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INTERNAL FORCES IN DYNAMICS OF A TRANSLATIONAL 3-PUU PARALLEL ROBOT

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Abstract. Matrix relations in dynamics analysis of a spatial 3-PUU translational parallel manipulator are established in this paper. Giving the evolution of the platform, an inverse dynamic problem is solved based on a set of recursive explicit equations. Finally, compact relations and graphs of simulation for the input forces of three actuators and also for some internal joint forces are obtained.

Key words: dynamics, joint forces, parallel robot, virtual velocities.

1. INTRODUCTION

Provided with closed-loop structures, the links of the parallel robots can be connected one to the other by spherical joints, universal joints, revolute joints or prismatic joints. Compared with traditional serial manipulators, the accuracy and precision in the direction of the tasks are essential for the parallel architectures [1, 2].

During last three decades, considerable efforts have been devoted to the dynamics of parallel robots. Among these, the class of manipulators known as Stewart-Gough platform, used in flight simulators, focused great attention [3, 4]. Delta parallel manipulator (Clavel [5], Tsai and Stamper [6]) and Star parallel robot (Hervé and Sparacino [7]) are equipped with three motors, which train on the mobile platform in a general translation motion. Angeles [8], Wang and Gosselin [9] analysed the kinematics, singularity loci and determined the workspace of spherical Agile Wrist robot with three concurrent actuators.

To generate a complete dynamic modelling, most traditional methods are used such as Newton-Euler formulation [10], Lagrange formalism [11] and the Principle of virtual work [12]. Based on the Kane's equations with undetermined multipliers of constraints, Salinic [13] presents an approach for determination of joint reaction forces in a symbolic form for planar and spatial mechanisms. Using three different numerical methods, Wojtyra [14] can uniquely specify some selected groups of constraint reactions.

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Taking into consideration the frictionless revolute and prismatic joints, a recursive method based on explicit equations of parallel robots dynamics is applied in the present paper to the analysis of a spatial 3-DOF mechanism, reducing together the number of equations and computation operations significantly by using a set of matrices for dynamics model. The joint reactions are required for the analysis of the stress state in joints on the basis of which dimensioning of the joint constructive elements is done.

2. KINEMATICS ANALYSIS

The structure of the 3-PUU parallel manipulator consists of a fixed triangular base, a circular mobile platform and three legs with identical architecture. The architecture of one of the three kinematical chains of the robot is made of an electrical motor, a helical joint that connect a lead screw to a translational nut and an intermediary rectilinear leg connecting finally the moving platform. As explained in our paper [15], driven by the lead screw linear actuator, each limb connects the fixed base to the moving platform by a prismatic joint followed by two universal joints in sequence (Fig. 1).

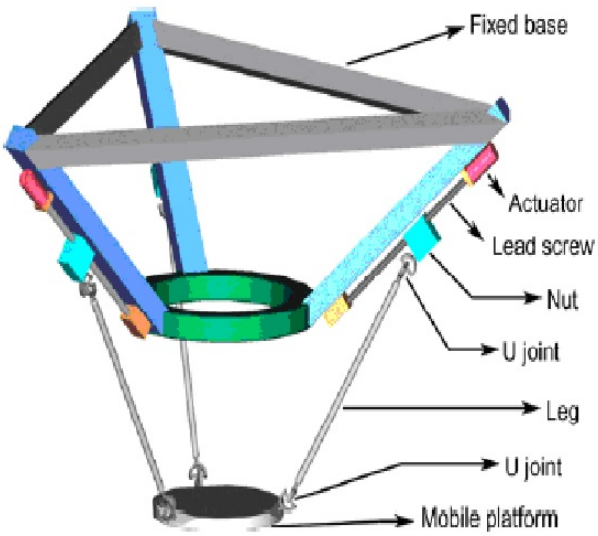

Fig. 1 – Virtual prototype for the 3-PUU parallel robot.

To describe the position of the manipulator, we assign a fixed Cartesian coordinate system $Ox_0y_0z_0(T_0)$ at the centred point O of the fixed base platform and a mobile frame $\varepsilon_{43}^{A} = \ddot{\varphi}_{43}^{A}$ on the moving platform at its mass centre G. The platform is initially located at a *central configuration*, where this is not translated with respect to the fixed base and the origin O of fixed frame is located at an elevation m_4 above the centre G. The lead screw of first leg $A_4x_4^Ay_4^Az_4^A$ ($\alpha_A = 0$) is typically contained within the $\varepsilon_{32}^A = \ddot{\varphi}_{32}^A$ vertical plane, whereas the planes of remaining lead screws of legs B, C make the angles $\omega_{32}^A = \dot{\varphi}_{32}^A$ and $\alpha_C = -120°$ with Ox_0z_0. The angle $A_3z_3^A$ of deviation of first joint's line GA_5 in the platform's plane, for example, is defined as *twist* angle of the manipulator. The lines of action of three prismatic joints may be inclined from the fixed base by a constant angle $\hat{J}_3$ as architectural parameter (Fig. 2).

