ESTIMATION OF OUT OF PLANE SHEAR MODULI FOR HONEYCOMB CORES WITH MODAL FINITE ELEMENT ANALYSES

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Abstract. The main problem in analyzing honeycomb structures is the substantial computational effort that has to be spent in modelling and analyzing such a structure with a multi-cell construction core by maintaining the actual geometry. Therefore, the common practice in the finite element modelling of such structures is to replace them by an equivalent orthotropic material. The determination of these equivalent properties of the homogenized core are based on analytical or numerical relationships which are approximate. If the honeycomb core is part of a sandwich panel then the most important elastic constants for out-of plane loading are the out of plane shear moduli. This paper presents a numerical method that can be used to correct these two equivalent elastic properties of the homogenized material. Other elastic constants needed for bending and torsion loading are established relying on modal analyses applied for a real honeycomb panel, to obtain a reference solution, and then on the homogenized orthotropic sandwich panel where these equivalent elastic constants are iteratively improved by an optimization algorithm to fit the reference solution. The analyzed types of honeycomb cores in this paper are the commercial ones with three cell configurations: square, regular hexagonal and re-entrant shapes. The material of the core and the skin is aluminium. The in plane dimensions of the analyzed panels in modal analysis for equivalent properties determination are around 400×400 mm^2 for two different thicknesses, 5 mm and 25 mm. The obtained results show that some of the equivalent elastic properties obtained in the classical way must be corrected as to obtain a proper response for bending and torsion loads.

Key words: sandwich panel, honeycomb, homogenization, equivalent properties, finite element, modal analysis.

1. INTRODUCTION

The use of sheet metals or laminate composites as skins and low density cellular materials as cores in sandwich constructions has enabled a very good utilization of the constituent materials also providing structural components with high stiffness

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and strength to weight ratios. These composite panels are incorporated in civil, automotive and aerospace engineering, where, usually, for structural analysis, the finite element method is used. The main problem in analyzing such honeycomb sandwich structures is the substantial computational effort that has to be spent in modelling and analyzing a sandwich structure with a multi-cell construction core by maintaining the authentic honeycomb core geometry. Therefore, the common practice in the finite element modelling of honeycomb sandwich structures is to replace the core by an equivalent two or three dimensional orthotropic material. Replacement of the actual honeycomb core by an equivalent continuum model (homogenization) works well, especially in problems involving global structural analyses such as deflection, eigenbuckling and vibration analysis.

Due to its array construction, the honeycomb core should exhibit orthotropic behaviour. Usually the constitutive model approximates the honeycomb core as a homogeneous material rather than a structural assembly of shells. The characterization of a constitutive model for an orthotropic material can be difficult due to the nine distinct parameters that must be determined (three Young's moduli, three shear moduli and three Poisson's ratios). There are some researches which present analytical relationships for these nine constants [1–4], obtained using homogenization theory and considering only axial and pure shearing loads. It was observed that using these analytical formulae, which are obtained using the beam theory, for relatively small core height, in a homogenized sandwich model which deforms in bending and torsion, the obtained results introduce large errors [5], due to the core thickness effect [6–10], therefore, considering the authors' previous experience [11, 12] in this field and some similar papers [13, 14], a procedure to extract some of the orthotropic elastic constants of analyzed honeycombs core using modal analysis is presented in this paper. In the recent papers [14–16], a similar procedure is used, but only for a regular honeycomb core panel, whereas in [13, 17, 18], a special design of a beam like structure of honeycomb core with two attached masses is considered. It was proved that these properties are dependent not only on the cell material and geometry, but also on the panel dimensions and skin thickness [7].

The normal mode of vibration may be obtained experimentally or numerically with good accuracy using finite element analysis (FEA), as in [19, 20], but the choice of finite element types and also the discretization must be carefully done [20–22].

2. CORE CONFIGURATIONS

In this paper, a commercial honeycomb type, defined by five geometrical parameters (Fig. 1), in three variants (Fig. 2), included in a symmetric sandwich panel with constant thickness skin, $t_f = 1$ mm, were analyzed.

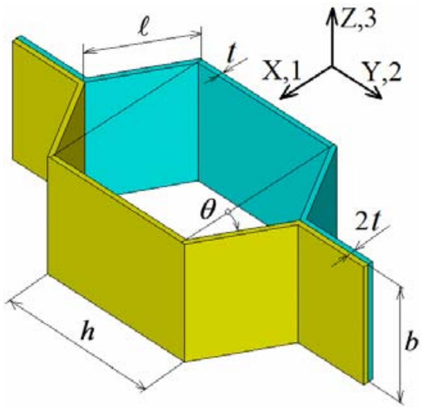


Fig. 1 – A representative volume element and honeycomb parameters.

The considered material for the honeycomb cores and also for the skins is an aluminium alloy (E_s = 70 000 MPa; ν_s = 0.33 and ρ_s = 2 700 kg/m^3). The cell parameters dimensions are: t = 0.1 mm for all three variants; ℓ = 5 mm, h = 10 mm and θ = 0º for *square honeycomb*; ℓ = h = 5.132 mm and θ = 30º for *regular honeycomb* and ℓ = 8.1407 mm, h = 15.0218 mm and θ = -30º for *re-entrant honeycomb*. Two sets of core thicknesses b are chosen: b = 5 mm, considered as relatively *thin thickness*, and then b = 25 mm, considered as *thick thickness*.

