

HELICAL SPRING WITH VARIABLE WIRE DIAMETER

DUMITRU POP, SIMION HARAGĂȘ, DORIAN POPA

Abstract. The geometrical and functional particularities of helical springs with variable wire diameter are studied in this paper. The spring's characteristic is determined by two factors. On one hand, by the minimum distance between two adjacent coils, for the unloaded spring, distance which decreases with increasing wire diameter. On the other hand, by the coils' deflection, which decreases with increasing wire diameter. Consequently, the actual distance between the adjacent coils represents the difference between the two sizes. As a result, the adjacent coils will successively touch each other, depending on the actual distance between them, and the spring stiffness shall be increasing.

Key words: variable wire diameter, distance between coils, rotation angle, deflection, spring stiffness, Archimedean spiral.

1. INTRODUCTION

Springs of various types have multiple applications in various fields, from industrial robots to assemblies of fine mechanics, from medical prosthetics to military equipment, from constructions addressing exploration of outer space to submersibles, from various aircraft to land vehicles, from orthodontics to furniture of various types, from pens to toys, from construction of industrial equipment and machinery to hydraulic operating gears and the list could go on.

The variety of springs' applications explains the diversity of purposes for which they are used, among which: shock absorption; for exerting a pushing force; performing a mechanical work by converting the potential energy accumulated by elastic deformation; limiting the forces or torques; measuring the forces or torques; charge equalization.

Helical springs represent a significant category from a constructive and functional point of view. The functions for which they are designed determine the prevalence of one or the other of their particularities. For example, the characteristic feature is important for some applications: linear or nonlinear. For others, spring's own frequency. Some applications require a certain stiffness of the spring, a certain index. Others relate to radial or axial spring gauge. In some cases a buckling check is necessary, in other cases the winding direction, right or left, matters. The types

Technical University of Cluj-Napoca, Romania

of springs' ends may also be relevant: open not ground or ground, squared not ground or ground. Some situations require to consider relaxation of the springs.

Therefore, all these features are found in the specialized papers. Paper [9] refers to aspects concerning geometry, coil stress, buckling, own frequency. The lateral vibrations of the axially loaded helical cylindrical spring, taking into account the way the spring ends are supported, are dealt with in [3]. In paper [1] the authors approach the issue of helical coiled spring relaxation. In [6], the equivalent stress from the cross section of the laterally loaded helical coiled spring is approached. A study of the static application of the helical coiled spring is developed in [5], assisted by ANSYS V14.5. Papers [7] and [8] aim to optimize, with genetic algorithms, the helical coiled springs intended the automotive suspension, respectively the tamping rammers.

In [10] the author studies under a dynamic aspect, the valve spring for an automobile engine, taking into account its non-linearity, having either the variable pitch, or the variable diameter of the wire. In [2] two versions of a helical spring with variable wire diameter are presented, designed for heavy trucks as well as military vehicles. The authors of paper [4] present the results obtained by certain researchers who experimentally studied the conical helical springs, respectively the helical springs with variable wire diameter. For both types of springs, they proposed an equivalent cylindrical helical spring.

The purpose of this paper is to present the result of the study on a helical spring with variable wire diameter, demonstrating the fact that the spring coils touch each other sequentially, resulting a variable spring stiffness. The proof requires the following two phases: a) calculating the minimum distance between adjacent active coils for the unloaded spring; b) calculating the corresponding deflection for each active coil.

2. MINIMUM DISTANCE BETWEEN ADJACENT COILS

The studied spring is presented in Fig. 1. The spring is materialized by winding the wire on a rod, so that the inner diameter is D_i , the pitch being t . The mean diameter of the first active coil is D_{m0} and of the last active coil n is D_{mn} .

In Figure 2, the successive sections of an arbitrary coil are represented, the coil with the number $k+1$. The wire's cross section, at the thinner end, has the radius r_k , and at the other end – the radius r_{k+1} . Based on Fig. 2 it is finally determined the computation of the minimum distance, e_{kmin} , between two successive sections of the coil's wire.

The equation of the circle with the centre in O_k in the system of coordinates $O_k x_k y_k$ is:

$$x_k^2 + y_k^2 = r_k^2 . \quad (1)$$

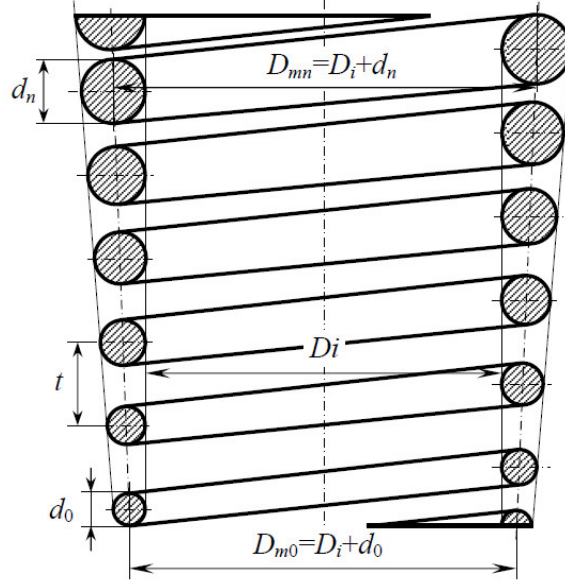


Fig. 1 – Helical spring with variable wire diameter.