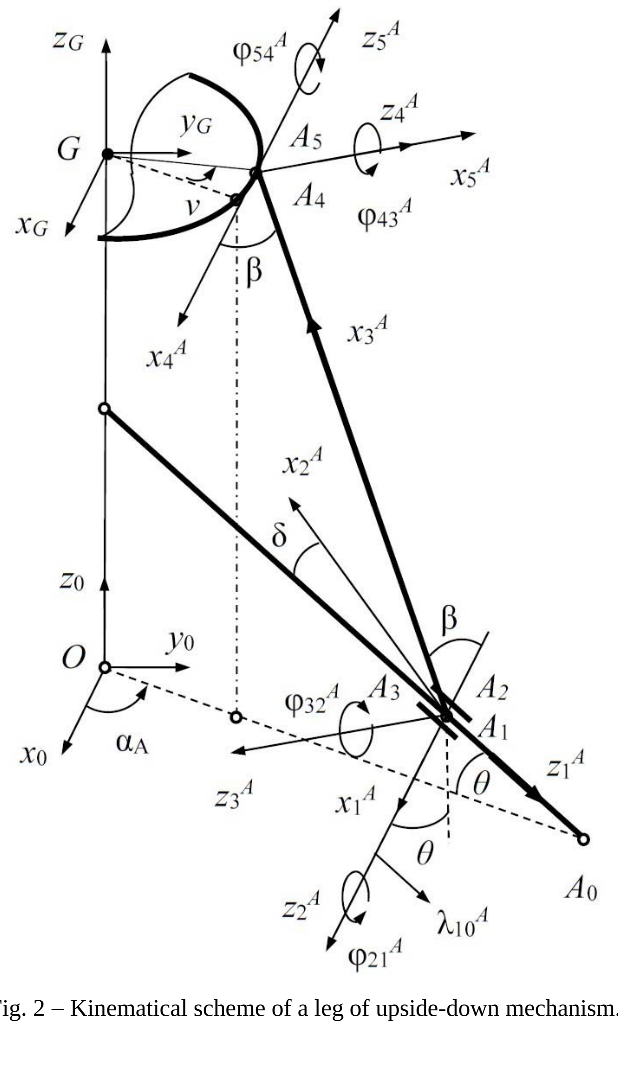

Fig. 2 – Kinematical scheme of a leg of upside-down mechanism.

The first active leg A, for example, consists of the homogenous *lead screw*, the prismatic joint with the nut of mass m_1 linked at the $A_1x_1^Ay_1^Az_1^A$ moving frame, having a translation with its absolute displacement λ_{10}^A proportional with the rotation angle of the screw, the velocity $v_{10}^A = \dot{\lambda}_{10}^A$ and the acceleration $\gamma_{10}^A = \ddot{\lambda}_{10}^A$, the cross of an universal joint $A_2x_2^Ay_2^Az_2^A$ characterized by the mass m_2, the angular velocity $\omega_{21}^A = \dot{\varphi}_{21}^A$ and the angular acceleration $\varepsilon_{21}^A = \ddot{\varphi}_{21}^A$ and the moving link $A_3x_3^Ay_3^Az_3^A$ of length $A_3A_4 = l_2$, mass m_3 and tensor of inertia $\hat{J}_3$, having a relative rotation about $A_3z_3^A$ axis with the angular velocity $\omega_{32}^A = \dot{\varphi}_{32}^A$ and the angular acceleration $\varepsilon_{32}^A = \ddot{\varphi}_{32}^A$. Finally, a second universal joint $A_4x_4^Ay_4^Az_4^A$ having the mass m_4, the angular velocity $\omega_{43}^A = \dot{\varphi}_{43}^A$ and the angular acceleration $\varepsilon_{43}^A = \ddot{\varphi}_{43}^A$ is introduced at the edge of a moving platform, which can be schematised as a circle of radius r and mass m_p in a relative rotation around $A_5z_5^A$ axis with the angular velocity $\omega_{54}^A = \dot{\varphi}_{54}^A$ and the angular acceleration $\varepsilon_{54}^A = \ddot{\varphi}_{54}^A$.

At the central configuration, we also consider that the three sliders are initially starting from same position $A_0A_1 = l_1 = h\sin\theta + (l_0 - r\cos\nu)\cos\theta - l_2\sin\beta\cos\delta$ and that the angles of orientation of the legs are given by

$$\alpha_A = 0, \quad \alpha_B = \frac{2\pi}{3}, \quad \alpha_C = -\frac{2\pi}{3}, \quad \theta = \frac{\pi}{6}, \quad \nu = \frac{\pi}{6}$$

$$l_2\cos\beta = h\cos\theta - (l_0 - r\cos\nu)\sin\theta, \; l_2\sin\beta\sin\delta = r\sin\nu. \tag{1}$$

Starting from the reference origin O and pursuing three independent legs $OA_0A_1A_2A_3A_4A_5$, $OB_0B_1B_2B_3B_4B_5$, $OC_0C_1C_2C_3C_4C_5$, we obtain the transformation matrices

$$p_{10} = a_\theta\theta_1a_\alpha^i, \; p_{21} = p_{21}^\varphi a_\delta\theta_1, \; p_{32} = p_{32}^\varphi a_\beta\theta_1\theta_2^{\mathrm{T}}, \; p_{43} = p_{43}^\varphi a_\beta\theta_3, \; p_{54} = p_{54}^\varphi\theta_1^{\mathrm{T}}$$

$$p_{n0} = \prod_{\tau=1}^{n} p_{n-\tau+1,n-\tau} \quad (p = a, b, c), (i = A, B, C), (n = 2, 3, 4, 5), \tag{2}$$

where we denote the matrices

$$\begin{gathered} a_\alpha^i = \mathrm{rot}(z,\alpha_i), \quad a_\beta = \mathrm{rot}(z,\beta), \quad a_\delta = \mathrm{rot}(z,\delta), \quad a_\theta = \mathrm{rot}(y,\theta) \\ \theta_1 = \mathrm{rot}(y,\pi/2), \quad \theta_2 = \mathrm{rot}(z,\pi/2), \quad \theta_3 = \mathrm{rot}(y,\pi), \end{gathered} \tag{3}$$

$$p^{\varphi}_{k,k-1} = \mathrm{rot}(z, \varphi^{i}_{k,k-1}) \quad (k = 1, 2, \dots, 5).$$

