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ON ENVIRONMENT MATHEMATICAL MODEL AND ON IMPROVED STABLE EVOLUTION IN THESE HYPOTHESES *

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Abstract. The subject of the paper is focused on the mathematical characterization of the environment through the mathematical model of the dynamic systems in general case when depend on parameters. A lot of results on the Liapunov stability of the dynamic system that depends on parameters, performed by us, are selected and explicitly accepted as properties that must describe the dynamic system of the environment. The property of separation between stable and unstable regions, in the domain of free parameters, on the matrix attached to the linear dynamic system mathematical model or to the "first approximation" of the nonlinear dynamic system was analysed. Our study is referred, as example, on particular case of biped walking robot model described by us in the paper that opened a way to perform the walking robot problems. Existence of the stable regions in the free parameters domain assures the possibility to realize stability control on each such region using a compatible criterion. A method for improved stable evolution of the environment's dynamic system is proposed and analyzed on our case of biped walking robot where an important problem is selection of the parameters domain such that the dynamic system there exists and another important problem is optimization of stable evolution.

Key words: environment, dynamic/kinematics system, free parameters, stability control, biped walking robot, dynamic/kinematics analyze.

1. INTRODUCTION

Environment mathematical model is described by the dynamic systems in general case as function of relevant parameters [1–19], without specifying its values, as geometrical parameters that describe the system, physical parameters (in

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particular mechanical parameters), possible chemical, economical, biological parameters etc. A phenomenon of the environment can be approached by the linear or non linear dynamic system mathematical model, discussed and here together with the fundamental notion on the stability, in sense of Liapunov, of the dynamic system evolution [12–15]. The linear dynamic system is defined through the matrix that has the component functions of the matrix assumed to be with real values, and the matrices that intervene in the exposure of the stability analysis method are also assumed with real values components [20, 21]. We called in the last papers on the subject the hypothesis that the matrices from description of the mathematical model have the complex values such that the real values are taken into account as special cases of complex values [22]. This hypothesis assures a simplified method of analysis, in the complex domain, on the linear dynamic system stability. For the stability analysis of the non linear dynamic system we called the linear dynamic system of “first approximation” or indirect method of non linear dynamic system analysis [1, 2].

The mathematical property that characterizes the evolution of all dynamic systems models from the literature that approaches phenomena of the environment is property of separation between stable and unstable regions of the free parameters domain [3, 7, 8]. We formulated, for the first time, the conditions imposed on the functions that defined the dynamic system, which assure the separation between stable and unstable regions from the free parameters domain [18]. The application of the theory is focused on our biped walking robot model that opened a new possibility to simplify the solving of the walking robot problems.

2. ON THE CONTINUITY OF THE REAL MATRIX EIGENVALUES

The real matrix from the discussion is considered matrix that defines the linear dynamic system or “first approximation” of the nonlinear dynamic system depending of parameters. The components of the real matrix are assumed continue on piecewise referred to system parameters (including time parameter).

The property of the continuity transmissibility from the real matrix components to the real matrix eigenvalues is discussed in this paragraph.

QR algorithm for Hessenberg form of the real matrix:

Let the matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$ be with the real elements a_{ij} , $i = 1,...,n$; $j = 1,...,n$. We assume that the real matrix $\mathbf{A}$ has the distinct eigenvalues, real or complex. The matrix $\mathbf{A}$ has a Hessenberg form if its elements $a_{ij} = 0$ in the cases $2 < i \le n,\ j < i-1$. A real matrix $\mathbf{A}$ can be substituted by a similar matrix of Hessenberg form.

$$\mathbf{M}_r = \begin{bmatrix} \mathbf{I}_{(r-1)\times(r-1)} & & & \mathbf{0}_{(r-1)\times(n-r+1)} & & \\ 0 & \ldots\ldots. & 1 & 0 & \ldots\ldots. & 0 \\ 0 & \ldots\ldots. & m_{r+1,r} & 1 & \ldots\ldots. & 0 \\ 0 & \ldots\ldots. & m_{r+2,r} & 0 & \ldots\ldots. & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots\ldots. & m_{n,r} & 0 & \ldots\ldots. & 1 \end{bmatrix}, \tag{1}$$

$$\mathbf{M}_r^{-1} = \begin{bmatrix} \mathbf{I}_{(r-1)\times(r-1)} & & & \mathbf{0}_{(r-1)\times(n-r+1)} & & \\ 0 & \ldots\ldots & 1 & 0 & \ldots\ldots. & 0 \\ 0 & \ldots\ldots & -m_{r+1,r} & 1 & \ldots\ldots. & 0 \\ 0 & \ldots\ldots & -m_{r+2,r} & 0 & \ldots\ldots. & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots\ldots & -m_{n,r} & 0 & \ldots\ldots. & 1 \end{bmatrix}. \tag{1'}$$

For each column $r = 2,...,n-1$ of the matrix **A**, to deduce the Hessenberg form of the matrix, we can substitute the matrix **A** with the similar matrix $\mathbf{M}_r\,\mathbf{A}\mathbf{M}_r^{-1}$, using the elementary matrices as above.

The notations from the matrices $\mathbf{M}_r, \mathbf{M}_r^{-1}$ signify the expressions: $m_{k,r} = -a_{k,r-1}/a_{r,r-1}$; $a_{r,r-1} \neq 0$, $r = 2,...,n-1$; $k = r+1,...,n$; $\mathbf{I}_{(r-1)\times(r-1)}$ signify the unity matrix of order $r-1$; $\mathbf{0}_{(r-1)\times(n-r+1)}$ signify zero matrix of order $(r-1)\times(n-r+1)$ and $\mathbf{M}_r^{-1}$ represents the inverse matrix of the matrix $\mathbf{M}_r$.