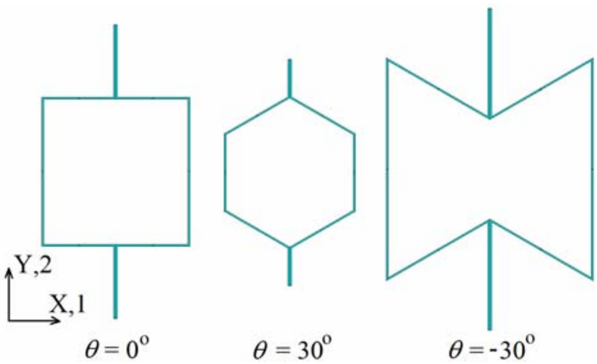


Fig. 2 –The analyzed configurations of constant relative density ρ and constant wall thickness t.

3. ANALITICAL BACKGROUND

For an equivalent *orthotropic material,* which may be a homogenized honeycombs, with the principal directions along the axes of the system O*XYZ* (Fig. 1), according to the generalized Hooke's law, it yields $\bar{\varepsilon} = \boldsymbol{S}\,\bar{\sigma}$, where $\bar{\varepsilon}$ and $\bar{\sigma}$ are the strain and respectively the stress vector of the homogenized structure. The compliance matrix, which includes the equivalent engineering mechanical elastic properties [1], is

$$\boldsymbol{S} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix} \tag{1}$$

Owing to the symmetry, the following equations must exist:

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}; \quad \frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}; \quad \frac{\nu_{32}}{E_3} = \frac{\nu_{23}}{E_2}. \tag{2}$$

Here Poisson's ratio v_{ij} is defined as the negative of the strain in the j direction divided by the strain in the i direction, for normal loading in the i direction ($\nu_{ij} = -\varepsilon_j / \varepsilon_i$). For example ν_{12} correspond to the negative ratio of the strain along direction 2 and the strain parallel to direction 1 when the axial loading direction is 1, so $\nu_{12} = -\varepsilon_2 / \varepsilon_1$.

The inverse matrix of the compliance matrix in (1) is the material stiffness matrix which must be positive definite. This condition can be easily obtained using linear algebra. According to the honeycombs in Fig. 1, where the absolute values of the Poisson's ratios ν_{21}, ν_{31} and ν_{32} are usually larger than ν_{13} and ν_{23} and close to ν_{12}, it is convenient that they are considered as inputs into finite element codes. Because the Young's moduli and shear moduli are always positive, the condition of a positive definite orthotropic material results

$$1 - \nu_{21}^2 \frac{E_1}{E_2} - \nu_{32}^2 \frac{E_2}{E_3} - \left(\nu_{31}^2 + 2\nu_{21}\nu_{31}\nu_{32}\right)\frac{E_1}{E_3} > 0. \tag{3}$$

The equivalent mechanical properties of honeycombs can be obtained starting with isotropic constants of the core material: ρ_s – mass density, E_s – Young's modulus and ν_s – Poisson's ratio. Using the beam theory [1], or more accurately a numerical method [5], all the engineering constants in (1), and supplementary the equivalent mass density ρ, can be computed. The analytical relationships are obtained in the beam theory hypotheses and are valid only for commercial honeycombs (Fig. 1) with $t << \ell$, and $b >> \ell$ because of supplementary hypotheses used in their derivations. The numerical estimations of equivalent mechanical properties of honeycombs are more accurate but they are also obtained only from pure axial and shear loading conditions. Even when these accurate equivalent orthotropic constants are used in plates or sandwiches that work in bending and/or torsion the results may be erroneous as it was shown in [6] and part of them, i.e. the in-plane elastic constant can be improved using a similar modal technique but considering only the honeycomb core as presented in [23]. This means that a part of the equivalent orthotropic properties in bending and/or torsion are different from the same properties in axial and/or simple shearing. Due to date, according to the author's knowledge, there are not accurate analytical relationships for obtaining the equivalent orthotropic shear moduli G_{23} and G_{13} for large variations of the input parameters.

To justify the chosen analyzed configuration of cells and to better understand the honeycombs behaviour the usual equivalent orthotropic constants of honeycombs are presented in the following. If we consider the relation of the mass density of the equivalent commercial honeycomb core, [1], the relative density of commercial honeycomb type (honeycombs with double thickness vertical walls and thin wall thickness) is described by

$$\frac{\rho}{\rho_s} = \frac{\frac{t}{\ell}\left(\frac{h}{\ell}+1\right)}{\left(\frac{h}{\ell}+\sin\theta\right)\cos\theta}. \tag{4}$$

In plane elastic properties. The Young's moduli parallel to OX and OY are:

$$E_1 = k_1 E_s \left(\frac{t}{\ell}\right)^3 \frac{\cos\theta}{\left(\frac{h}{\ell}+\sin\theta\right)\sin^2\theta}, \quad E_2 = k_2 E_s \left(\frac{t}{\ell}\right)^3 \frac{\frac{h}{\ell}+\sin\theta}{\cos^3\theta}, \tag{5}$$

where

$$k_1 = \frac{1}{1+\left(\frac{t}{\ell}\right)^2\left(2.4+1.5\nu_s+\cot^2\theta\right)}, \tag{6}$$

and

$$k_2 = \frac{1}{1+\left(\frac{t}{\ell}\right)^2\left(2.4+1.5\nu_s+\tan^2\theta+2\frac{h}{\ell\cos^2\theta}\right)} \tag{7}$$

are correction coefficients.