In the inverse geometric problem, the position of the mechanism is completely given through the coordinates x_0^G, y_0^G, z_0^G of the mass center G. Consider, for example, that during three seconds the moving platform remain in same orientation and the motion of the centre G along a *rectilinear trajectory* is expressed in fixed frame $Ox_0y_0z_0$ through the following analytical functions

$$\frac{x_0^G}{x_0^{G*}} = \frac{y_0^G}{y_0^{G*}} = \frac{h - z_0^G}{z_0^{G*}} = 1 - \cos\frac{\pi}{3}t, \tag{4}$$

where the values $2x_0^{G*}, 2y_0^{G*}, 2z_0^{G*}$ denote the final position of the moving platform.

The *translation conditions* concerning the absolute orientation of the moving platform are given by the following identities

$$\begin{aligned} &p^{\circ \mathrm{T}}_{50}\, p_{50} = R = I\,, \quad (p = a, b, c), \quad (i = A, B, C) \\ &p^{\circ}_{50} = p_{50}(t = 0) = \theta_1^{\mathrm{T}} a_\beta \theta_3 a_\beta \theta_1 \theta_2^{\mathrm{T}} a_\delta \theta_1 a_\theta \theta_1 a^{i}_{\alpha}\,, \end{aligned} \tag{5}$$

where $R = I$ is the diagonal identity matrix. From these conditions we obtain the relations between the rotation angles $\varphi^i_{43} = \varphi^i_{32}, \varphi^i_{54} = \varphi^i_{21}$ $(i = A, B, C)$.

Pursuing the kinematical modelling developed in our published paper [15], nine independent variables $\lambda^A_{10}, \varphi^A_{21}, \varphi^A_{32}, \lambda^B_{10}, \varphi^B_{21}, \varphi^B_{32}, \lambda^C_{10}, \varphi^C_{21}, \varphi^C_{32}$ are determined from nine analytical equations.

Now, we compute the relative velocities $\vec{V}_i = [v^i_{10} \;\; \omega^i_{21} \;\; \omega^i_{32}]^{\mathrm{T}}$ in terms of the velocity of centre G, starting from the *matrix conditions of connectivity* [16]:

$$\vec{V}_i = [Q_i]^{-1} \vec{P}_i\,, \tag{6}$$

where some terms determines the contents of 3×3 invertible square matrix $[Q_i]$ and the column matrix $\vec{P}_i$:

$$\begin{aligned} &q^i_{j1} = u^{\mathrm{T}}_j p^{\mathrm{T}}_{10} \vec{u}_3\,, \quad q^i_{j2} = \vec{u}^{\mathrm{T}}_j p^{\mathrm{T}}_{20} \tilde{u}_3 p^{\mathrm{T}}_{32} \vec{r}^{\,i}_{43}\,, \quad q^i_{j3} = \vec{u}^{\mathrm{T}}_j p^{\mathrm{T}}_{30} \tilde{u}_3 \vec{r}^{\,i}_{43}\,, \\ &\vec{P}_i = \dot{\vec{r}}^{\,G}_0 = [\dot{x}^G_0 \quad \dot{y}^G_0 \quad \dot{z}^G_0]^{\mathrm{T}} \quad (j = 1, 2, 3). \end{aligned} \tag{7}$$

The matrix kinematical relations (6) will be further required in the computation of virtual velocity distribution of the elements of the manipulator. Concerning the first leg A, for example, the characteristic *virtual velocities* are expressed as functions of the pose of the mechanism at any time by the general equations (6), where we add the contributions of some virtual translations during the fictitious displacements of the revolute joints A_2, A_3, A_4, A_5 as follows:

$$v_{10a}^{Av}\vec{u}_j^{\mathrm{T}} a_{10}^{\mathrm{T}}\vec{u}_3 + v_{21a}^{Av}\vec{u}_j^{\mathrm{T}} a_{10}^{\mathrm{T}}\vec{u}_1 + \omega_{21a}^{Av}\vec{u}_j^{\mathrm{T}} a_{20}^{\mathrm{T}}\tilde{u}_3 a_{32}^{\mathrm{T}}\vec{r}_{43}^{A} + v_{32a}^{Av}\vec{u}_j^{\mathrm{T}} a_{20}^{\mathrm{T}}\vec{u}_2 +$$
$$+\omega_{32a}^{Av}\vec{u}_j^{\mathrm{T}} a_{30}^{\mathrm{T}}\tilde{u}_3\vec{r}_{43}^{A} + v_{43a}^{Av}\vec{u}_j^{\mathrm{T}} a_{30}^{\mathrm{T}}\vec{u}_3 + v_{54a}^{Av}\vec{u}_j^{\mathrm{T}} a_{40}^{\mathrm{T}}\vec{u}_1 = \vec{u}_j^{\mathrm{T}}\vec{v}_0^{Gv} \quad (j = 1, 2, 3)\,. \tag{8}$$