Another operation that can intervene in deducing the Hessenberg form of the matrix $\mathbf{A}$ is permutation of two matrix lines, assumed i and j lines. The matrix $\mathbf{A}$ is substituted in this goal by the similar matrix $\mathbf{P}_{ij}\mathbf{A}\mathbf{P}_{ij}$ where the matrix $\mathbf{P}_{ij}$ is deduced from the unity matrix by permutation of i and j lines. The inverse of the matrix $\mathbf{P}_{ij}$ is also $\mathbf{P}_{ij}$. In the matrix $\mathbf{P}_{ij}\mathbf{A}\mathbf{P}_{ij}$ the lines and the columns i and j from the matrix $\mathbf{A}$ have been permuted.

The QR algorithm [20–23] is formulated in hypothesis that the matrix $\mathbf{A}$ has Hessenberg form to facilitate that the complex eigenvalues $\alpha \pm i\beta$, if there exists, to be represented in real final Schur form of the matrix **A**, explained below, deduced by QR algorithm convergence, using the real matrix of second order $\begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$ situated on the diagonal for each distinct complex conjugate eigenvalues and each distinct real eigenvalue situated also on the diagonal of the real final Schur form of the matrix $\mathbf{A}$, similar with initial matrix. We justify the similar Schur form of the matrix $\mathbf{A}$ by the following reason.

Let λ be a real eigenvalue and $\mathbf{x} \in \mathbb{R}^{n\times 1}$ the corresponding real eigenvector of the matrix $\mathbf{A}$ so that $\mathbf{Ax} = \lambda\mathbf{x}$, $\mathbf{x} \neq 0$. Let $\mathbf{Q} = [\mathbf{x}, \mathbf{Y}]$, $\mathbf{x} \in \mathbb{R}^{n\times 1}$, $\mathbf{Y} \in \mathbb{R}^{n\times(n-1)}$ be an orthogonal base of vectors in $\mathbb{R}^n$ that include the eigenvector $\mathbf{x} \in \mathbb{R}^{n\times 1}$ so that $\mathbf{Q}\mathbf{Q}^{\mathrm{T}} = \mathbf{I}_n$.

We have the relations $\mathbf{Y}^{\mathrm{T}}\mathbf{x} = 0$, $\mathbf{Y}^{\mathrm{T}} \in \mathbb{R}^{(n-1)\times n}$ and also the below relations:

$$\mathbf{AQ} = [\lambda\mathbf{x}, \mathbf{AY}] = \left(\mathbf{Q}\mathbf{Q}^{\mathrm{T}}\right)\mathbf{AQ} = \mathbf{Q}\left(\mathbf{Q}^{\mathrm{T}}\mathbf{AQ}\right) = \mathbf{QA}';$$

$$\mathbf{A}' = \mathbf{Q}^{\mathrm{T}}\mathbf{AQ} = \begin{bmatrix} \mathbf{x}^{\mathrm{T}} \\ \mathbf{Y}^{\mathrm{T}} \end{bmatrix} [\lambda\mathbf{x}, \mathbf{AY}] = \begin{bmatrix} \lambda & \left(\mathbf{x}^{\mathrm{T}}\mathbf{AY}\right) \\ \mathbf{0} & \left(\mathbf{Y}^{\mathrm{T}}\mathbf{AY}\right) \end{bmatrix} \Rightarrow$$

$$\Rightarrow \mathbf{A} = \mathbf{QA}'\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}\begin{bmatrix} \lambda & \left(\mathbf{x}^{\mathrm{T}}\mathbf{AY}\right) \\ \mathbf{0} & \left(\mathbf{Y}^{\mathrm{T}}\mathbf{AY}\right) \end{bmatrix}\mathbf{Q}^{\mathrm{T}}, \tag{2}$$

In the relations (2) the matrices $\mathbf{x}^{\mathrm{T}}\mathbf{AY} \in \mathbb{R}^{1\times(n-1)}$, $\mathbf{0} \in \mathbb{R}^{(n-1)\times 1}$, $\mathbf{B} = \mathbf{Y}^{\mathrm{T}}\mathbf{AY} \in$ $\in \mathbb{R}^{(n-1)\times(n-1)}$ are used where the dimension $n > 2$.

In the matrix $\mathbf{A}'$ on the diagonal the real eigenvalue λ of the matrix $\mathbf{A}$ that represents an intermediary Schur form of the matrix $\mathbf{A}$ appears. The eigenvalues of the matrix $\mathbf{B}$ are also eigenvalues of the matrix $\mathbf{A}$ because if $\mathbf{BX} = \gamma\mathbf{X}$, $\gamma \neq 0$ then $(\mathbf{Y}^{\mathrm{T}}\mathbf{AY})\mathbf{X} = \gamma\mathbf{X}$ and $\mathbf{A}(\mathbf{YX}) = \gamma(\mathbf{YX})$. The value γ can be real or complex and the corresponding vector $\mathbf{X}$ can also be real respectively complex. The eigenvalues of the matrix $\mathbf{B}$ are the same as for the matrix $\mathbf{A}$ excepting the assumed real eigenvalue λ of the matrix $\mathbf{A}$.

