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INCREMENTAL NUMERICAL METHOD USED FOR THE KINEMATIC ANALYSIS OF THE FOUR-BAR LINKAGE MECHANISM

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Abstract. Kinematics deals with the study of geometrics aspects of the movement without taking into consideration the mass of the rigid solids (their inertia) and neither the causes that provoked the movement namely the forces. The present paper presents a comparative kinematical survey of the articulated quadrilateral mechanism. More precisely, in this paper is presented an incremental numerical method used for kinematics analysis of the articulated quadrilateral mechanism (four bar linkage mechanism). Further on, the kinematics analysis of the same mechanism is effectuated using an analytical method. Finally the results obtained by using those two methods are compared.

Key words: kinematics, kinematical analysis, incremental numerical method.

NOMENCLATURE

$\mathbf{0}$ – matrix with “*m*” rows and “*n*” columns

$\varphi_1, \varphi_2, \varphi_3$ – angles of self rotation of the mechanism’s elements

$d\varphi_1, d\varphi_2, d\varphi_3$ – infinite small variations of the angles $\varphi_1, \varphi_2, \varphi_3$

$\Delta\varphi_1, \Delta\varphi_2, \Delta\varphi_3$ – finite variations of the angels $\varphi_1, \varphi_2, \varphi_3$

Δt – the increment of time

$\varphi_1^*, \varphi_2^*, \varphi_3^*$ – values of the angles of self-rotation at a certain moment “*t*”

1. INTRODUCTION

Kinematical analysis of each mechanism is important therefore it needs to be given much attention to this thing. It is considered the articulated quadrilateral mechanism (four bar linkage mechanism) shown in the figure below (Fig.1). It

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consists of three rigid solid bodies linked each other by joints. Two of the three rigid solid bodies are linked by joints with the element which is supposed to be fixed and describe motions of rotation.

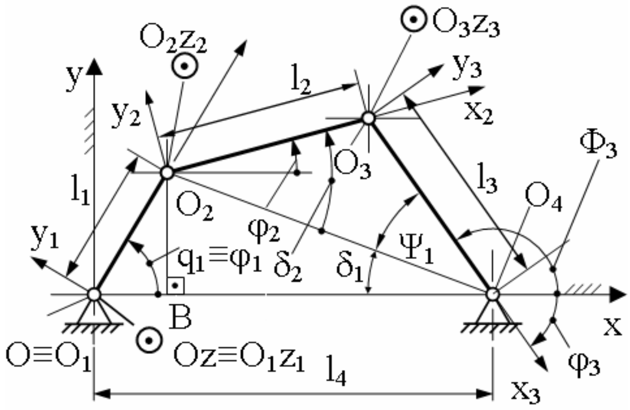

Fig. 1 – Articulated quadrilateral mechanism.

The third one is linked with the other two and describes a plane-parallel motion [1–3].

2. KINEMATIC ANALYSIS OF THE FOUR BARS LINKAGE MECHANISM USING ANALYTICAL METHOD

The analytical method involves the expression of the kinematical parameters which describe the motion of the rigid solid bodies which make up the system as a function of the kinematical parameter describing the motion of the leader element. In this case the leader element is considered to be the element denoted on the figure by O_1O_2 (Fig.1). It may be observed that the length of this element is "l_1". The value of the angle is given by the following relationship [2]:

$$\varphi_2 = \delta_2 - \delta_1. \tag{1}$$

In the relationship (1) the values of the angles "δ_1" and "δ_2" are given by the relations below [4]:

$$\delta_1 = \mathrm{arctg}\left[l_1 \cdot \sin(\varphi_1) \big/ \left(l_4 - l_1 \cdot \cos(\varphi_1) \right) \right] \tag{2}$$

$$\delta_2 = \arccos\left(N_1 / N_2 \right). \tag{3}$$

In the relationship (3) the quantities involved have the followings expressions:

$$N_1 = \mathrm{O_2O_4^2} + l_2^2 - l_3^2 \tag{4}$$

$$N_2 = 2 \cdot \mathrm{O_2O_4} \cdot l_2 \tag{5}$$

$$\mathrm{O_2O_4} = \sqrt{l_1^2 + l_4^2 - 2 \cdot l_1 \cdot l_4 \cdot \cos(\varphi_1)}. \tag{6}$$