Let us assume that the robot has successively virtual motions expressed through following velocities:

$$\begin{aligned}
&v_{10a}^{Av} = 1,\ v_{10a}^{Bv} = 0,\ v_{10a}^{Cv} = 0,\ v_{21a}^{iv} = 0,\ v_{32a}^{iv} = 0,\ v_{43a}^{iv} = 0,\ v_{54a}^{iv} = 0\\
&v_{10a}^{iv} = 0,\ v_{21a}^{Av} = 1,\ v_{21a}^{Bv} = 0,\ v_{21a}^{Cv} = 0,\ v_{32a}^{iv} = 0,\ v_{43a}^{iv} = 0,\ v_{54a}^{iv} = 0\\
&v_{10a}^{iv} = 0,\ v_{21a}^{iv} = 0,\ v_{32a}^{Av} = 1,\ v_{32a}^{Bv} = 0,\ v_{32a}^{Cv} = 0,\ v_{43a}^{iv} = 0,\ v_{54a}^{iv} = 0\\
&v_{10a}^{iv} = 0,\ v_{21a}^{iv} = 0,\ v_{32a}^{iv} = 0,\ v_{43a}^{Av} = 1,\ v_{43a}^{Bv} = 0,\ v_{43a}^{Cv} = 0,\ v_{54a}^{iv} = 0\\
&v_{10a}^{iv} = 0, v_{21a}^{iv} = 0, v_{32a}^{iv} = 0, v_{43a}^{iv} = 0, v_{54a}^{Av} = 1, v_{54a}^{Bv} = 0, v_{54a}^{Cv} = 0\\
&\qquad (i = A, B, C)\,.
\end{aligned} \tag{9}$$

The virtual velocities of the component elements of the mechanism are given by the general kinematical conditions (8).

Based on another conditions of connectivity, expressions of relative accelerations $\vec{\Gamma}_i = [\gamma_{10}^i \quad \varepsilon_{21}^i \quad \varepsilon_{32}^i]^{\mathrm{T}}$ are obtained from the column matrix

$$\vec{\Gamma}_i = [Q_i]^{-1}\vec{S}_i\,, \tag{10}$$

where following terms determine the contents of column matrix $\vec{S}_i = \dot{\vec{P}}_i - [\dot{Q}_i]\vec{V}_i$:

$$s_j^i = \vec{u}_j^{\mathrm{T}}\ddot{\vec{r}}_0^{G} - \omega_{21}^i\omega_{21}^i\vec{u}_j^{\mathrm{T}} p_{20}^{\mathrm{T}}\tilde{u}_3\tilde{u}_3 p_{32}^{\mathrm{T}}\vec{r}_{43}^{i} - \omega_{32}^i\omega_{32}^i\vec{u}_j^{\mathrm{T}} p_{30}^{\mathrm{T}}\tilde{u}_3\tilde{u}_3\vec{r}_{43}^{i}$$
$$-2\omega_{21}^i\omega_{32}^i\vec{u}_j^{\mathrm{T}} p_{20}^{\mathrm{T}}\tilde{u}_3 p_{32}^{\mathrm{T}}\tilde{u}_3\vec{r}_{43}^{i} \quad (j = 1, 2, 3)\,. \tag{11}$$

3. DYNAMICS MODELLING

In the context of a real-time control, neglecting the frictions forces and considering the gravitational effects, the relevant objective of the dynamics is first to determine the input torques or forces, which must be exerted by the actuators in order to produce a given trajectory of the end-effector, but also to calculate all internal joint reaction forces or torques.

Some exact relations that give in real-time the position, velocity and acceleration of each element of a two-module hybrid parallel robot have been established in the present paper. Choosing the appropriate serial kinematical circuits connecting many moving platforms, the present concept and the procedure above developed can be immediately extended to analysis of a complex robotic system composed of a multitude serially arranged similar parallel modules, but are also applicable to study of hybrid robots that are composed of different structures of parallel modules, where the number of links of the mechanisms is increased and the value of total degrees-of-freedom is augmented.

A lot of works have focused on the dynamics of Stewart platform. Dasgupta and Mruthyunjaya [10] used the Newton-Euler approach to develop closed-form dynamic equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints. Tsai [1] presented an algorithm to solve the inverse dynamics for a Delta platform-type using Newton-Euler equations. This commonly known approach requires computation of all constraint forces and moments between the links. Geng [11] developed Lagrange equations of motion under some simplifying assumptions regarding the geometry and inertia distribution of the manipulator.

Knowing the kinematics state of each link as well as the external forces acting on the 3-PUU parallel manipulator and neglecting the frictional forces at the joints, in the present paper we apply some explicit recursive matrix equations for the inverse dynamic problem in order to obtain the input forces of actuators and also the internal reaction forces in the joints.

Three independent mechanical systems control the motion of the moving platform, using lead screws which generate the forces $\vec{f}_{10}^{A} = f_{10}^{A}\vec{u}_3$, $\vec{f}_{10}^{B} = f_{10}^{B}\vec{u}_3$, $\vec{f}_{10}^{C} = f_{10}^{C}\vec{u}_3$ along the directions z_1^A, z_1^B, z_1^C of three rails. The parallel robot has the form of a multi-body structure which moves in a uniform gravitational field. This spatial mechanism can artificially be transformed in a set of three open chains C_i $(i = A, B, C)$ subject to the constraints. This is possible by imaginary cutting of each joint for the moving platform, and takes its effect into account by introducing the corresponding constraint conditions.