In the case in which the matrix $\mathbf{B}$ and implicit matrix $\mathbf{A}$ admit two complex conjugate eigenvalues $\alpha \pm \mathrm{i}\beta$ for these eigevalues are associated two complex conjugate vectors $\mathbf{u} \pm i\mathbf{v}$ of the matrix $\mathbf{B}$ with $\mathbf{u}, \mathbf{v} \in \mathbb{R}^{(n-1)\times 1}$ linear independent real vectors. We can write:

$$\mathbf{B}(\mathbf{u} \pm \mathrm{i}\mathbf{v}) = (\alpha \pm \mathrm{i}\beta)(\mathbf{u} \pm \mathrm{i}\mathbf{v}) \;\Rightarrow\; \mathbf{B}[\mathbf{u} \;\; \mathbf{v}] = [\mathbf{u} \;\; \mathbf{v}]\begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$$

$$\mathbf{X}^* = [\mathbf{u} \;\; \mathbf{v}],\; \mathbf{M} = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \;\Rightarrow\; \mathbf{BX}^* = \mathbf{X}^*\mathbf{M} \tag{3}$$

Let an orthogonal base $\mathbf{Q}^* = \left[\mathbf{X}^*, \mathbf{Y}^*\right]$ be from $\mathbb{R}^{n-1}$ where $\mathbf{X}^* = [\mathbf{u}, \mathbf{v}] \in$ $\in \mathbb{R}^{(n-1)\times 2}$, $\mathbf{Y}^* \in \mathbb{R}^{(n-1)\times(n-3)}$ and $\mathbf{Q}^*\mathbf{Q}^{*\mathrm{T}} = \mathbf{I}_{n-1}$. Then

$$\mathbf{BQ}^* = \mathbf{B}\left[\mathbf{X}^*, \mathbf{Y}^*\right] = \left[\mathbf{X}^*\mathbf{M}, \mathbf{BY}^*\right] = \left(\mathbf{Q}^*\mathbf{Q}^{*\mathrm{T}}\right)\mathbf{BQ}^* = \mathbf{Q}^*\left(\mathbf{Q}^{*\mathrm{T}}\mathbf{BQ}^*\right) = \mathbf{Q}^*\mathbf{B}^* \quad (4)$$

$$\mathbf{B}^* = \mathbf{Q}^{*\mathrm{T}}\left(\mathbf{BQ}^*\right) = \begin{bmatrix} \mathbf{X}^{*\mathrm{T}} \\ \mathbf{Y}^{*\mathrm{T}} \end{bmatrix}\left[\mathbf{X}^*\mathbf{M}, \mathbf{BY}^*\right] = \begin{bmatrix} \mathbf{M} & \left(\mathbf{X}^{*\mathrm{T}}\mathbf{BY}^*\right) \\ 0 & \left(\mathbf{Y}^{*\mathrm{T}}\mathbf{BY}^*\right) \end{bmatrix} \quad (5)$$

The matrix **A** with the real eigenvalue λ and two complex eigenvalues $\alpha \pm i\beta$ has the form:

$$\mathbf{A} = \mathbf{Q}\begin{bmatrix} \lambda & \left(\mathbf{x}^{\mathrm{T}}\mathbf{AY}\right) \\ \mathbf{0} & \mathbf{B} \end{bmatrix}\mathbf{Q}^{\mathrm{T}}, \quad \mathbf{B} = \mathbf{Q}^*\begin{bmatrix} \mathbf{M} & \left(\mathbf{X}^{*\mathrm{T}}\mathbf{BY}^*\right) \\ \mathbf{0} & \left(\mathbf{Y}^{*\mathrm{T}}\mathbf{BY}^*\right) \end{bmatrix}\mathbf{Q}^{*\mathrm{T}} \quad (6)$$

The matrix **A** with distinct eigenvalues is, in conclusion, similarly with the matrix in Hessenberg form with the real eigenvalues on the diagonal and with the matrices of order two on the diagonal corresponding to each complex conjugate eigenvalues of the matrix **A**. This form is the mentioned Schur form of the matrix **A**.

The QR algorithm [20], by the Wilkinson's manner, is described below where the initial real matrix **A** is denoted $\mathbf{A}_1$ in the algorithm:

$$\mathbf{A}_s = \mathbf{Q}_s\mathbf{R}_s, \quad \mathbf{A}_{s+1} = \mathbf{R}_s\mathbf{Q}_s, \ s = 1,2,...; \ s \to \infty$$
$$\mathbf{A}_s = \mathbf{R}_{s-1}\mathbf{Q}_{s-1}; \quad \mathbf{A}_{s-1} = \mathbf{Q}_{s-1}\mathbf{R}_{s-1}, \ s = 2,3,...;$$
$$\mathbf{R}_{s-1} = \mathbf{Q}_{s-1}^{-1}\mathbf{A}_{s-1}; \quad \mathbf{A}_s = \mathbf{Q}_{s-1}^{-1}\mathbf{A}_{s-1}\mathbf{Q}_{s-1}, \ s = 2,3,...;$$
$$\mathbf{A}_s = \mathbf{Q}_{s-1}^{-1}...\mathbf{Q}_1^{-1}\mathbf{A}_1\mathbf{Q}_1...\mathbf{Q}_{s-1} = \left(\mathbf{Q}_1...\mathbf{Q}_{s-1}\right)^{-1}\mathbf{A}_1\mathbf{Q}_1...\mathbf{Q}_{s-1}, \ s = 2,3,...;$$
$$\mathbf{Q}_1...\mathbf{Q}_{s-1}\mathbf{A}_s = \mathbf{A}_1\mathbf{Q}_1...\mathbf{Q}_{s-1}; \quad \mathbf{Q}_1...\mathbf{Q}_{s-1}\mathbf{Q}_s\mathbf{R}_s = \mathbf{A}_1\mathbf{Q}_1...\mathbf{Q}_{s-1}; \ s = 2,3,...;$$
$$\mathbf{Q}_1...\mathbf{Q}_{s-1}\mathbf{Q}_s\mathbf{R}_s\mathbf{R}_{s-1}...\mathbf{R}_1 = \mathbf{A}_1\mathbf{Q}_1...\mathbf{Q}_{s-1}\mathbf{R}_{s-1}...\mathbf{R}_1, \ s = 2,3,...;$$
$$\mathbf{Q}_1...\mathbf{Q}_{s-1}\mathbf{Q}_s\mathbf{R}_s\mathbf{R}_{s-1}...\mathbf{R}_1 = \mathbf{A}_1^s, \ s = 1,2,... \ . \quad (7)$$

The matrices $\mathbf{Q}_k, k = 1,2,...$ are orthogonal and the matrices R_k are upper triangular, invertible. The matrices $\mathbf{A}_k, \mathbf{A}_{k+1}, k = 1,2,...$ are also of Hessenberg form and similar.