The value of angle "φ_3" is given by the following relationship:

$$\varphi_3 = -(\Psi_1 + \delta_1). \tag{7}$$

In the relationship (7) the quantities involved have the followings expressions:

$$\Psi_1 = \arccos\left(N_3 / N_4\right). \tag{8}$$

In the relationship (8) the values of "N_3" and "N_4" have the followings expressions:

$$N_3 = l_3^2 + (\mathrm{O_2O_4})^2 - l_2^2 \tag{9}$$

$$N_4 = 2 \cdot l_3 \cdot \mathrm{O_2O_4}. \tag{10}$$

3. KINEMATIC ANALYSIS OF THE FOUR BARS LINKAGE MECHANISM USING INCREMENTAL NUMERICAL METHOD

Any mechanism including the articulated quadrilateral mechanism is a closed kinematics chain. Therefore we will write the kinematical closing equations in projections on the axes of the fix reference frame. The mathematical expressions of these equations are the followings [2]:

$$l_1 \cdot \cos(\varphi_1) + l_2 \cdot \cos(\varphi_2) + l_3 \cdot \cos(\varphi_3) - l_4 = 0 \tag{11}$$

$$l_1 \cdot \sin(\varphi_1) + l_2 \cdot \sin(\varphi_2) + l_3 \cdot \sin(\varphi_3) = 0. \tag{12}$$

The equations (11) and (12) may be written under differential form as follows:

$$\mathbf{A} \cdot \mathrm{d}\boldsymbol{\varphi} = \underset{2\times1}{\underset{\sim}{\mathbf{0}}} = [0 \mid 0]^{\mathrm{T}} \tag{13}$$

In the relationship (13) the terms that occur have the followings expressions:

$$\mathbf{A}=\left[\begin{array}{c:c:c} l_1\sin(\varphi_1) & l_2\sin(\varphi_2) & l_3\sin(\varphi_3) \\ \hdashline l_1\cos(\varphi_1) & l_2\cos(\varphi_2) & l_3\cos(\varphi_3) \end{array}\right] \tag{14}$$

$$\mathrm{d}\boldsymbol{\varphi}=\left[\mathrm{d}\varphi_1 \;\vdots\; \mathrm{d}\varphi_2 \;\vdots\; \mathrm{d}\varphi_3\right]^{\mathrm{T}}. \tag{15}$$

Relation (13) may be written as follows:

$$\mathbf{A}_1\cdot\mathrm{d}\varphi_1+\mathbf{A}_2\cdot\mathrm{d}\mathbf{q}_2=\underset{2\times1}{\underbrace{\mathbf{0}}}=\left[0 \;\vdots\; 0\right]^{\mathrm{T}}. \tag{16}$$

Relation (16) under the following form:

$$\mathbf{A}_2\cdot\mathrm{d}\mathbf{q}_2=-\mathbf{A}_1\cdot\mathrm{d}\varphi_1. \tag{17}$$

In the relation (17) the terms have the followings expressions:

$$\mathbf{A}_1=\left[l_1\cdot\sin\varphi_1 \;\vdots\; l_1\cdot\cos\varphi_1\right]^{\mathrm{T}} \tag{18}$$

$$\mathbf{A}_2=\left[\begin{array}{c:c} l_2\cdot\sin\varphi_2 & l_3\cdot\sin\varphi_3 \\ \hdashline l_2\cdot\cos\varphi_2 & l_3\cdot\cos\varphi_3 \end{array}\right] \tag{19}$$

$$\mathrm{d}\mathbf{q}_2=\left[\mathrm{d}\varphi_2 \;\vdots\; \mathrm{d}\varphi_3\right]^{\mathrm{T}}. \tag{20}$$