Using a recursive form of velocities $\vec{\omega}_{k0}^{A}, \vec{v}_{k0}^{A}$ and accelerations $\varepsilon_{k0}^{A}, \vec{\gamma}_{k0}^{A}$, in terms of vectors of preceding body [15]:

$$\vec{\omega}_{k0}^{A} = a_{k,k-1}\vec{\omega}_{k-1,0}^{A} + \dot{\varphi}_{k,k-1}^{A}\vec{u}_3\,, \quad \vec{v}_{k0}^{A} = a_{k,k-1}\vec{v}_{k-1,0}^{A} + a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}\vec{r}_{k,k-1}^{A} + \dot{\lambda}_{k,k-1}^{A}\vec{u}_3$$

$$\vec{\varepsilon}_{k0}^{A} = a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{A} + \ddot{\varphi}_{k,k-1}^{A}\vec{u}_3 + \dot{\varphi}_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{\mathrm{T}}\vec{u}_3$$

$$\begin{aligned}\vec{\gamma}_{k0}^{A} = {} & a_{k,k-1}\vec{\gamma}_{k-1,0}^{A} + a_{k,k-1}\{\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A} + \tilde{\varepsilon}_{k-1,0}^{A}\}\vec{r}_{k,k-1}^{A} + \\ & + 2\dot{\lambda}_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{\mathrm{T}}\vec{u}_3 + \ddot{\lambda}_{k,k-1}^{A}\vec{u}_3\end{aligned} \tag{12}$$

the force of inertia and the resulting moment of the forces of inertia of an arbitrary rigid body T_k^A, for example,

$$\vec{f}_{k0}^{\,inA} = -m_k^A\left[\vec{\gamma}_{k0}^A + \left(\tilde{\omega}_{k0}^A\tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A\right)\vec{r}_k^{\,CA}\right]$$
$$\vec{m}_{k0}^{\,inA} = -[m_k^A\tilde{r}_k^{CA}\vec{\gamma}_{k0}^A + \hat{J}_k^A\vec{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A\hat{J}_k^A\vec{\omega}_{k0}^A] \tag{13}$$

can be determined with respect to the centre of joint A_k. On the other hand, the wrench of two vectors $\vec{f}_k^{\,*A}$ and $\vec{m}_k^{\,*A}$ evaluates the influence of the action of the weight $m_k^A\vec{g}$ and of other external and eventually internal forces applied to the same element T_k^A of the manipulator:

$$\vec{f}_k^{\,*A} = m_k^A g a_{k0}\vec{u}_3,\quad \vec{m}_k^{\,*A} = m_k^A g\tilde{r}_k^{CA} a_{k0}\vec{u}_3 \quad (k = 1, 2, 3, 4)\,. \tag{14}$$

Applying the explicit form of the *equations of parallel robots dynamics* [16, 17], following compact matrix relations results

$$\begin{aligned} f_{10}^A = \vec{u}_3^{\mathrm{T}}\vec{F}_1^A + \vec{u}_3^{\mathrm{T}}\{&\omega_{21a}^{Av}\vec{M}_2^A + \omega_{32a}^{Av}\vec{M}_3^A + \omega_{43a}^{Av}\vec{M}_4^A + \\ &+\omega_{21a}^{Bv}\vec{M}_2^B + \omega_{32a}^{Bv}\vec{M}_3^B + \omega_{43a}^{Bv}\vec{M}_4^B + \omega_{21a}^{Cv}\vec{M}_2^C + \\ &+\omega_{32a}^{Cv}\vec{M}_3^C + \omega_{43a}^{Cv}\vec{M}_4^C + v_{10a}^{Gv}\vec{F}_1^G + v_{21a}^{Gv}\vec{F}_2^G + v_{32a}^{Gv}\vec{F}_3^G\} \end{aligned} \tag{15}$$

for the *input force* required by the first active prismatic joint,

$$\begin{aligned} f_{21}^A = \vec{u}_1^{\mathrm{T}} a_{21}^{\mathrm{T}}\vec{F}_2^A + \vec{u}_3^{\mathrm{T}}\{&\omega_{21a}^{Av}\vec{M}_2^A + \omega_{32a}^{Av}\vec{M}_3^A + \omega_{43a}^{Av}\vec{M}_4^A + \\ &+\omega_{21a}^{Bv}\vec{M}_2^B + \omega_{32a}^{Bv}\vec{M}_3^B + \omega_{43a}^{Bv}\vec{M}_4^B + \omega_{21a}^{Cv}\vec{M}_2^C + \\ &+\omega_{32a}^{Cv}\vec{M}_3^C + \omega_{43a}^{Cv}\vec{M}_4^C + v_{10a}^{Gv}\vec{F}_1^G + v_{21a}^{Gv}\vec{F}_2^G + v_{32a}^{Gv}\vec{F}_3^G\} \end{aligned} \tag{16}$$