The convergence of QR algorithm for the matrix **A** to the Schur form of the matrix, where the real matrix **A** is considered in the Hessenberg form, is described by Parlet [22].

The matrix $\mathbf{A} - \lambda\mathbf{I}$, where λ is real or complex value, is also a matrix in Hessenberg form. The value λ defines "the shift of origin" for the matrix. The shift of origin for the matrix is important because allows the transposition of the real matrix that defines the dynamic system in the complex domain through the complex value λ.

The QR algorithm for the matrix **A** with the shift of origin is described by the relations [22]:

$$\mathbf{Q}_s(\mathbf{A}_s - k_s\mathbf{I}) = \mathbf{R}_s, \quad \mathbf{A}_{s+1} = \mathbf{R}_s\mathbf{Q}_s^{\mathrm{T}} + k_s\mathbf{I} = \mathbf{Q}_s\mathbf{A}_s\mathbf{Q}_s^{\mathrm{T}}, \ \ s = 1,2,... \tag{8}$$

In the above relations by $\mathbf{A}_1$ is denoted initial matrix **A** of the system in Hessenberg form, k_s is the shift of origin, $\mathbf{Q}_s$ is orthogonal matrix, $\mathbf{R}_s$ is upper triangular matrix, $\mathbf{A}_s$, $s \geq 2$ is also in the Hessenberg form.

The shift of origin, with the initial value λ sufficient close to one initial matrix eigenvalue, real or complex, imposes acceleration of the QR algorithm convergence to respectively eigenvalue on the similar diagonal form of the matrix. This is another important motivation for using QR algorithm with the shift of origin.

The matrix **A** with distinct eigenvalues is similarly with the matrix in Hessenberg form and QR algorithm with the shift of origin can facilitate the convergence of the initial matrix to similar diagonal form of the matrix with real or complex eigenvalues on the diagonal.

The above study is performed in hypothesis that all eigenvalues of the real matrix are distinct. For the extension of the results in the case of real matrix multiple eigenvalues [1], we call to the results from matrix theory described as follows.

Definition 1. Let $L(\mathbb{R}^n)$ be the set of matrices of dimension *n*, or, similar, the linear maps set from $\mathbb{R}^n$. The distance between the matrices of dimension n is introduced using the distance between the vectors in $\mathbb{R}^n$, where the matrices of the set $L(\mathbb{R}^n)$ are considered as vectors from the set $\mathbb{R}^m$ with $m = n^2$.

The normed space $L(\mathbb{R}^n)$, using the distance defined above, is verified that is a linear normed space.

Hirsch, Smale and Devaney have demonstrated, on the matrix set $L(\mathbb{R}^n)$, the below theorem [1].

THEOREM 1. *The set of matrices with distinct eigenvalues from linear normed space $L(\mathbb{R}^n)$ is open and dense set in linear space $L(\mathbb{R}^n)$.*

The above theorem creates the possibilities to justify the transmission of some properties from the real matrices set with distinct eigenvalues to the real matrices set including multiple eigenvalues that can intervene in stability analysis of linear (can be of "first approximation") dynamic systems.

Transmissibility of the continuity from the matrix elements to the eigenvalues:

The components of the real matrix **A** that define the linear dynamic system depending on parameters are assumed continue on piecewise referred to the free parameters. We formulate below our theorem on the continuity transmissibility.

THEOREM 2. *If the components of the matrix* **A** *are continuous on piecewise and the sequence of Hessenberg form matrices* $\mathbf{A}_s$, $s = 1, 2, \ldots$ *from QR algorithm that started with the matrix* **A** *is uniform convergent to the Schur form of the matrix* **A** *then the eigenvalues of the matrix* **A** *are continuous on piecewise.*

The above property is capitalized in our study using the following property of continuous functions, formulated here for functions of one variable.

THEOREM 3. *Let the function* $f : E \to \mathbb{R}$, $E \subset \mathbb{R}$ *be a continuous function in the point* $x_0 \in E$ *and the function value* $f(x_0)$ *so that the inequalities* $\alpha < f(x_0) < \beta$; $\alpha, \beta \in \mathbb{R}$ *are satisfied; then there is a neighbourhood of the point* $x_0 \in E$ *where the function values respect the same inequalities.*

Remark. Theorem 3 assures that the function f, continuous in the point $x_0 \in E$ preserve, in the neighborhood of x_0, the function sign from x_0.

Sufficient mathematical conditions that assure the separation between stable and unstable regions for the linear dynamic system are deduced using also the classical property formulated below.