Passing from infinite small displacements to finite but small displacements we may write the kinematical closing equations as follows:

$$\mathbf{A}_2^*\cdot\Delta\mathbf{q}_2=-\mathbf{A}_1^*\cdot\Delta\varphi_1. \tag{21}$$

In the relation (21) the terms involved have the followings expressions:

$$\mathbf{A}_2^*=\left[\begin{array}{c:c} l_2\cdot\sin(\varphi_2^*) & l_3\cdot\sin(\varphi_3^*) \\ \hdashline l_2\cdot\cos(\varphi_2^*) & l_3\cdot\cos(\varphi_3^*) \end{array}\right] \tag{22}$$

$$\mathbf{A}_1^*=\left[l_1\cdot\sin(\varphi_1^*) \;\vdots\; l_1\cdot\cos(\varphi_1^*)\right]^{\mathrm{T}} \tag{23}$$

$$\Delta\mathbf{q}_2=\left[\Delta\varphi_2 \;\vdots\; \Delta\varphi_3\right]^{\mathrm{T}}. \tag{24}$$

From equation (21) it may be deduced:

$$\Delta\mathbf{q}_2 = -\left(\mathbf{A}_2^*\right)^{-1} \cdot \mathbf{A}_1^* \cdot \Delta\varphi_1. \tag{25}$$

By performing the calculations we will obtain the followings:

$$\Delta\mathbf{q}_2 = \mathbf{B} \cdot \Delta\varphi_1. \tag{26}$$

In the relation (26) the matrix **B** has the following expression:

$$\mathbf{B} = \left[B_{11} \mid B_{12}\right]^{\mathrm{T}} = -\left(\mathbf{A}_2^*\right)^{-1} \cdot \mathbf{A}_1^* \tag{27}$$

$$B_{11} = \left(l_1/l_2\right) \cdot \left(C_1^*/D_1^*\right) \tag{28}$$

$$B_{12} = \left(l_1/l_3\right) \cdot \left(C_2^*/D_1^*\right). \tag{29}$$

In the relations (28) and (29) the terms involved have the followings expressions:

$$C_1^* = \sin\left(\varphi_1^* - \varphi_3^*\right) \tag{30}$$

$$D_1^* = \sin\left(\varphi_3^* - \varphi_2^*\right) \tag{31}$$

$$C_2^* = \sin\left(\varphi_1^* - \varphi_2^*\right). \tag{32}$$

The mathematical expressions for $\Delta\varphi_2$ and $\Delta\varphi_3$ are the followings:

$$\Delta\varphi_2 = \left(l_1/l_2\right) \cdot \left(C_1^*/D_1^*\right) \cdot \Delta\varphi_1 \tag{33}$$

$$\Delta\varphi_3 = \left(l_1/l_3\right) \cdot \left(C_2^*/D_1^*\right) \cdot \Delta\varphi_1. \tag{34}$$

Equations (25) are valid only for an interval of time Δt. Therefore they must be remade for the following interval of time.

4. NUMERICAL APPLICATION

In this chapter it will be presented a concrete numerical calculus for a certain four bars linkage mechanism. First time the calculus will be performed using the analytical method and then using numerical incremental method. Then, the results will be compared. In order to perform the analytical calculus we will establish first the variation interval of the independent variable which is denoted with "φ_1" and represents the angle of rotation of the element "1" which is part of the mechanism presented in the figure above (Fig. 1). We will consider that the element "1" of the mechanism executes two complete rotations around the axis O*z*:

$$\varphi_1 \in [0\,,\; 4 \cdot \pi]. \tag{35}$$

We will ascribe arbitrary values to the variable denoted with "φ_1". These arbitrary values belong to the above mentioned interval and they must cover the entire interval:

$$\varphi_{1,\mathrm{i}} = \left[(i-1)/n\right] \cdot (4 \cdot \pi). \tag{36}$$

In the relation above the index "*i*" is a natural number and it takes values into the following range:

$$i \in [1\,,\; n+1]. \tag{37}$$

The values of angle "φ_2" may be calculated using the following relation:

$$\varphi_{2,\mathrm{i}} = \delta_{2,\mathrm{i}} - \delta_{1,\mathrm{i}}. \tag{38}$$