for the *internal joint reaction force* acting along the axis $A_2 z_2^A$,

$$\begin{aligned} f_{32}^A = \vec{u}_2^{\mathrm{T}} a_{32}^{\mathrm{T}}\vec{F}_3^A + \vec{u}_3^{\mathrm{T}}\{&\omega_{21a}^{Av}\vec{M}_2^A + \omega_{32a}^{Av}\vec{M}_3^A + \omega_{43a}^{Av}\vec{M}_4^A + \\ &+\omega_{21a}^{Bv}\vec{M}_2^B + \omega_{32a}^{Bv}\vec{M}_3^B + \omega_{43a}^{Bv}\vec{M}_4^B + \omega_{21a}^{Cv}\vec{M}_2^C + \\ &+\omega_{32a}^{Cv}\vec{M}_3^C + \omega_{43a}^{Cv}\vec{M}_4^C + v_{10a}^{Gv}\vec{F}_1^G + v_{21a}^{Gv}\vec{F}_2^G + v_{32a}^{Gv}\vec{F}_3^G\} \end{aligned} \tag{17}$$

for the *internal joint reaction force* acting along the axis $A_3 z_3^A$,

$$\begin{aligned} f_{43}^A = \vec{u}_3^{\mathrm{T}} a_{43}^{\mathrm{T}}\vec{F}_4^A + \vec{u}_3^{\mathrm{T}}\{&\omega_{21a}^{Av}\vec{M}_2^A + \omega_{32a}^{Av}\vec{M}_3^A + \omega_{43a}^{Av}\vec{M}_4^A \\ &+\omega_{21a}^{Bv}\vec{M}_2^B + \omega_{32a}^{Bv}\vec{M}_3^B + \omega_{43a}^{Bv}\vec{M}_4^B + \omega_{21a}^{Cv}\vec{M}_2^C + \\ &+\omega_{32a}^{Cv}\vec{M}_3^C + \omega_{43a}^{Cv}\vec{M}_4^C + v_{10a}^{Gv}\vec{F}_1^G + v_{21a}^{Gv}\vec{F}_2^G + v_{32a}^{Gv}\vec{F}_3^G\} \end{aligned} \tag{18}$$

for the *internal joint reaction force* acting along the axis $A_4 z_4^A$, and

$$f_{54}^A = \vec{u}_3^{\mathrm{T}}\{\omega_{21a}^{Av}\vec{M}_2^A + \omega_{32a}^{Av}\vec{M}_3^A + \omega_{43a}^{Av}\vec{M}_4^A + \omega_{21a}^{Bv}\vec{M}_2^B + \omega_{32a}^{Bv}\vec{M}_3^B + \omega_{43a}^{Bv}\vec{M}_4^B + \\ + \omega_{21a}^{Cv}\vec{M}_2^C + \omega_{32a}^{Cv}\vec{M}_3^C + \omega_{43a}^{Cv}\vec{M}_4^C + v_{10a}^{Gv}\vec{F}_1^G + v_{21a}^{Gv}\vec{F}_2^G + v_{32a}^{Gv}\vec{F}_3^G\} \qquad (19)$$

for the *internal joint reaction force* acting along the axis $A_5 z_5^A$ from the first leg *A* (Fig. 3).

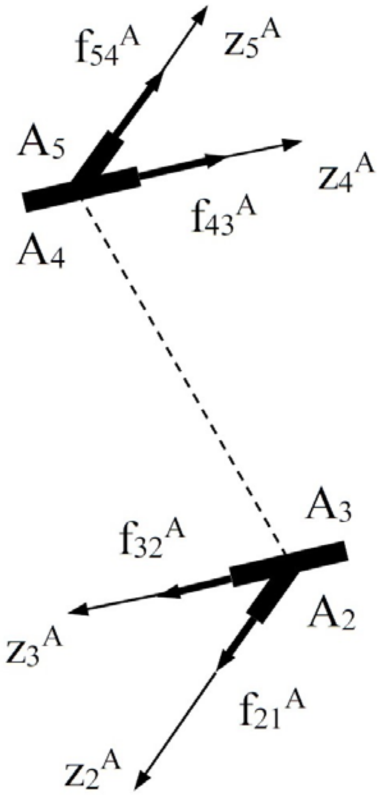

Fig. 3 – Internal joint reaction forces.

Two significant recursive relations generate the vectors for the leg *A*:

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^A, \quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^{\mathrm{T}}\vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^{\mathrm{T}}\vec{F}_{k+1}^A \qquad (20)$$

with the notations

$$\vec{F}_{k0}^A = -\vec{f}_{k0}^{inA} - \vec{f}_k^{*A}, \quad \vec{M}_{k0}^A = -\vec{m}_{k0}^{inA} - \vec{m}_k^{*A}. \qquad (21)$$

As example, starting from (20), we develop a set of eight recursive matrix relations

$$\vec{F}_4^A = \vec{F}_{40}^A\,,\quad \vec{F}_3^A = \vec{F}_{30}^A + a_{43}^{\mathrm{T}}\vec{F}_4^A\,,\quad \vec{F}_2^A = \vec{F}_{20}^A + a_{32}^{\mathrm{T}}\vec{F}_3^A\,,\quad \vec{F}_1^A = \vec{F}_{10}^A + a_{21}^{\mathrm{T}}\vec{F}_2^A$$

$$\vec{M}_4^A = \vec{M}_{40}^A\,,\ \ \vec{M}_3^A = \vec{M}_{30}^A + a_{43}^{\mathrm{T}}\vec{M}_4^A + \tilde{r}_{43}^A a_{43}^{\mathrm{T}}\vec{F}_4^A\,,\ \ \vec{M}_2^A = \vec{M}_{20}^A + a_{32}^{\mathrm{T}}\vec{M}_3^A + \tilde{r}_{32}^A a_{32}^{\mathrm{T}}\vec{F}_3^A$$

$$\vec{M}_1^A = \vec{M}_{10}^A + a_{21}^{\mathrm{T}}\vec{M}_2^A + \tilde{r}_{21}^A a_{21}^{\mathrm{T}}\vec{F}_2^A\,. \tag{22}$$