THEOREM 4. *Let the linear dynamic system be defined by the differential equation of the form* $\mathrm{d}\mathbf{y}/\mathrm{d}t = \mathbf{A}\mathbf{y}(t)$, $\mathbf{y}(t) = (y_1(t), \ldots, y_n(t))^{\mathrm{T}}$, $\mathbf{A} = (a_{ij})$, $i = 1, \ldots, n$; $j = 1, \ldots, n$, *the symbol* $^{\mathrm{T}}$ *signifying transposition of the matrix and where the values* a_{ij} *are assumed constants. If the real part of all eigenvalues of the matrix* **A** *is strictly negative then the solution of the differential equation is asymptotic stable in origin. If the real part at least one eigenvalue of the matrix* **A** *is strictly positive then the solution of the differential equation is unstable in origin. If the real part of the eigenvalues of the matrix* **A** *is strictly negative with the exception of at least one eigenvalue that has null real part then the stability of the dynamic system in origin is unknown (possible stable or unstable).*

3. ON THE SEPARATION OF THE DYNAMIC SYSTEM STABLE REGIONS

The possible structure of the stable and unstable points from the dynamic system free parameters domain is described by the following cases:

– If the dynamic system is stable in one point from the domain of free parameter then there is a neighborhood around this point where the dynamic system is also stable in each point from the neighborhood. This neighborhood represents a stable region of the dynamic system that can be extended up to maximal stable region from the domain of free parameters. The analog possible case can be described for one unstable point of the free parameters domain.

– We denote that a maximal stable or unstable region from the free parameter domain can be compounded only by one isolate stable or unstable point in the unstable respectively stable neighborhood. With other words we underline the possibility of singular (isolate) stable or unstable point existence in the free parameters domain.

– The maximal stable or unstable regions are separated in the free parameters domain by the frontier compounded from stable and unstable points.

Our theorem on sufficient conditions of separation between stable and unstable regions in the free parameters domain, in the case of distinct eigenvalues of the real matrix **A** that defines the linear dynamic system, is formulated below:

THEOREM 5 (Separation theorem). *If the linear dynamic system defined by the real matrix* **A**, *in the Hessenberg form, has the continuous on piecewise components of the matrix as functions of dynamic system free parameters and the convergent QR algorithm assures that the real part of eigenvalue functions of the matrix* **A** *are also continuous on piecewise, then these conditions impose the separation between stable and unstable regions of the dynamic system in the domain of free parameters.*

Remark. We discuss on the necessity to substitute in practice the infinite QR algorithm by finite one because the infinite process is not perceived by one observer from the environment and so that the conditions needed for the Theorem 5 applications will be simplified.

The separation studied by nonlinear system "first approximation":

The stability study for no null solution of nonlinear equation of the form $\mathrm{d}\mathbf{y}/\mathrm{d}t = \mathbf{h}(t,\mathbf{y})$, $\mathbf{y} \neq 0$, can be similar to one corresponding to null solution.

In this goal we consider $\tilde{\mathbf{y}}(t) \neq 0$ solution of the equation $\mathrm{d}\mathbf{y}/\mathrm{d}t = \mathbf{h}(t,\mathbf{y})$ and equation $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{f}(t,\mathbf{x})$ with transformed function $\mathbf{f}(t,\mathbf{x}) = \mathbf{h}(t,\mathbf{x}+\tilde{\mathbf{y}}(t)) - \mathbf{h}(t,\tilde{\mathbf{y}}(t))$. The function $\mathbf{x}(t) \equiv 0$ is a solution of the transformed equation $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{f}(t,\mathbf{x})$. If the equation $\mathrm{d}\mathbf{y}/\mathrm{d}t = \mathbf{h}(t,\mathbf{y})$ has the solution $\mathbf{y}(t)$ then the equation $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{f}(t,\mathbf{x})$ has the solution $\mathbf{x}(t) = \mathbf{y}(t) - \tilde{\mathbf{y}}(t)$. Analogue if the equation $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{f}(t,\mathbf{x})$ has the solution $\mathbf{x}(t) = \mathbf{y}(t) - \tilde{\mathbf{y}}(t)$, with $\tilde{\mathbf{y}}(t)$ defined above, and then $\mathbf{y}(t) = \mathbf{x}(t) + \tilde{\mathbf{y}}(t)$ is a solution of the equation $\mathrm{d}\mathbf{y}/\mathrm{d}t = \mathbf{h}(t,\mathbf{y})$. The study of the stability for the fixed solution $\tilde{\mathbf{y}}(t)$ of the equation $\mathrm{d}\mathbf{y}/\mathrm{d}t = \mathbf{h}(t,\mathbf{y})$, $\mathbf{y} \neq 0$ is equivalent with the study of the stability for the solution $\mathbf{x}(t) \equiv 0$ of the equation $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{f}(t,\mathbf{x})$. This is the aspect for which is analyzed only the stability for null solution of nonlinear dynamic system. Another assumption is that the equation of the dynamic system is of the autonomous form $\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{f}(\mathbf{x})$. Many of the dynamic systems from the literature are of the autonomous form.

The function $\mathbf{f}(\mathbf{x})$ is assumed with the variable $\mathbf{x}=(x_1,\ldots,\ x_n)^{\mathrm{T}}$ and with the function value denoted $\mathbf{f}(\mathbf{x})=(f_1(\mathbf{x}),\ldots,\ f_n(\mathbf{x}))^{\mathrm{T}}$. The components $f_i(\mathbf{x})$, $i=1,\ldots,n$, are assumed that can be developed in series around origin as follows:

$$f_i(\mathbf{x})=f_i(\mathbf{0})+\sum\nolimits_{j=1}^{n}\left(\partial f_i(\mathbf{x})/\partial x_j\right)\Big|_{\mathbf{x}=0}x_j+$$
$$+\sum\nolimits_{j=1}^{n}\sum\nolimits_{k=1}^{n}\left(\partial^2 f_i(\mathbf{x})/\partial x_j\partial x_k\right)\Big|_{\mathbf{x}=0}x_jx_k+\ldots,\qquad i=1,\ldots,n. \tag{9}$$