In the relation (38) the values of the angles "δ_1" and "δ_2" will be calculated using the followings formulas:

$$\delta_{1,\mathrm{i}} = \mathrm{arctg}\left[l_1 \cdot \sin\left(\varphi_{1,i}\right) \Big/ \left(l_4 - l_1 \cdot \cos\left(\varphi_{1,i}\right)\right)\right] \tag{39}$$

$$\delta_{2,\mathrm{i}} = \arccos\left(N_{1,i} / N_{2,i}\right) \tag{40}$$

$$N_{1,i} = \left(\mathrm{O_2O_4}\right)_i^2 + l_2^2 - l_3^2 \tag{41}$$

$$N_{2,i} = 2 \cdot \left(\mathrm{O_2O_4}\right)_i \cdot l_2 \tag{42}$$

$$\left(\mathrm{O_2O_4}\right)_i = \sqrt{l_1^2 + l_4^2 - 2 \cdot l_1 \cdot l_4 \cdot \cos\left(\varphi_{1,i}\right)}. \tag{43}$$

The values of the angle "φ_3" are given by the following relationship:

$$\phi_3 = -\left(\Psi_{1,i} + \delta_{1,i}\right). \tag{44}$$

In the relationship (44) the values of the angle "δ_1" are given by the relation (39) and those of the angle "ψ_1" by the following relation:

$$\Psi_{1,i} = \arccos\left(N_{3,i} / N_{4,i}\right). \tag{45}$$

In the above relationship the values of the variables “N_3” and “N_4” may be calculated using the followings formulas:

$$N_{3,i} = l_3^2 + \left(\mathrm{O_2O_4}\right)_i^2 - l_2^2 \tag{46}$$

$$N_{4,i} = 2 \cdot l_3 \cdot \left(\mathrm{O_2O_4}\right)_i . \tag{47}$$

In the relationships (46) and (47) the values of the variable $\mathrm{O_2O_4}$ are given by the relationship (43).

The greater the value of the natural number “n” is the more accurate the calculus will be.

The value of the $\Delta\varphi_1$ will be calculated using the following formula:

$$\Delta\varphi_1 = \varphi_{1,n+1} / n = 4 \cdot \pi / n . \tag{48}$$

In order to perform the kinematics survey of the mechanism we must know its initial configuration. In other words we have to know the initial position of each element which is part of the mechanism. We will suppose that the initial value of the angle φ_1^* is known and we will express the kinematical parameters of the other elements which make up the mechanism as a function of the values of angle φ_1^*. For this we will use the relationships (1–10):

$$\varphi_2^* = \delta_2^* - \delta_1^*. \tag{49}$$

In the relationship (49) the quantities involved have the followings expressions:

$$\delta_1^* = \mathrm{arctg}\left[l_1 \cdot \sin(\varphi_1^*) \Big/ \left(l_4 - l_1 \cdot \cos\varphi_1^* \right) \right] \tag{50}$$

$$\delta_2^* = \arccos\left(N_1^* / N_2^* \right) \tag{51}$$

$$N_1^* = \left(\mathrm{O_2O_4^*}\right)^2 + l_2^2 - l_3^2 \tag{52}$$

$$N_2^* = 2 \cdot \mathrm{O_2O_4^*} \cdot l_2 \tag{53}$$

$$\mathrm{O_2O_4^*} = \sqrt{l_1^2 + l_4^2 - 2 \cdot l_1 \cdot l_4 \cdot \cos(\varphi_1^*)}. \tag{54}$$

