j
$\tilde{\varepsilon}_{\tau,\tau-1}^j$	– skew symmetric matrix associated to the vector $\vec{\varepsilon}_{\tau,\tau-1}^j$

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$\vec{r}^{A}_{k,k-1}$ $(k=1,2,3)$	– relative position vector of the centre of A_k joint
$\vec{v}^{A}_{k,k-1}$	– relative velocity of the centre A_k
$\vec{\gamma}^{A}_{k,k-1}$	– relative acceleration of the centre A_k
$m_p, \hat{J}_p$	– mass and tensor of inertia of two moving platforms
$m^j_k, \hat{J}^j_k$	– mass and tensor of inertia of T^j_k rigid body
f^j_{32}, P^j_{32}	– input forces and powers of six prismatic actuators

1. INTRODUCTION

Parallel robots are closed-loop structures presenting very good potential in terms of accuracy, stiffness and ability to manipulate large loads. The links of the robot are connected by spherical joints, universal joints, revolute joints or prismatic joints. Compared with traditional serial manipulators, the compactness, accuracy and precision in the direction of the tasks are essential for the parallel architectures [1–3].

The mechanisms of parallel robots can be found in practical applications, in which it is desired to orient a rigid body in space at high speed, such as flight simulators and positional trackers [4–6].

A hybrid manipulator is a combination of closed-chain and open-chain planar and spatial mechanisms or a sequence of parallel robots [7–11]. A serial-parallel manipulator has several modules with parallel structure that are connected serially. This kind of hybrid architectures posses the advantages of both serial and parallel manipulators from rigidity, accuracy and workspace point of view.

Shahinpoor [12] obtained and solved numerically a system of nonlinear equations for the inverse kinematics problem of a hybrid robotic system consisting of two serially connected parallel manipulators. Cheng *et al.* [13] used a numerical Newton-Raphson method to obtain the solution of the direct kinematics problem of a 10-DOF hybrid redundant manipulator, containing a closed-loop slider-crank mechanism and a parallel-driven mechanism. Considering the lower module as a positional mechanism and the upper as an orientation device, Zhang and Song [14] analyzed the geometry and the position of a hybrid manipulator composed of two serially connected parallel robots, each mechanism having three degrees of freedom. Based on screw theory and principle of virtual work, Gallardo et al. [15] addressed a complete kinematics analysis of a modular spatial hyper-redundant manipulator built with a variable number of serially connected mechanical modules.

Taking into consideration the frictionless revolute and prismatic joints, a recursive method based on explicit equations of parallel robots dynamics is applied in the present paper to the inverse dynamics of a new spatial hybrid manipulator, to prove that the number of equations and computational operations reduces significantly by using a set of matrices for dynamics modelling.

2. KINEMATICS REVIEWS

The hybrid robot here analyzed is made up of two 3-DOF similar parallel modules, which are serially connected to a fixed base [16]. Practically, the degree-of-freedom value $F = 6$ of the hybrid mechanism is equal to the degrees of freedom associated with all the moving links $\nu = 42$ minus the total number of independent constraint relations $l = 36$ imposed by the joints.

We assign the fixed frame $Ox_0y_0z_0$ at the point O and two appropriate mobile frames $G_4x_4^Gy_4^Gz_4^G$ and $H_4x_4^Hy_4^Hz_4^H$ on the moving platforms at their mass centers $G_3 = G_4$ and $H_3 = H_4$. The legs A ($\alpha_A = 0$) and D ($\alpha_D = 0$) of lower and upper modules are initially contained within the Ox_0z_0 vertical plane, whereas the planes of legs B,C and E,F make the angles $\alpha_B = \alpha_E = 120^0$ and $\alpha_C = \alpha_F = -120°$ with Ox_0z_0 (Fig. 1).