The relations (15–22) represent the *complete inverse dynamics model* of the 3-PUU parallel robot. Various dynamical effects, including the inertial Coriolis forces, the centrifugal forces coupling and the gravitational actions are considered. Based on the free-body-diagram method and applying the Newton-Euler procedure, a set of analytical equations established for each compounding rigid body could eventually constitute verification for the results obtained by this novel approach based on recursive explicit equations.

As application let us consider same spatial parallel manipulator 3-PUU analysed in [15], which has the following architectural and mechanical characteristics:

$$x_0^{G*} = 0.05\,\mathrm{m}\,,\ \ y_0^{G*} = 0.05\,\mathrm{m}\,,\ \ z_0^{G*} = 0.15\,\mathrm{m}$$

$$r = 0.1\mathrm{m},\ \ OA_0 = l_0 = 0.3\mathrm{m},\ \ l_2 = 0.3\mathrm{m},\ \ h = 0.4\,\mathrm{m},\ g = 9.807\,\mathrm{ms}^{-2},\ \Delta t = 3\mathrm{s}$$

$$m_1 = 1.297\ \mathrm{kg},\quad m_2 = 0.2\ \mathrm{kg},\quad m_3 = 0.906\ \mathrm{kg},\quad m_4 = 0.2\ \mathrm{kg},\quad m_p = 3.982\ \mathrm{kg}$$

$$A_0A_1 = l_1 = h\sin\theta + (l_0 - r\cos\nu)\cos\theta - l_2\sin\beta\cos\delta\,.$$

Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the 3-PUU parallel robot, supposing that there are no additional external forces and moments acting on the moving platform. To develop the algorithm, it is assumed that the platform starts at rest from a central configuration and moves pursuing successively two rectilinear translations.

Some examples are solved to illustrate the algorithm. For the first example, the platform moves along the z_0 *vertical direction* with variable acceleration while other positional parameters are held equal to zero.

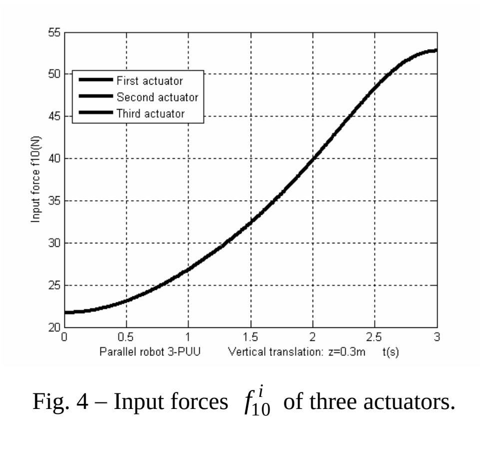

Fig. 4 – Input forces f_{10}^i of three actuators.

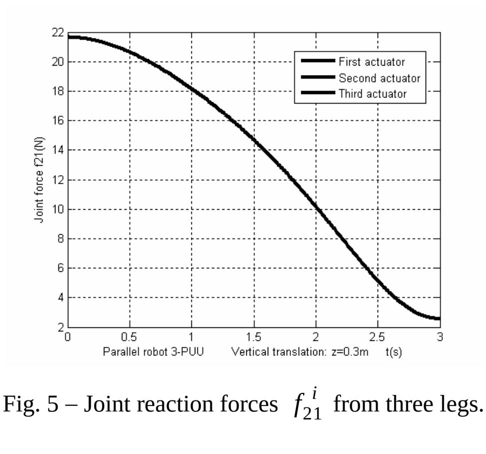

Fig. 5 – Joint reaction forces f_{21}^i from three legs.

The time-history for the input forces f_{10}^{i} (Fig. 4) of prismatic actuators and also the axial joint reaction forces f_{21}^{i} (Fig. 5), f_{32}^{i} (Fig. 6), f_{43}^{i} (Fig. 7) and f_{54}^{i} (Fig. 8) exerted along the axes $A_2z_2^A$, $B_2z_2^B$, $C_2z_2^C$, $A_3z_3^A$, $B_3z_3^B$, $C_3z_3^C$, $A_4z_4^A$, $B_4z_4^B$, $C_4z_4^C$, $A_5z_5^A$, $B_5z_5^B$ and $C_5z_5^4$ respectively, are derived for a period of $\Delta t = 3s$ in terms of equations (4). It is proved to be true that all actuating forces and the internal reaction forces in joints are permanently equal to one another.

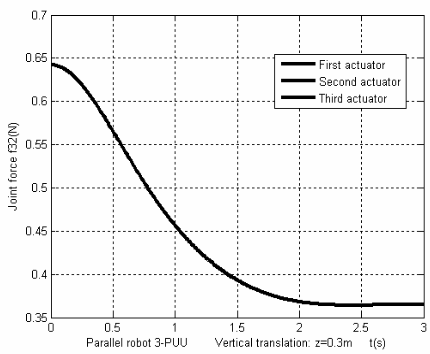

Fig. 6 – Joint reaction forces f_{32}^{i} from three legs.