The in plane Poisson's ratios can be calculated using

$$\nu_{12} = c_{12}\frac{\cos^2\theta}{\left(\frac{h}{\ell}+\sin\theta\right)\sin\theta}, \quad \nu_{21} = c_{21}\frac{\left(\frac{h}{\ell}+\sin\theta\right)\sin\theta}{\cos^2\theta}, \tag{8}$$

where

$$c_{12} = \frac{1+\left(\frac{t}{\ell}\right)^2\left(1.4+1.5\nu_s\right)}{1+\left(\frac{t}{\ell}\right)^2\left(2.4+1.5\nu_s+\cot^2\theta\right)}, \tag{9}$$

and

$$c_{21} = \frac{1+\left(\frac{t}{\ell}\right)^2\left(1.4+1.5\nu_s\right)}{1+\left(\frac{t}{\ell}\right)^2\left(2.4+1.5\nu_s+\tan^2\theta+2\frac{h}{\ell\cos^2\theta}\right)} \tag{10}$$

are also correction coefficients to include the axial and shearing forces in beam formulation problem.

The in plane shear modulus (without correction for axial and shearing forces), [5], was demonstrated by the authors of this paper and its corrected form is

$$G_{12} = E_s\left(\frac{t}{\ell}\right)^3\frac{\frac{h}{\ell}+\sin\theta}{\left(\frac{h}{\ell}\right)^2\left(1+\frac{h}{4\ell}\right)\cos\theta}. \tag{11}$$

Out of plane elastic properties. The Young's modulus parallel to OZ, and Poisson's ratios due to a load along OZ axis are simply set to

$$E_3 = E_s\frac{\rho}{\rho_s}, \quad \nu_{31} = \nu_{32} = \nu_s, \tag{12}$$

and then, according to the reciprocal relations, it results the remainder of Poisson's ratios

$$\nu_{13} = \nu_s\frac{E_1}{E_3}, \quad \nu_{23} = \nu_s\frac{E_2}{E_3}. \tag{13}$$

The following relationships, obtained also in the beam theory hypotheses, are valid only for $t << \ell$, and $b >> \ell$ because of supplementary hypotheses as the shear stresses are uniform within the cell walls. The two out of plane shear moduli can be obtained from:

$$G_{13} = G_s\left(\frac{t}{\ell}\right)\frac{\cos\theta}{\left(\frac{h}{\ell}+\sin\theta\right)}, \quad G_{23} = G_{23}^L + \frac{\alpha}{\left(\frac{b}{\ell}\right)}\left(G_{23}^U - G_{23}^L\right). \tag{14}$$

The second relation in (14) was deduced in [4] using FEA in some particular hypothesis ($t/\ell = 0.08$, $h/\ell = 1$ and $0 < \theta < 30°$) for which $\alpha = 0.787$. The upper and lower bounds of the G_{23} shear modulus are obtained from:

$$G_{23}^U = G_s\left(\frac{t}{\ell}\right)\frac{\frac{h}{\ell}+\sin^2\theta}{\left(\frac{h}{\ell}+\sin\theta\right)\cos\theta}; \quad G_{23}^L = G_s\left(\frac{t}{\ell}\right)\frac{\frac{h}{\ell}+\sin\theta}{\left(\frac{h}{\ell}+1\right)\cos\theta}. \tag{15}$$

Using the relations (4)–(15) and numerical approach based on finite element analysis presented in [5], Table 1 presents the equivalent elastic properties of the honeycomb core of thin thickness (b = 5mm), whereas the Table 2 presents the same elastic properties for thick thickness (b = 25mm).

Table 1

The equivalent properties of thin thickness honeycombs (b = 5 mm)

Mechanical property	*Square honeycomb*		*Regular honeycomb*		*Re-entrant honeycomb*	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
ρ [kg/m^3]	81	81	81	81	81	81
E_1 [MPa]	700	708.8	1.193	1.314	0.3338	0.3639
E_2 [MPa]	1.117	1.229	1.193	1.314	0.2684	0.2927
E_3 [MPa]	2100	2100	2100	2100	2100	2100
G_{12} [MPa]	0.1867	0.2037	0.7176	0.7869	0.04051	0.0438
G_{23} [MPa]	294.4	482.0	342.1	477.0	124.2	365.0
G_{13} [MPa]	203.4	263.0	227.9	296.0	143.6	208.0
ν_{12} [–]	0	0.01641	0.9978	0.9983	–1.114	–1.114
ν_{21} [–]	0	0	0.9985	0.9989	–0.896	–0.8963
ν_{32} [–]	0.33	0.33	0.33	0.33	0.33	0.33
ν_{31} [–]	0.33	0.33	0.33	0.33	0.33	0.33

Table 2

The equivalent properties of thick thickness honeycombs (b = 25 mm)