Above assumptions permit us to consider $f_i(\mathbf{0})=0$, $i=1,\ldots,n$ and using the notations for the derivatives of the first order $a_{ij}=\partial f_i(\mathbf{x})/\partial x_j\big|_{\mathbf{x}=0}$; $i,j=1,\ldots,n$ we can formulate the equation:

$$\mathrm{d}\mathbf{x}/\mathrm{d}t=\left[a_{ij}\right]\mathbf{x}+\mathbf{g}(\mathbf{x});\quad i,j=1,\ldots,n \tag{10}$$

The linear system of “first approximation” deduced from (10) is of the form:

$$\mathrm{d}\mathbf{x}/\mathrm{d}t=\left[a_{ij}\right]\mathbf{x};\quad i,j=1,\ldots,n \tag{11}$$

The following Liapunov theorems are mentioned:

THEOREM 6. *The evolution of non linear dynamic system* (10) *is asymptotic stable in origin if the real part of all eigenvalues of the matrix* $\mathbf{A}=\left[a_{ij}\right]$, $i,j=1,\ldots,n$*, is strictly negative.*

THEOREM 7. *The evolution of the non linear dynamic system* (10) *is unstable in origin if the real part of at least one eigenvalue of the matrix* $\mathbf{A}=\left[a_{ij}\right]$, $i,j=1,\ldots,n$*, is strictly positive.*

The separation studied on nonlinear system by indirect method:

The indirect method of stability analysis consists in using of the differential equation solution that describes evolution of the dynamic system.

The equation $\mathrm{d}\mathbf{x}/\mathrm{d}t=\mathbf{f}(\mathbf{x})$ is considered again with the solution $\mathbf{x}(t)\equiv\mathbf{0}$, $\mathbf{x}=(x_1,\ldots,\ x_n)^{\mathrm{T}}$ and the assumption that the functions $f_i(\mathbf{x})$, $i=1,\ldots,n$, can be developed in series around origin so that the above equation can be expressed in the form (10) where is assumed that the function $\mathbf{x}(t)$ is of at least C^2 class so that the function $\mathbf{g}(\mathbf{x})=\mathrm{d}\mathbf{x}/\mathrm{d}t-\mathbf{A}\mathbf{x}$ is of at least C^1 class.

Because the matrix $\mathbf{A}$ is Jacobian matrix in origin $\mathbf{x}(t)\equiv\mathbf{0}$ of the function $\mathbf{f}(\mathbf{x})$ then $\mathbf{g}(\mathbf{x})$ has the property that for each $\gamma>0$ there is $\delta(\gamma)>0$ such that if $|\mathbf{x}|<\delta(\gamma)$ then $|\mathbf{g}(\mathbf{x})|<\gamma|\mathbf{x}|$. This property means that $\mathbf{g}(\mathbf{x})$ who corresponds to "higher order terms" in series developing around origin becomes negligible reported to linear order terms for sufficient small **x**.

In the following we mention a theorem on the stability regions separation of the nonlinear dynamic systems in the free parameters domain, calling indirect method and using the results performed by Halanay and Răsvan [2].

THEOREM 8. *Let the dynamic system be defined by the equation:*

$$\mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{A}\mathbf{x} + \mathbf{g}(\mathbf{x}) . \qquad (12)$$

The real matrix **A**, *of dimension* $n \times n$, *is assumed that is compounded from constant elements, the variable* $\mathbf{x} = (x_1,..., x_n)^{\mathrm{T}}$ *is of dimension* n, *the function* $\mathbf{x}(t) \equiv \mathbf{0}$ *is a solution of the equation, the function* $\mathbf{g}(\mathbf{x})$ *is assumed continuous and with the property that for each* $\gamma > 0$ *there is* $\delta(\gamma) > 0$ *such that if* $|\mathbf{x}| < \delta(\gamma)$ *then* $|\mathbf{g}(\mathbf{x})| < \gamma|\mathbf{x}|$. *It is also assumed that the matrix* **A** *has the property that all roots* λ_i, $i = 1,...,n$ *of the characteristic polynomial have the real part strictly negative such that* $\mathrm{Real}\,\lambda_i \le -2\alpha < 0$, $i = 1,...,n$.

Then there is $\delta_0 > 0$, $\beta \ge 1$ *such that for each* $|\mathbf{x}_0| < \delta_0$ *is true the inequality:*

$$|\mathbf{x}(t; t_0, \mathbf{x}_0)| \le \beta e^{-\alpha(t-t_0)/2} |\mathbf{x}_0|, \quad t \ge t_0 \qquad (13)$$

If the dynamic system conditions of Theorem 8 are respected then we remark that stability in origin assures stability in neighborhood of origin and thus assures stable region separation in neighborhood of origin.

4. PHYSICAL AND MATHEMATICAL MODEL OF BIPED WALKING ROBOT

In the following we describe our physical and mathematical model of biped walking robot, with one physical model component of the robot compounded from a pivot point B_t assumed fixed in initial study and two legs that are simultaneously moved in the same plane that is firstly discussed (Fig. 1). Each of the robot leg with an articulated extremity attached to the robot body is compounded from two arms B_tP and PQ also articulated in the point P denoted "knee joint" of the leg. The point P, in the case of fixed pivot point, describes a circle arc route in a cycled evolution of the robot leg and the Q base point describes a close route compounded from the superior ellipse arc Q_BQ_A with semi axes length a, b and with point O_E centre of the ellipse, and also we remark closure of the leg cycle evolution by horizontal segment Q_AQ_B traversed by the base point Q.