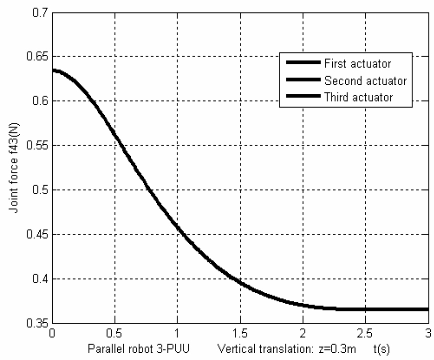

Fig. 7 – Joint reaction forces f_{43}^{i} from three legs.

For the case when the platform's center G moves along a *rectilinear horizontal trajectory* without any rotation of the platform, the graphs are depicted in Figs. 9–13.

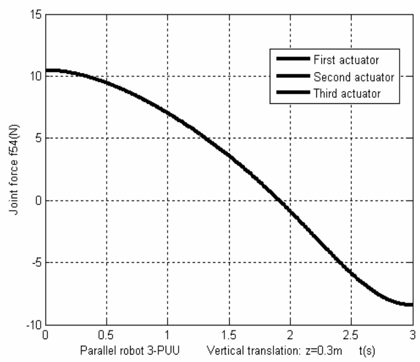

Fig. 8 – Joint reaction forces f_{54}^{i} from three legs.

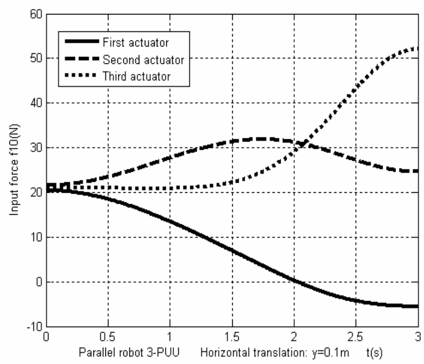

Fig. 9 – Input forces f_{10}^{i} of three actuators.

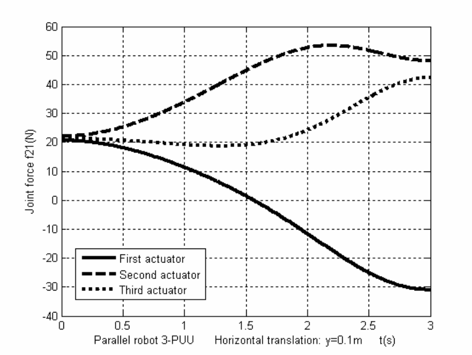

Fig. 10 – Joint reaction forces f_{21}^{i} from three legs.

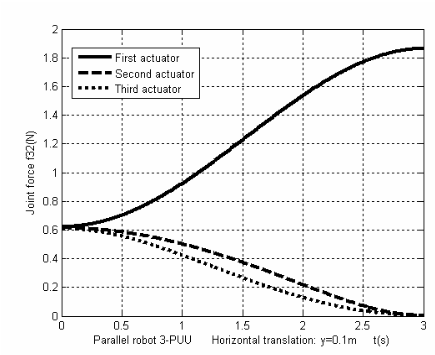

Fig. 11 – Joint reaction forces f_{32}^{i} from three legs.

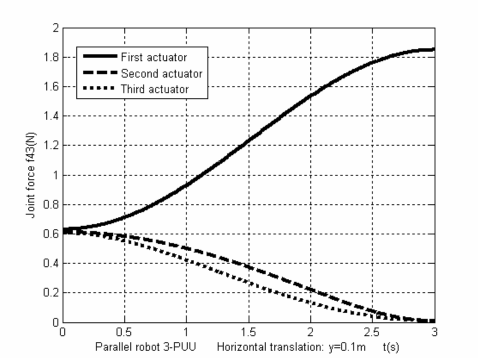

Fig. 12 – Joint reaction forces f_{43}^{i} from three legs.

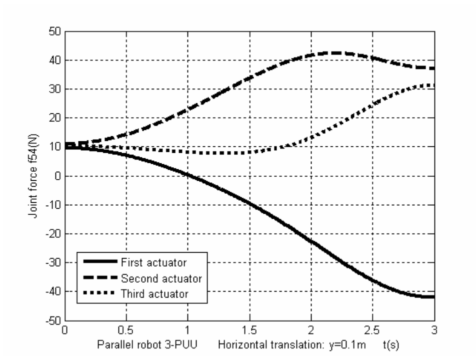

Fig. 13 – Joint reaction forces f_{54}^{i} from three legs.

The simulation through the program certifies that one of the major advantages of the current matrix recursive formulation is accuracy and a smaller processing time of numerical computation.

4. CONCLUSIONS

Using a set of explicit recursive matrix equations, based on the principle of virtual work, already implemented in dynamics of parallel robots, a novel algorithm takes into consideration the masses and forces of inertia introduced by all component elements and establishes the time-history evolution of the active forces and particularly the intensities of the internal reaction forces or torques in joints.

The current matrix recursive formulation can easily be transformed in a model which is successfully expected to be deployed for automatic robotic control of the parallel robots and that one of its major advantages is the effectiveness and accuracy of numerical computation. The present procedure can be extended in mechanics of parallel hybrid manipulators, where the number of links of the mechanisms is increased and the value of total degrees-of-freedom is augmented.

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