Mechanical property	*Square honeycomb*		*Regular honeycomb*		*Re-entrant honeycomb*	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
ρ [kg/m^3]	81	81	81	81	81	81
E_1 [MPa]	700	737.5	1.193	1.334	0.3338	0.3725
E_2 [MPa]	1.117	1.249	1.193	1.334	0.2684	0.2996
E_3 [MPa]	2100	2100	2100	2100	2100	2100
G_{12} [MPa]	0.1867	0.2081	0.7176	0.7996	0.04051	0.04509
G_{23} [MPa]	327.4	389.0	400.8	452.0	162.8	314.0
G_{13} [MPa]	218.6	263.0	245.9	296.0	172.0	208.0
ν_{12} [–]	0	–0.01898	0.9978	0.9983	–1.114	–1.114
ν_{21} [–]	0	$-3.216\cdot10^{-5}$	0.9985	0.9989	–0.8960	–0.8963
ν_{32} [–]	0.33	0.33	0.33	0.33	0.33	0.33
ν_{31} [–]	0.33	0.33	0.33	0.33	0.33	0.33

One can observe good correlations between analytical and numerical results, anyway small discrepancies exist, but they can be easily explained [5]. In the homogenization of honeycomb cores, the 3D Shell numerical results were used because the reference results, as will be see next, for the sandwich panels are also obtained using Shell models. These constants obtained from axial and pure shear loadings, with correction only for E_1, E_2 and G_{12} in bending and torsion loads established previously in [23], were used for the following optimization analyses as initial material input data.

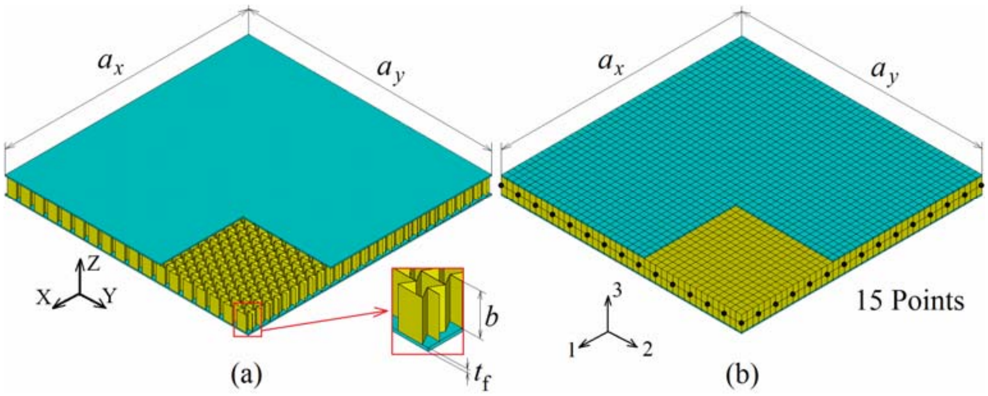


Fig. 3 – A real honeycomb sandwich panel (a) and its equivalent homogenized layered panel (b). The global dimensions are preserved, also the principal directions of equivalent orthotropic material 123 correspond to global system of coordinate O*XYZ*. Location of 15×15 points for obtaining displacements considered in the eigenmodes pairs algorithm are also presented in (b).

The main idea of the homogenization process is presented in Fig. 3. The real sandwich panel including the honeycomb cores was accurately modelled and analyzed in ANSYS using the Shell 181 element type. The main characteristics of six variants of the analyzed honeycomb panels are presented in Table 3.

Table 3

Main parameters of analyzed sandwiches with honeycomb cores

Honeycomb core type	Core thickness b [mm]	Number of cells		Global size		Core mass [g]
		Along X	Along Y	a_x [mm]	a_y [mm]	
Square	5	40	20	400	400	64.80
	25					324.0
Regular	5	45	26	400	400.3	64.85
	25					324.2
Re-entrant	5	28	18	394.8	394.2	63.04
	25					315.2

4. OPTIMIZATION ALGORITHM

The natural frequencies and mode shapes, obtained in the reference model (Fig. 3a) and in the homogenized model (Fig. 3b), must be the same, if the equivalent orthotropic elastic constants are correctly obtained from analytical relationships. But if we consider only the core, it was observed that only in-plane normal modes maintain the natural frequencies in the two models as in [23]. The out of plane normal modes (bending and torsion) present a shift in the natural frequencies from the two sets of models, meaning that the elastic equivalent properties in bending and torsion are different from those obtained in axial and shearing loads. If we consider the equivalent elastic properties as design variables, and define an objective function as the sum of these shifts in absolute or square values for a number of N arbitrarily out-of-plane normal modes, by minimizing this objective function it is possible to obtain the effective elastic properties in bending [13, 24].

The optimization algorithm should not take into account parameters that are not sensitive to the change in frequencies, and thus a sensitivity analysis is to be carried out first. So, the objective function may be defined as the total square relative errors between the reference solutions and the current ones, or called also optimization model solutions, by calculating

$$\text{Obj\_function} = \sum_{i=1}^{N} c_i \left(1 - \frac{f_{X,i}}{f_{A,i}}\right)^2 , \tag{16}$$

where c_i are weight factors (in this paper $c_i = 1$); $f_{A,i}$ and $f_{X,i}$ are pairs of natural frequencies from the reference model (or experimental one if possible) and respectively from the optimized model of the homogenized structure.