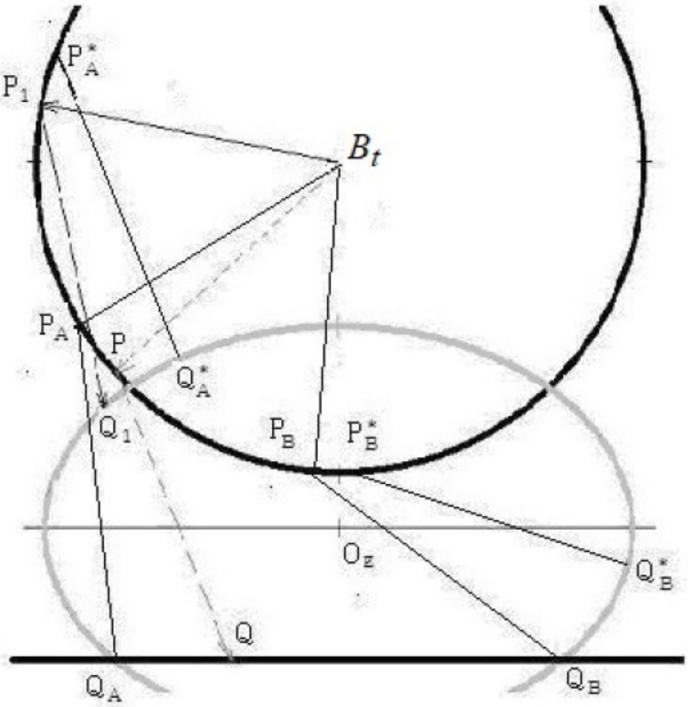

Fig. 1 – Physical model of the walking robot leg.

The orthogonal system of coordinates and parameters signification is identified from the following coordinates on the figure points: $F_t(a,h)$, $O_E(a,b_1)$, $P(x_P,y_P)$, $Q(x_Q,y_Q)$, $Q_A(x_A,0)$, $Q_B(x_B,0)$.

The points P_A^* and P_B^* define the extremities of the maximal domain on circle arc where the knee joint P is moving because in these points, geometrical identified by the property that the segments $P_A^*Q_A^*$ and $P_B^*Q_B^*$ are normally on the ellipse arc, the direction of movement is changed. The positions P_A^* and P_B^* of the knee joint identified by us on the particular case of Figure 1, are named by us the critical points from the leg evolution. In other cases of robot leg with fixed pivot point, defined by the values of the geometrical parameters or in cases where the pivot point is moved in the biped walking robot evolution, is important to search the possible existence of the knee joint critical positions where the direction of movement is changed and where the speed of the knee joint must to be zero for the continuous evolution of the knee joint.

The mathematical model deduced from the physical model, suggested by the particular case represented in Fig. 1, is defined through two implicit functions described by the equations:

$$(x-a)^2+(y_P-h)^2-a^2=0; \quad \frac{(x-a)^2}{a^2}+\frac{(y_Q-b_1)^2}{b^2}=1. \tag{14}$$

Between the parameters' values, there are conditions $a>b>b_1>0;\ 2a>h$ where a and b are the ellipse semi axes. The covering domain of variables from (14) of our problem is defined below.

$$x\in[0,2a], \quad y_P\in[h-a,\ h+a), \quad y_Q\in[-b,b]. \tag{15}$$

Explicit functions deduced from (14), are:

$$y_P=h\mp(2ax-x^2)^{1/2}; \quad y_Q=\pm\frac{b}{a}(2ax-x^2)^{1/2}+b_1. \tag{16}$$

Let $P(x_P,y_P)$ and $Q(x_Q,y_Q)$ be points on the circle arc respectively on the ellipse arc that correspond for one leg position from the evolution. The condition on the distance PQ namely $(x_P-x_Q)^2+(y_P-y_Q)^2-a^2=0$ appears.

The uniform linear evolution of the variable x between 0 and $2a$, excepting a neighbourhood around possible critical points, in the case of fixed pivot point of the leg, is assumed as below, where the selected constant speed ω and initial condition x_0 are considered:

$$x(t)=\omega t+x_0. \tag{17}$$

One cycle of evolution for the robot leg with fixed pivot point can be started from the point Q_B, moving on the superior ellipse arc up to the point Q_A, respecting the evolution law of the type (17), and returns by the linear uniform evolution on the horizontal axle, in the point Q_B, excepting a neighbourhood around the positions Q_A^*, Q_B^*, where is defined a selected evolution.

We remark that the domain of parameters' values $x,\ y_P,\ y_Q,\ h,\ b_1,\ \omega,\ x_0,\ t$ with fixed values of positive parameters a,b,b_1, in this analyzed case, for which the robot leg evolution exists, is an interval for each free parameter. The domain of existence coincides, in these formulated cases, with the domain of stability, such that we can affirm that there is a separation between stable (existence) and unstable (inexistence) regions of the free parameters values of the described robot leg model. The analyzed case is a kinematics analyze. We can intuitively conclude that the analysis is true and for robot leg with uniform distributed mass on the leg.

In the case of moved pivot point the leg is compounded from the superior component B_tQ_t defined by the extremities points denoted here B_t, Q_t jointed in pivot point B_t attached to the body of the robot and inferior component Q_tP_t with "knee joint" Q_t and base point P_t.

For the length of components B_tQ_t and Q_tP_t a constant value has been assumed.

The evolution imposed for one leg B_t, Q_t, P_t of the biped robot, described here, is inspired from the method proposed for multi-legged walking robot by Cononovici (2016).