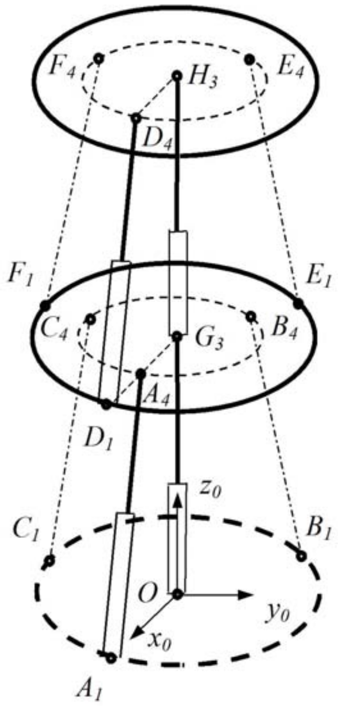

Fig. 1 – Spatial hybrid parallel robot.

The active leg A of lower module, for example, consists of a little cross of a fixed universal joint of mass m_1 linked at the frame $A_1x_1^Ay_1^Az_1^A$ which has the angular velocity $\omega_{10}^A = \dot{\varphi}_{10}^A$ and the angular acceleration $\varepsilon_{10}^A = \dot{\omega}_{10}^A$, connected at a moving cylinder

$A_2x_2^Ay_2^Az_2^A$ of length l_2, mass m_2 and tensor of inertia $\hat{J}_2$, having a relative rotation around $A_2z_2^A$ axis with the angle φ_{21}^A, so that $\omega_{21}^A = \dot{\varphi}_{21}^A$, $\varepsilon_{21}^A = \dot{\omega}_{21}^A$. An actuated prismatic joint is as well as a piston of length l_3, mass m_3 and tensor of inertia $\hat{J}_3$, linked to the $A_3x_3^Ay_3^Az_3^A$ frame, having a relative velocity $v_{32}^A = \dot{\lambda}_{32}^A$ and acceleration $\gamma_{32}^A = \dot{v}_{32}^A$. The fourth constraining chain consists of a prismatic joint attached to the fixed base and a moving link of length l_1, mass m_1^G and tensor of inertia $\hat{J}_1^G$, having a known purely relative vertical displacement λ_{10}^G, velocity $v_{10}^G = \dot{\lambda}_{10}^G$ and acceleration $\gamma_{10}^G = \dot{v}_{10}^G$.

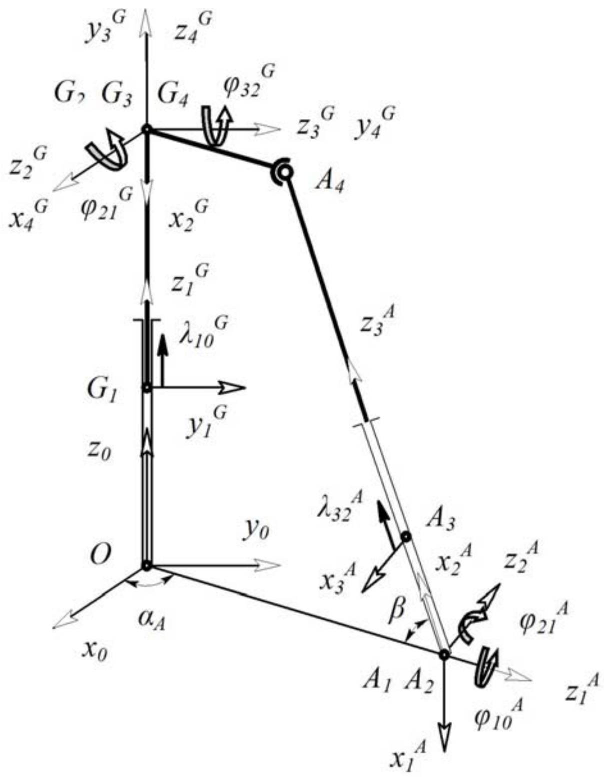

Fig. 2 – Kinematical scheme of first leg *A* of lower module.