A global indicator of the cumulative frequency errors can be obtained with the equation below, which is used mainly in post processing the results

$$\text{Obj\_freq} = 100\sum_{i=1}^{N}\left|1-\frac{f_{X,i}}{f_{A,i}}\right| \quad [\%]. \tag{17}$$

A main problem in the optimization algorithm is to obtain the correct pairs of natural modes of vibration and that the MAC (**M**odal **A**ssurance **C**riterion) is used. The MAC is a widely used technique to estimate the degree of correlation between mode shape vectors. The MAC between a reference mode $\boldsymbol{\phi}_{Aj}$ and a current mode $\boldsymbol{\phi}_{Xk}$ is defined as

$$\text{MAC}_{jk} = \frac{\left|\boldsymbol{\phi}_{Aj}^{\mathrm{T}}\,\boldsymbol{\phi}_{Xk}\right|^2}{\left\|\boldsymbol{\phi}_{Aj}\right\|^2\left\|\boldsymbol{\phi}_{Xk}\right\|^2}; \quad j=1,\ldots,n_A; \quad k=1,\ldots,n_X. \tag{18}$$

The value of MAC is between zero and one. A value of one means that one mode shape vector is a multiple of the other. So, if the MAC matrix for equal number of reference and optimized total modes is the identity matrix, the two sets of vectors denote the same mode shape. It must be mentioned that the degrees of freedom in the two sets must be identically arranged. If for one column (or/and row) there is not a value greater than a limit of correlations, for example 0.8, we can say that there is a mode shape that cannot be paired with the reference set mode. In this work a 15×15 uniform distributed points in the cores (Fig. 3b) were retained in the modal vectors used in relation (18) to build the MAC matrix.

For the effective optimization was used the Design Optimization Module from ANSYS. A general text file for optimization, describing parametrically the homogenized model using APDL commands, was created to be used in sub-problem and first order optimization tools which are available in Ansys [25]. A special attention in the algorithm was devoted to relation (3) by the introduction of some particular restrictions, to obtain the positively defined material stiffness matrix for all combinations of the inputs.

The reference results, first 30 elastic natural frequencies and corresponding mode shapes, were obtained for all six geometries presented in Table 3 (see for example details of Fig. 3a). The all six models were meshed with Shell 181 finite element type (a four nodded Shell element) in ANSYS, and were obtained over 200,000 finite elements for each model. The results were saved in text files to be read in the optimization algorithm. The first eight elastic natural modes of vibration for a sandwich with regular honeycomb core of thin thickness, all out-of-plane modes, are presented in Fig. 4, as an example.

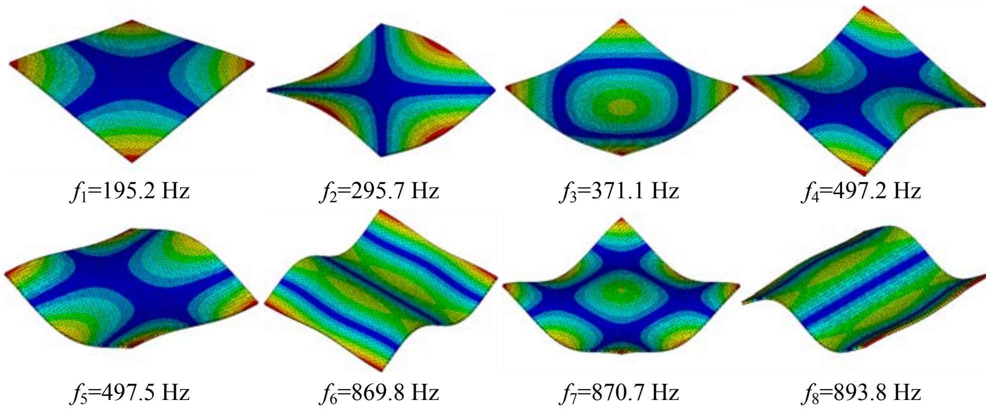


Fig. 4 – First eight natural modes of vibration for sandwich with regular honeycomb core (b = 5 mm) obtained from a fine mesh of the reference model using shell finite element type.

5. SENSITIVITY ANALYSIS

For the sensitivity analyses, all the elastic material properties from Eq. (1) were considered as design parameters (variables). Two different homogenized models are considered: the first one uses three layers (skins and core) and an eight nodded Shell elements (Shell281) in ANSYS – this is why the accuracy in modelling composite shells is governed by the first-order shear-deformation theory (Mindlin-Reissner shell theory); the second one was built with Shell281 for the skins and Brick186 elements for the core. Using the convergence criteria for the first elastic 30 normal modes of vibration for the two models it was concluded that an average element size of 15 mm is enough for discretization. Because very small differences in the two models were obtained, the layered Shell element model was used for the results reported in this paper.

As reported in [15] and [16] it was found that only some parameters are sensitive to the considered objective function (Fig. 5) and natural frequencies variations (Fig. 6).

Only two elastic parameters G_{13} and G_{23}, have an important contribution to change the objective function or lower natural frequencies of the homogenized plate model, regardless if this is modelled using composite Shell281 only or composite Solid186 elements with Shell281 elements. The parameters E_1, E_2 and G_{12} have a smaller contribution to the objective frequency function – Obj_freq by about two orders of magnitude relative to the out of plane shear moduli. The rest of the parameters do not have a real influence except the core thickness, the thickness of the skin and mass density but they are known, thus leaving only G_{13} and G_{23} as having a real influence.