The base point P_t is moving on the ellipse arc between points P_I and P_F, in cycling evolution of biped walking robot in vertical plane, using uniform accelerated displacement on the horizontal direction up to the median point P_M, for one leg of biped robot, and symmetric displacement assured up to the final point P_F of the robot leg. The joint point B_t attached to the body of the robot is moving simultaneously with point P_t, having linear route parallel to the axis Ox, using uniform displacement between initial point B_I up to the median point B_M in traverse of one leg and symmetric displacement assured between the median point B_M up to final point B_F through the same leg of biped walking robot.

The trajectory of the “knee joint” point Q_t, uniquely identified in the vertical plane, for suitable values of the biped walking robot parameters (selected so that the evolution of the walking robot to be possible), identified by the points B_t, Q_t, P_t evolution, at each time *t*, with the length of segment B_tP_t dependent on time *t*, is also studied for possible critical points identification, similarly as in two dimensional evolution of one leg with fixed pivot point described above (Fig. 1). The following formulas, calling the geometrical and physical data, are used.

The uniform accelerated displacement of the point P_t, on horizontal direction, from the initial point P_I identified by abscise denoted $x_{P_{tI}}$ up to middle of the segment P_IP_F, is described by:

$$x_{P_t}(t) = x_{P_{tI}} + a_P \frac{t^2}{2}. \tag{18}$$

The assumed uniform displacement of the pivot point B_t, on parallel line with axis Ox, for successive evolution of each leg of biped robot, from the initial position identified by abscise denoted $x_{B_{tI}}$ up to middle of the corresponding one semi route deduced by bisection of the route B_IB_F, is described by the relation:

$$x_{B_t}(t) = x_{B_{tI}} + v_B\, t \tag{19}$$

The parameter a_P is a constant acceleration and the parameter v_B is constant speed in the displacement of the points P_t respectively B_t.

The moving of biped walking robot is considered so that to close one cycle of evolution, the time needed for the point B_t to traverse, for each leg, the semi route from the route $B_I B_F$ is equal with the time needed for the point P_t to traverse the ellipse arc route $P_I P_F$. This relation is a result from the hypothesis on successive displacement of each leg in cycling evolution.

The necessary time for simultaneously arriving in the middle of the route $P_I P_F$ or in the corresponding middle of semi route from the route $B_I B_F$ by point P_t respectively B_t, is denoted by t_M. The following relations are imposed to some from this biped walking robot parameters:

$$v_B t_M = \overline{B_I B_F}/4; \quad a_P t_M^2/2 = \overline{P_I P_F}/2. \tag{20}$$

The mathematical model of the two dimensional legs evolution for biped walking robot, explained above, permits to identify possible existence of the critical position for the knee joint point Q_t, where the direction of movement is changed, by calling specialized computer program that is proposed for next study.

5. PROPOSED METHOD FOR SYSTEM STABLE EVOLUTION IMPROVEMENT

The method is described on biped walking robot with some assumptions.

The algorithm for our case of biped walking robot evolution exposed in previous capitol can be used for analyzing proposed method on walking robot stable evolution improvement. Some aspects are related in Figs. 2 and 3.

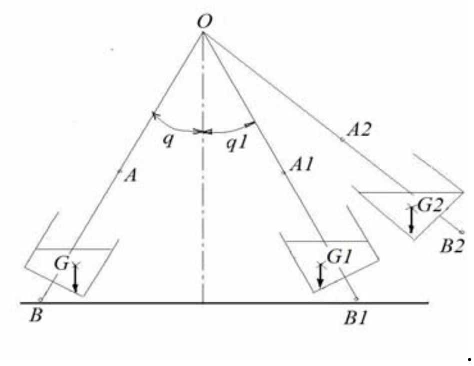

Fig. 2 – The biped with attached box of balls.

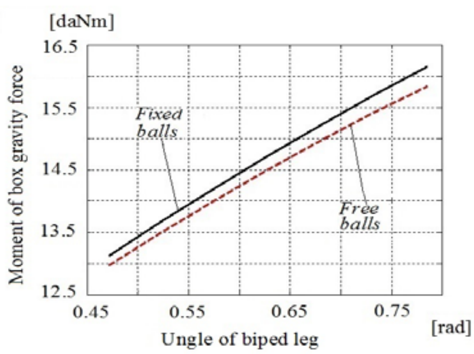

Fig. 3 – Moment of biped box gravity force.

The first idea of the method, described on the biped walking robot, consist in concentration of majority mass of the robot in two equal boxes attached at inferior component of legs. The evolution of the robot is analyzed in two cases, the first case in which mass of the boxes is compounded from balls with negligible friction and the second case in which the equivalent mass of boxes is of fixed form. The first case is exposed in Fig. 2 where the first leg of the biped is denoted OB, with point B the leg base point, and the second leg is considered in some evolution positions OB_1, OB_2,... . For each position of second leg is evaluated the moment of second box gravity force reported at the leg base point B. The diagram for one case that describes the general tendency is exposed in Fig. 3. An elaborate study is proposed for the future.

6. CONCLUSIONS

The general case of the dynamic systems that depend on parameters of the environment is analyzed for mathematical characterization of its. The separation between stable and unstable regions from the free parameters domain of the dynamic system is a fundamental mathematical property of the dynamic systems that approach a phenomenon from the reality, the property that can be accepted as first axiom of the environment. We mention with this occasion a classical law that a system of axioms sufficient expressive is incomplete. A method for stable evolution improvement of environment's dynamic system by substitution the mass of the system by equivalent mass in evolution time, and by assistance of the environment's gravitation force that action on the equivalent mass, is proposed and analyzed on simplified case. Our study is not exhausted the problem of stability control of the environments' dynamic systems but an interesting new way of research is opened.

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