A little cross of a new universal joint of mass m_2^G is attached to the center of the first moving platform which can be schematised as a circle of radius r_p, mass m_p and central tensor of inertia $\hat{J}_p$. Two successive concurrent rotations of this platform are defined in the local coordinates $G_2x_2^Gy_2^Gz_2^G$ and $G_3x_3^Gy_3^Gz_3^G$ by given angles $\varphi_{21}^G, \varphi_{32}^G$ (Fig. 2).

Six sliders and two passive central sliders are starting from initial positions $A_2A_3 = B_2B_3 = ... = F_2F_3 = l_5$, $OG_1 = G_3H_1 = l_6$, with following angles

$$\alpha_A = \alpha_D = 0, \alpha_B = \alpha_E = \frac{2\pi}{3}, \alpha_C = \alpha_F = -\frac{2\pi}{3}$$

$$(l_3 + l_5)\cos\beta = l_0 - l_4\,,\ (l_3 + l_5)\sin\beta = l_1 + l_6\,. \tag{1}$$

Considering the passive constraining legs along the *principal kinematical chain* $OG_1G_2G_3G_4H_1H_2H_3H_4$, which connect the moving platforms, relative passing matrices are derived

$$p_{10} = I, \quad p_{21} = p_{21}^{\varphi}\theta_1, \quad p_{32} = p_{32}^{\varphi}\theta_1\theta_2, \quad p_{43} = \theta_2\theta_1\theta_2,$$
$$p_{20} = p_{21}p_{10}, \quad p_{30} = p_{32}p_{20}, \quad p_{40} = p_{43}p_{30} \ (p = g, h), \tag{2}$$

where the angles $\varphi_{21}^i, \varphi_{32}^i, (i = G, H)$ characterise the sequence of concurrent rotations around the universal joints G_2 and H_2 through following matrices

$$p_{\sigma,\sigma-1}^{\varphi} = \mathrm{rot}(z, \varphi_{\sigma,\sigma-1}^i) \quad (\sigma = 2, 3)\,. \tag{3}$$

Now, starting successively from two reference origins O, G_4 and pursuing six legs $OA_1A_2A_3$, $OB_1B_2B_3$, $OC_1C_2C_3$, $G_4D_1D_2D_3$, $G_4E_1E_2E_3$, $G_4F_1F_2F_3$, we obtain new transformation matrices

$$q_{10} = q_{10}^{\varphi}\theta_1 a_{\alpha}^j, \quad q_{21} = q_{21}^{\varphi}a_{\beta}\theta_1\theta_2, \quad q_{32} = \theta_1, \quad q_{20} = q_{21}q_{10}, q_{30} = q_{32}q_{20} \tag{4}$$

$$(q = a, b, c, d, e, f) \quad (j = A, B, C, D, E, F),$$

where we denote the matrices

$$q_{\tau,\tau-1}^{\varphi} = \mathrm{rot}(z, \varphi_{\tau,\tau-1}^j) \quad (\tau = 1,\ 2) \tag{5}$$

$$a_{\alpha}^j = \mathrm{rot}(z, \alpha_j)\,,\ a_{\beta} = \mathrm{rot}(z, \beta)\,,\ \theta_1 = \mathrm{rot}(y, \pi/2)\,,\ \theta_2 = \mathrm{rot}(z, \pi/2)\,.$$

In a forward geometric problem, the orientation and the position of the serial-parallel robotic system is completely given through the angles of rotation $\varphi_{\sigma,\sigma-1}^i$ and the relative coordinates $z_0^i = l_1 + l_6 + \lambda_{10}^i$ of the centers of mobile platforms. Consider, for example, that during three seconds the coordinates of the centers and the angles of orientation can describe the relative motions of the platforms through the following analytical functions

$$\frac{\lambda_{10}^i}{\lambda_{10}^{i*}} = \frac{\varphi_{\sigma,\sigma-1}^i}{\varphi_{\sigma,\sigma-1}^{i*}} = 1 - \cos\frac{\pi}{3}t \quad (\sigma = 2, 3) \quad (i = G, H)\,. \tag{6}$$

Pursuing the kinematical modelling developed in my recently published paper [16], a set of 18 independent variables $\varphi^{j}_{\tau,\tau-1}, \lambda^{j}_{32}$ characterising the kinematics of two modules will be determined from 18 analytical equations.