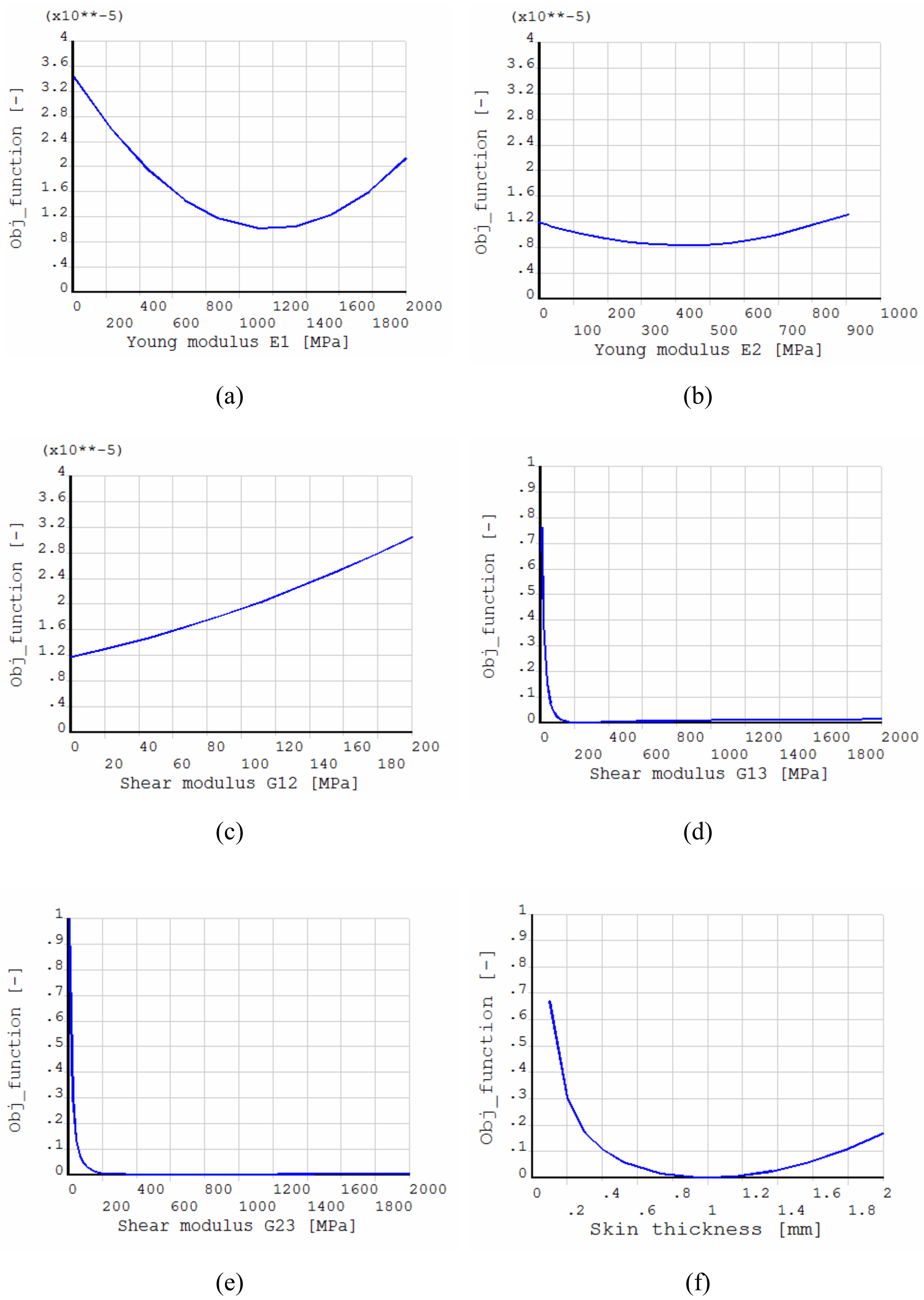
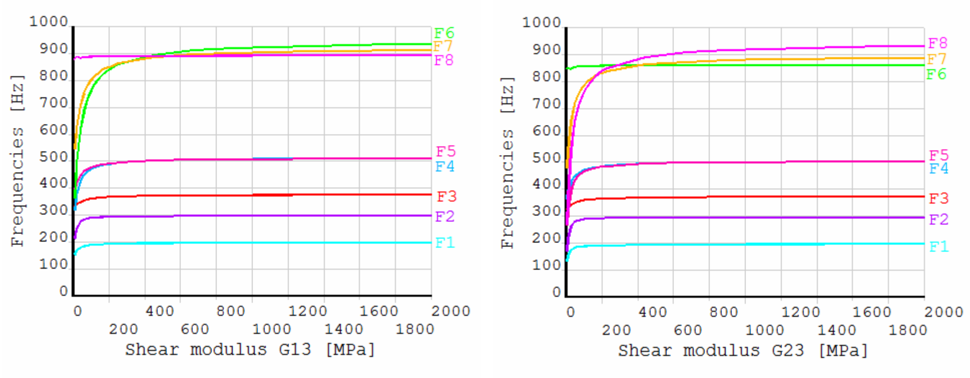


Fig. 5 – Sensitivity of the objective functions for: E_1 (a), E_2 (b), G_{12} (c), G_{13} (d), G_{23} (e) of the core and the skin thickness (f), for square honeycomb panel (b = 5 mm).