The value of the angle φ_3^* may be calculated using the following formula:

$$\varphi_3^* = -\left(\Psi_1^* + \delta_1^*\right). \tag{55}$$

In the relation (55) the quantities involved have the followings expressions:

$$\Psi_1^* = \arccos\left(N_3^* / N_4^*\right) \tag{56}$$

$$N_3^* = l_3^2 + \left(\mathrm{O}_2\mathrm{O}_4^*\right)^2 - l_2^2 \tag{57}$$

$$N_4^* = 2 \cdot l_3 \cdot \mathrm{O}_2\mathrm{O}_4^*. \tag{58}$$

Based on the formulae above and using a MATLAB software a calculus program has been elaborated. The calculus algorithm is represented by the formulae (33) and (34). In order to effectuate a numerical calculus we will consider the following input data:

$$l_1 = 1\text{m} \tag{59}$$

$$L_2 = 4\text{m} \tag{60}$$

$$L_3 = 6\text{m} \tag{61}$$

$$l_4 = 8\text{m}. \tag{62}$$

The initial configuration of the mechanism is given by the angles φ_1^*, φ_2^* and φ_3^* as followings:

$$\varphi_1^* = 0 \text{ rad} \tag{63}$$

$$\varphi_2^* = 1.0265 \text{ rad} \tag{64}$$

$$\varphi_3^* = 2.5347 - \pi = -0.60689 \text{ rad}. \tag{65}$$

The coefficients of the algebraic system (25) will be recalculated for the next increment of time as followings:

$$\varphi_1^* = \varphi_1^* + \Delta\varphi_1 \ [\text{rad}] \tag{66}$$

$$\varphi_2^* = \varphi_2^* + \Delta\varphi_2 \ [\text{rad}] \tag{67}$$

$$\varphi_3^* = \varphi_3^* + \Delta\varphi_3 \; [\mathrm{rad}]. \tag{68}$$

The increment of time "Δt" is calculated using the following formula:

$$\Delta t = t_{i+1} - t_i. \tag{69}$$

The values of $\Delta\varphi_2$ and $\Delta\varphi_3$ will be calculated using the relations (33) and (34). The relationship between the values of angle Φ_3 and those of the angle φ_3 (Fig.1) are given by the following relation:

$$\Phi_3 = \varphi_3 + \pi. \tag{70}$$

The results are represented in Fig. 2 and Fig. 3.

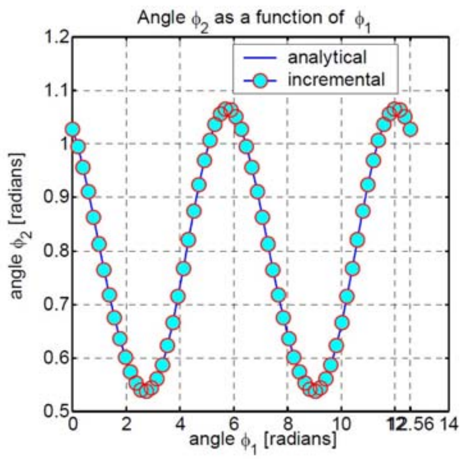

Fig. 2 – Variation of angle φ_2 as a function of φ_1.

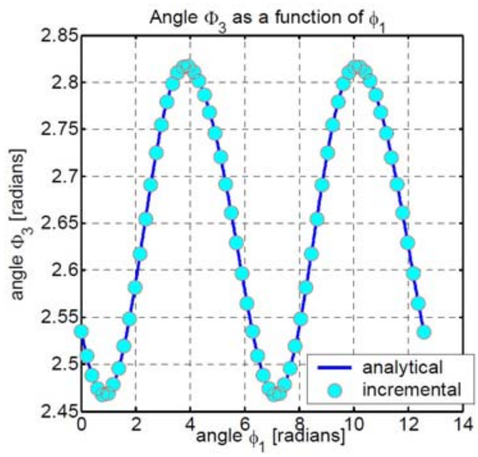

Fig. 3 – Variation of angle φ_3 as a function of φ_1.

As it can be seen in the figures above are represented both the results obtained using analytical method and incremental method so it can be compared.

5. CONCLUSIONS

The results obtained using analytical method are practically identical with those obtained by incremental method.

The numerical method presented in the paper is very general and for this reason it may be used to perform the kinematics survey of any mechanical system no matter how complex it might be. The mechanism the kinematics of which has been presented in this paper is only an example so that the numerical method shown in the paper could be understood easily.

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