Using the skew-symmetric matrices $\tilde{\omega}^{i}_{40} = \dot{\varphi}^{i}_{21} p_{43} p_{32} \tilde{u}_3 p^{\mathrm{T}}_{32} p^{\mathrm{T}}_{43} + \dot{\varphi}^{i}_{32} p_{43} \tilde{u}_3 p^{\mathrm{T}}_{43}$, which are associated to the angular velocities $\vec{\omega}^{i}_{40}$ of two moving platforms, we obtain the *matrix conditions of connectivity* [17], and the relative velocities

$$\vec{V}_j = [N_j]^{-1} \vec{P}_j, \quad \vec{P}_j = v^{i}_{10} \vec{u}^{\mathrm{T}}_l \vec{u}_3 + \vec{u}^{\mathrm{T}}_l p^{\mathrm{T}}_{40} \tilde{\omega}^{i}_{40} \vec{r}^{\,j_4}_{i}, \tag{7}$$

where following terms determines the contents of 3×3 invertible square matrix $[N_j]$:

$$n^{j}_{l1} = u^{\mathrm{T}}_l q^{\mathrm{T}}_{10} \tilde{u}_3 q^{\mathrm{T}}_{21} \vec{r}^{\,j}_{42}, \quad n^{j}_{l2} = \vec{u}^{\mathrm{T}}_l q^{\mathrm{T}}_{20} \tilde{u}_3 \vec{r}^{\,j}_{42}, \quad n^{j}_{l3} = \vec{u}^{\mathrm{T}}_l q^{\mathrm{T}}_{20} \vec{u}_1, \quad \vec{r}^{\,j}_{42} = \vec{r}^{\,j}_{32} + q^{\mathrm{T}}_{32} \vec{r}^{\,j}_{43} \tag{8}$$

$$(p = g, h) \ (i = G, H) \ (q = a, b, c, d, e, f) \quad (j = A, B, C, D, E, F) \ (l = 1, 2, 3).$$

Expressions of relative accelerations $\vec{\Gamma}_i = [\varepsilon^{i}_{10} \ \ \varepsilon^{i}_{21} \ \ \gamma^{i}_{32}]^{\mathrm{T}}$ are obtained using new conditions of connectivity $\vec{\Gamma}_i = [Q_i]^{-1} \vec{S}_i$ with the column matrix $\vec{S}_i = \dot{\vec{P}}_i - [\dot{Q}_i] \vec{V}_i$ [16].

3. INVERSE DYNAMICS MODEL

Considering all gravitational effects and neglecting the frictions forces, the relevant objective of the inverse dynamics is to determine the input torques or forces, which must be exerted by the actuators in order to produce a given trajectory of the end-effector.

A lot of works have focused on the dynamics of Stewart platform. Dasgupta and Mruthyunjaya [18] used the Newton-Euler approach to develop closed-form dynamics equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints. Tsai and Stamper [19] presented an algorithm to solve the inverse dynamics for the Delta translational robot, using Newton-Euler equations and Lagrange formalism.

Knowing the position and kinematics state of each link as well as the external forces acting on the hybrid parallel manipulator, in the present paper we apply some explicit recursive matrix equations for the inverse dynamic problem in order to obtain the input forces and powers required by all prismatic actuators.