(a) (b)

Fig. 6 – Sensitivity of the first eight natural frequencies for square honeycomb panel (b = 5 mm) by considering G_{13} (a) and G_{23} (b).

It can be observed from Fig. 6, that some modes as Mode No. 8 and Mode No. 6 are not dependent on G_{13} and G_{23}, respectively, so it is a good practice to include more natural modes into the optimization algorithm or to use these particular modes to the first estimation of the out-of-plane shear moduli.

6. OPTIMIZATION RESULTS AND DISCUSSIONS

All the results reported below (Table 4 and Table 5), correspond to the case in which only the first eight elastic out-of-plane normal modes of vibrations ($N = 8$ in Eq. (16)) are fit to the reference modes using the proposed optimization algorithms for all six different case studies. From paper [23] it can be observed that in bending and torsion, the elastic properties E_1, E_2 and G_{12} are different from their corresponding values obtained from axial and shearing behaviours (Table 1 and Table 2) and these corrected values were used as the initial values in the optimization algorithm to obtain G_{13} and G_{23}, but also E_1, E_2 and G_{12}, as all five were considered as variable parameters.

Table 4

The initial estimation and the optimized equivalent properties of thin thickness honeycombs (b=5 mm)

Parameter	*Square honeycomb*		*Regular honeycomb*		*Re-entrant honeycomb*	
	Initial	Optimized	Initial	Optimized	Initial	Optimized
E_1 [MPa]	868.0	868.1	3.46	3.66	1.86	1.704
E_2 [MPa]	3.16	3.16	3.46	3.66	1.50	1.374
G_{12} [MPa]	1.13	1.13	3.45	3.45	0.353	0.313
G_{13} [MPa]	263.0	255.0	296	298.0	208	205.0
G_{23} [MPa]	482.0	443.0	477	472.7	518	365.0
Obj_freq [%]	2.23	0.88	0.60	0.51	6.54	0.69

Table 5

The initial estimation and the optimized equivalent properties of thick thickness honeycombs (b = 25 mm)

Parameter	*Square honeycomb*		*Regular honeycomb*		*Re-entrant honeycomb*	
	Initial	Optimized	Initial	Optimized	Initial	Optimized
E_1 [MPa]	912.6	904.7	1.420	1.426	0.423	0.420
E_2 [MPa]	1.364	1.209	1.420	1.426	0.340	0.338
G_{12} [MPa]	0.249	0.253	0.900	0.909	0.0571	0.0519
G_{13} [MPa]	263	272.4	296	301.9	208	218.3
G_{23} [MPa]	389	389.7	452	468.6	314	250.0
Obj_freq [%]	3.25	1.86	4.16	1.82	12.53	3.65

It can be noticed that the initial values used in the optimization algorithm are relatively close to the ones obtained after optimization. In fact the in-plane elastic constants E_1, E_2 and G_{12} used in the optimization don't change compared to those obtained by the modal optimization of the behaviour in bending of the core [23]. The values of the initially established out-of-plane moduli G_{13} and G_{23} were corrected through optimization so that the cumulated error of the frequencies of the first 8 eigenmodes was reduced less than 1% for the panels with thin core and under 2–4% for panels with thick core. It is to be underlined that the greatest error of 3.65% was obtained for the panel with re-entrant core, probably due to the fact that the reference modes were influenced by the border introduced to eliminate some local modes of the core obtained due to the large size of the cell.

In Fig. 7 the errors obtained for the frequencies in the 8 modes considered in the process of identification of the material constants, before and after optimization for one of the analyzed cases (thick panel with re-entrant core) are presented. One can notice that initially mode eight has an error above 6% and after optimization all the modes have errors bellow 1%. Although the optimization was done only for the first 8 eigenmodes, after optimization it is found that in fact all the monitored modes in the reference models are obtained quite correctly. As an example, for the panel with thin (b = 5 mm) regular core, in Fig. 8 are presented the relative errors of the 30 modes monitored in the reference models and they don't exceed 0.6% for any of the modes.

After performing supplementary analyses it was found that there are changes for the optimized moduli G_{13} and G_{23} for thicknesses of the skins smaller and greater than the one considered initially as t_f = 1 mm, but this discussion will be done elsewhere.

If there are considered the estimated moduli G_{13} and G_{23} by using the analytical relations (14) and (15), as well as the ones obtained through modal analysis the results presented in Table 6 lead to the conclusion that for G_{23} obtained for a thick core the analytical equations give correct values, but for thin cores the errors are significant, mainly for the re-entrant core.

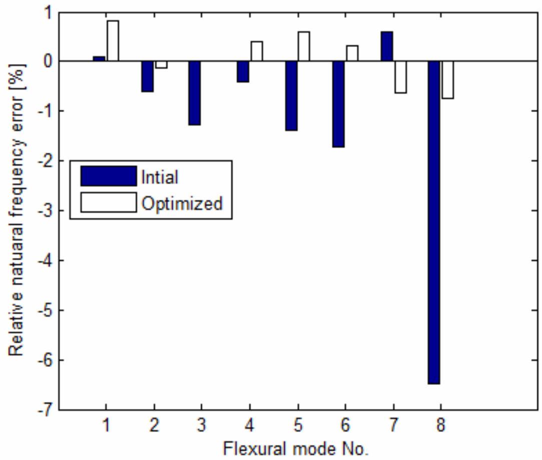


Fig. 7 – Natural first eight frequencies errors for the initial and after optimization elastic properties of the re-entrant core (b = 25 mm) of the panel.