Six independent pneumatic or hydraulic systems that generate active force $\vec{f}^{\,j}_{32} = f^{j}_{32} \vec{u}_3$ $(j = A, B, C, D, E, F)$ control the motion of six moving pistons of the legs. The serial-parallel hybrid robot can artificially be transformed in a set of *seven open trees* subject to the constraints. This is possible by imaginary cutting each joint for the moving platforms and taking its effect into account by introducing the corresponding constraint conditions.

The force of inertia and the resulting moment of the forces of inertia of an arbitrary rigid body T_k^A, for example,

$$\vec{f}_{k0}^{\,\mathrm{in}A} = -m_k^A\left[\vec{\gamma}_{k0}^A + (\tilde{\omega}_{k0}^A\tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A)\vec{r}_k^{\,CA}\right],\ \ \vec{m}_{k0}^{\,\mathrm{in}A} = -(m_k^A\tilde{r}_k^{CA}\vec{\gamma}_{k0}^A + \hat{J}_k^A\vec{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A\hat{J}_k^A\vec{\omega}_{k0}^A) \tag{9}$$

are determined with respect to the centre of joint A_k. On the other hand, the wrench of two vectors $\vec{f}_k^{\,*A}$ and $\vec{m}_k^{\,*A}$ evaluates the influence of the action of the weight $m_k^A\vec{g}$ and of external and eventually internal forces applied to the same element T_k^A of the manipulator:

$$\vec{f}_k^{\,*A} = m_k^A g a_{k0}\vec{u}_3\,,\quad \vec{m}_k^{\,*A} = m_k^A g\tilde{r}_k^{CA}a_{k0}\vec{u}_3\,. \tag{10}$$

Pursuing the first leg *A*, two *significant recursive relations* generate the vectors

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^A,\quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^{\mathrm{T}}\vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^A, \tag{11}$$

with the notations

$$\vec{F}_{k0}^A = -\vec{f}_{k0}^{\,inA} - \vec{f}_k^{\,*A},\quad \vec{M}_{k0}^A = -\vec{m}_{k0}^{\,inA} - \vec{m}_k^{\,*A}\,. \tag{12}$$

As example, starting from (11), we develop a set of six recursive relations:

$$\vec{F}_3^A = \vec{F}_{30}^A\,,\ \vec{F}_2^A = \vec{F}_{20}^A + a_{32}^{\mathrm{T}}\vec{F}_3^A\,,\ \vec{F}_1^A = \vec{F}_{10}^A + a_{21}^{\mathrm{T}}\vec{F}_2^A\,, \tag{13}$$

$$\vec{M}_3^A = \vec{M}_{30}^A\,,\ \ \vec{M}_2^A = \vec{M}_{20}^A + a_{32}^{\mathrm{T}}\vec{M}_3^A + \tilde{r}_{32}^A a_{32}^{\mathrm{T}}\vec{F}_3^A\,,\ \ \vec{M}_1^A = \vec{M}_{10}^A + a_{21}^{\mathrm{T}}\vec{M}_2^A + \tilde{r}_{21}^A a_{21}^{\mathrm{T}}\vec{F}_2^A\,.$$

The characteristic *virtual velocities* are expressed as functions of the pose of the hybrid robot by the matrix kinematical equations (7). Considering some *independent virtual motions* of the spatial mechanism, virtual displacements and velocities should be compatible with the virtual motions imposed by all kinematical constraints and joints at any instant in time. Let us assume that the robot has successively six independent virtual motions determined by following sets of velocities:

$$\begin{gathered} v_{32a}^{Av} = 1,\ v_{32a}^{jv} = 0\ \ (j \neq A);\ \ v_{32b}^{Bv} = 1,\ v_{32b}^{jv} = 0\ \ (j \neq B);\ \ v_{32c}^{Cv} = 1,\ v_{32c}^{jv} = 0\ \ (j \neq C) \\ v_{32d}^{Dv} = 1,\ v_{32d}^{jv} = 0\ (j \neq D);\ v_{32e}^{Ev} = 1,\ v_{32e}^{jv} = 0\ (j \neq E); \\ v_{32f}^{Fv} = 1,\, v_{32f}^{jv} = 0\ \ (j \neq F) \end{gathered} \tag{14}$$

Applying the explicit form of the *equations of parallel robots dynamics* [20], [21], following compact matrix relations results

$$f_{32}^{A} = \vec{u}_3^T \Big(\vec{F}_3^A + \omega_{10a}^{Av} \vec{M}_1^A + \omega_{21a}^{Av} \vec{M}_2^A + \omega_{10a}^{Bv} \vec{M}_1^B + \omega_{21a}^{Bv} \vec{M}_2^B + \\ + \omega_{10a}^{Cv} \vec{M}_1^C + \omega_{21a}^{Cv} \vec{M}_2^C + v_{10a}^{Gv} \vec{F}_1^G + \omega_{21a}^{Gv} \vec{M}_2^G + \omega_{32a}^{Gv} \vec{M}_3^G \Big), \tag{15}$$

for the *input force* required by the first active prismatic joint A_3 along the axis $A_3 z_3^A$ of lower module, and

$$f_{32}^{D} = \vec{u}_3^T \Big(\vec{F}_3^D + \omega_{10d}^{Dv} \vec{M}_1^D + \omega_{21d}^{Dv} \vec{M}_2^D + \omega_{10d}^{Ev} \vec{M}_1^E + \omega_{21d}^{Ev} \vec{M}_2^E + \\ + \omega_{10d}^{Fv} \vec{M}_1^F + \omega_{21d}^{Fv} \vec{M}_2^F + v_{10d}^{Hv} \vec{F}_1^H + \omega_{21d}^{Hv} \vec{M}_2^H + \omega_{32d}^{H} \vec{M}_3^H \Big). \tag{16}$$

for the *input force* required by the first active prismatic joint D_3 along the axis $D_3 z_3^D$ of upper module, for example.

The relations (11–16) represent the *inverse dynamics model* of the hybrid parallel robot. Various dynamical effects, including the Coriolis forces, the centrifugal forces coupling and the gravitational actions are considered.

As application let us consider same serial-parallel hybrid mechanism analysed in [16], which has the following mechanical and architectural characteristics:

$$\varphi_{21}^{G*} = \varphi_{32}^{H*} = \pi/18 \,, \;\; \varphi_{32}^{G*} = \varphi_{21}^{H*} = \pi/36 \,, \;\; \lambda_{10}^{G*} = 0.05\,\text{m} \,, \;\; \lambda_{10}^{H*} = 0.05\,\text{m} \,,$$

$$l_1 = 0.65\text{m} \,, \; l_2 = 0.85\text{m} \,, \; l_3 = 0.85\text{m} \,, \; G_3 A_4 = H_3 D_4 = l_4 = 0.45\text{m} \,, \; l_5 = l_6 = 0.25\text{m} \,,$$

$$\sin\beta = (l_1 + l_6)/(l_3 + l_5) \,, \;\; OA_1 = G_3 D_1 = l_0 = r_p = (l_3 + l_5)\cos\beta + l_4 \,, \quad g = 9.81\,\text{ms}^{-2} \,,$$

$$m_1 = m_2^G = m_2^H = 0.15\,\text{kg} \,, \;\; m_2 = 1.5\,\text{kg} \,, \;\; m_1^G = m_1^H = m_3 = 0.75\,\text{kg} \,, \;\; m_p = 2\,\text{kg} \,.$$

Assuming that the robot starts at rest from a central configuration and that are no additional external forces and moments acting on the moving platforms, the MATLAB software for a computer program was developed to solve the inverse dynamics of the hybrid parallel mechanism.

Two examples are solved to illustrate the algorithm.