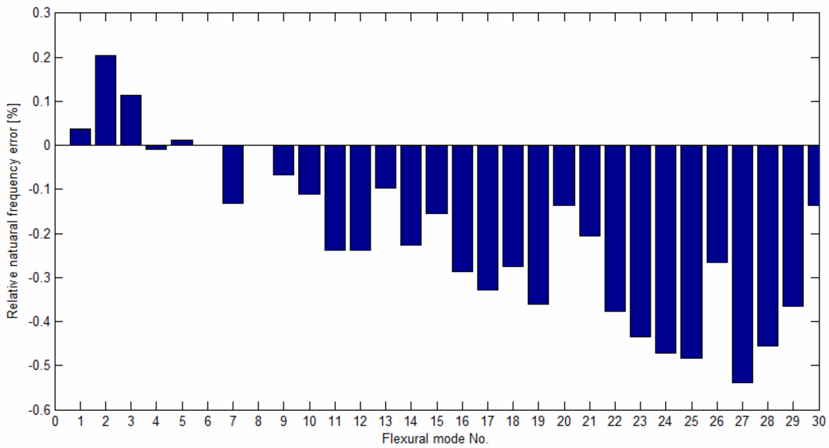


Fig. 8 – Natural frequencies errors for first 30 elastic modes after the optimization of the elastic properties of the regular core of the panel (b = 5 mm).

The analytical values obtained for G_{13} are giving a correct estimate compared to the numerical results for both core thicknesses. For G_{23} the values obtained by numeric modal analysis are in between the lower and upper bounds of the analytical models, and more important is that the values decrease with the increase of the core thickness, as estimated by Grediac's formula. In fact the comparison is only qualitative as here the calculus is not applicable because Grediac's formula was fitted for particular honeycombs, i.e. $0 < \theta < 30°$, $h/\ell = 1$ and $t/\ell = 0.08$, and, in our case t/ℓ is less than four times lower and $h/\ell = 1$ is satisfied only for a regular honeycomb.

Table 6

Analytical and modal analysis numerical results obtained for a skin thickness $t_f = 1$ mm

Shear modulus	Core thickness [mm]		*Square honeycomb* Analytic	*Square honeycomb* Numeric	*Regular honeycomb* Analytic	*Regular honeycomb* Numeric	*Re-entrant honeycomb* Analytic	*Re-entrant honeycomb* Numeric
G_{13} [MPa]	$b = 5$		263.2	255.0	296.1	298.0	208.1	205.0
G_{13} [MPa]	$b = 25$		263.2	272.4	296.1	301.9	208.1	218.3
G_{23} [MPa]	*Lower*		350.9	–	444.1	–	176.5	–
G_{23} [MPa]	*Upper*		526.3	–	493.4	–	581.4	–
G_{23} [MPa]	*Grediac*	$b = 5$	488.9	443.0	483.9	472.7	695.3*	365.0
G_{23} [MPa]	*Grediac*	$b = 25$	378.5	389.7	452.1	468.9	280.3*	250.0

* This value is an estimate obtained with Grediac's formula for a particular geometry of the honeycomb panel.

7. CONCLUSIONS

There are a number of advantages in using modal testing as a validation procedure for material properties estimations as compared to traditional static material testing. For this modal test structure, a number of modes (usually five to maximum 30–50) can be easily computed, or better, if possible, measured, thus producing more independent test data from a single structure to validate the model or to obtain some elastic constants. The modes of vibration are influenced by several of the elastic constants of the material, thus providing overlapping validation paths for the elastic constants. With modal testing, the analyzed structure can be gently supported, thus accurately simulating free boundary conditions. This is in contrast with material testing, for which the structure needs to be gripped with some fixture in order to apply loads creating some unknown or difficult to simulate boundary conditions.

The optimization algorithm used to extract the elastic properties of the analyzed honeycomb cores is similar with the algorithm presented in [12], but in this paper, instead of the experimental determination of modal characteristics of the analyzed cores, they are obtained numerically using a finite element model which completely and accurately replicates the honeycomb cores.

In conclusion, starting with a unit-cell geometry and using the periodicity, a finite element model of the honeycomb core can be generated and then analyzed in modal analysis for free-free boundary conditions to simulate an experiment. A number of maximum 30, in-plane and out-of-plane natural frequencies and normal modes of vibration were saved considering only 15×15 points uniformly distributed in the middle of the core and analyzing their three translations. Based on these virtual modal data set of results and a similar simplified orthotropic homogenized shell or combination shell-solid finite element model, some of the elastic properties of the honeycomb core in bending, G_{13} and G_{23} were successfully identified based on an optimization algorithm generated in ANSYS APDL. Further, to extend this procedure, the same concepts can be used to identify other parameters which are sensitive to the considered objective function for different geometric honeycomb core structures or sandwiches.

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