For the first example, the platforms move along the *vertical direction* z_0 with variable accelerations while all the other positional parameters are held equal to zero. Considering the weights of compounding elements as passive external forces acting on the spatial hybrid manipulator, the time-history evolution for the input forces f_{32}^j (Fig. 3) and powers P_{32}^j (Fig. 4) of six prismatic actuators are calculated for a period of $\Delta t = 3$ seconds in terms of given equations (6).

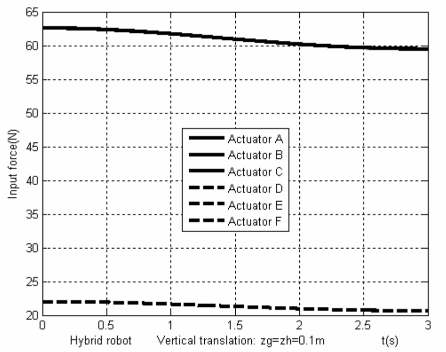

Fig. 3 – Input forces f_{32}^{j} of six actuators.

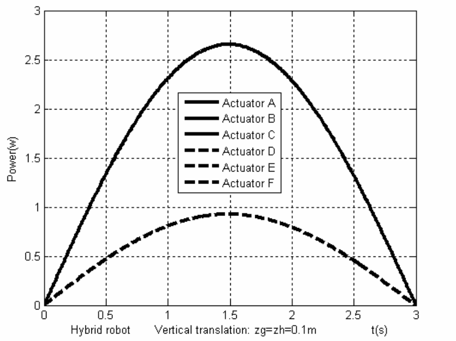

Fig. 4 – Powers P_{32}^{j} of six actuators.

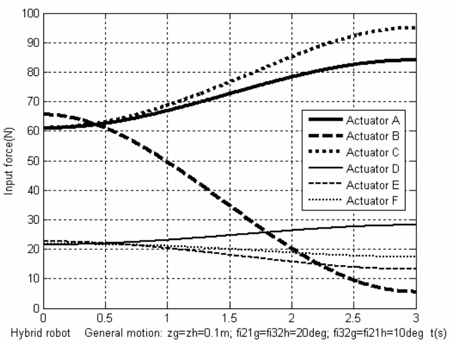

Fig. 5 – Input forces f_{32}^{j} of six actuators.

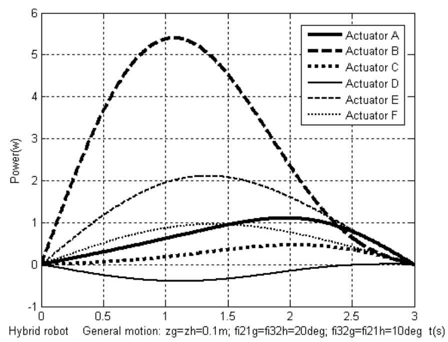

Fig. 6 – Powers P_{32}^{j} of six actuators.

As can be seen, it is proved to be true that all active forces and powers of lower module are permanently equal to one another but different to the forces and the powers of upper module.

In the case when the platforms make simultaneously two given *general relative motions*, the graphs are plotted and illustrated in Fig. 5 and Fig. 6.

The simulation through the program certifies that one of the major advantages of the current matrix recursive formulation is accuracy and a smaller processing time of numerical computation.

4. CONCLUSIONS

Using a set of explicit recursive matrix equations, already implemented in dynamics of parallel robots, a novel algorithm establishes the time-history evolution of input forces and powers developed by the actuators of the serial-parallel hybrid manipulator, where the number of links of the mechanisms is increased and the value of total degrees-of-freedom is augmented.

The simulation through the program certifies that the current matrix recursive formulation can easily be transformed in a model which is successfully expected to be deployed for automatic robotic control of hybrid parallel robots and that one of its major advantages is the effectiveness and accuracy of numerical computation.

The concept and the procedure above developed can be immediately extended to analysis of a complex robotic system composed of a multitude serially arranged similar parallel modules, but are also applicable to study of hybrid robots that are composed of different structures of parallel modules.

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