The equation of the circle with the centre in O_{k+1} in the system of coordinates $O_{k+1}x_{k+1}y_{k+1}$ is:

$$x_{k+1}^2 + y_{k+1}^2 = r_{k+1}^2. \quad (2)$$

To transfer the equation of the circle O_{k+1} in the system of coordinates $O_kx_ky_k$, it can be noticed that:

$$x_{k+1} = x_k + \Delta r = x_k + t \cdot \tan \beta \quad (3)$$

$$y_{k+1} = t - y_k - e_{k+1} \quad (4)$$

$$r_{k+1} = r_k + t \cdot \tan \beta. \quad (5)$$

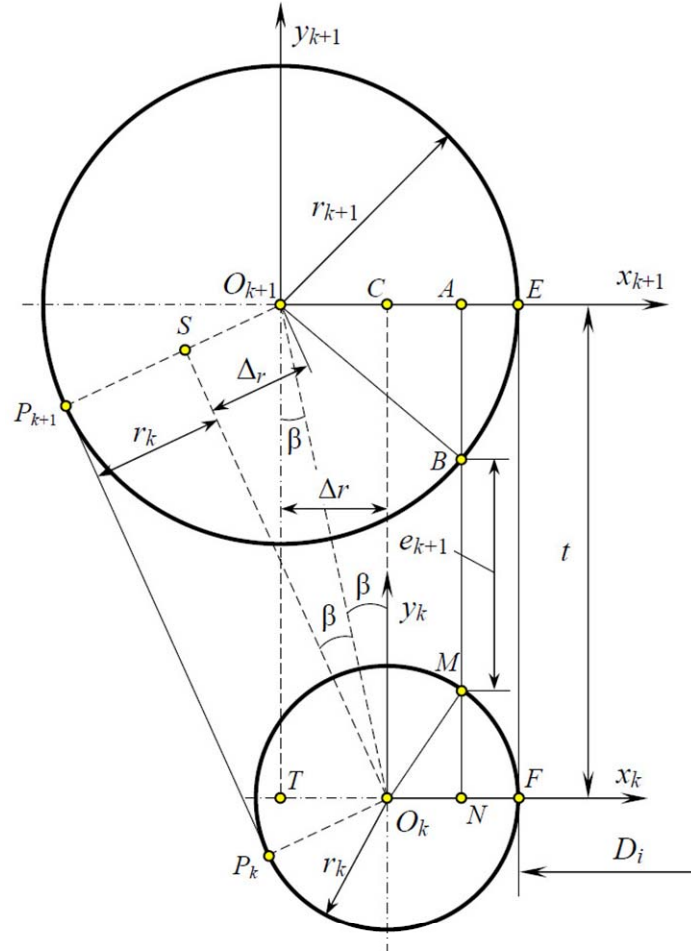
In these formulas, t is spring's pitch, and e_{k+1} is the distance between the two cross sections for a certain x_k .

Taking into consideration (3–5) the equation (2) becomes:

$$(x_k + t \cdot \tan \beta)^2 + (t - y_k - e_{k+1})^2 = (r_k + t \cdot \tan \beta)^2. \quad (6)$$

Equations (1) and (6) form the system:

$$\begin{cases} x_k^2 + y_k^2 = r_k^2 \\ (x_k + t \cdot \tan \beta)^2 + (t - y_k - e_{k+1})^2 = (r_k + t \cdot \tan \beta)^2 \end{cases} \quad (7)$$

Fig. 2 – Coil $k+1$.

For any particular case, the sizes r_k , r_{k+1} , t , β are known. From system (7) we calculate the distance e_{k+1} according to x_k :

$$(x_k + t \cdot \tan \beta)^2 + \left(t - \sqrt{r_k^2 - x_k^2} - e_{k+1} \right)^2 = (r_k + t \cdot \tan \beta)^2 \quad (8)$$

$$e_{k+1} = t - \sqrt{r_k^2 - x_k^2} \pm \sqrt{(r_k + t \cdot \tan \beta)^2 - (x_k + t \cdot \tan \beta)^2} . \quad (9)$$

In order to finally determine, the order in which the loaded spring's coils touch each other, it is necessary to know the minimum value e_{k+1}^{\min} of the distance e_{k+1} , expressed in terms of x_k . The minimum distance value e_{k+1}^{\min} can be found out

either by cancelling the first derivative of e_{k+1} (formula 9), or by deriving (8) and cancelling e'_{k+1} . The derivative of the equation (8) is:

$$2 \cdot (x_k + t \cdot \tan \beta) + 2 \cdot \left(t - \sqrt{r_k^2 - x_k^2} - e_{k+1} \right) \cdot \left(\frac{x_k}{\sqrt{r_k^2 + x_k^2}} - e'_{k+1} \right) = 0. \quad (10)$$

By making $e'_{k+1} = 0$ in formula (10) we obtain:

$$(x_k + t \cdot \tan \beta) + \left(t - \sqrt{r_k^2 - x_k^2} - e_{k+1} \right) \cdot \frac{x_k}{\sqrt{r_k^2 + x_k^2}} = 0. \quad (11)$$

From (8) and (10) the final result is:

$$r_k \cdot \frac{x_k + t \cdot \tan \beta}{x_k} = \pm (r_k + t \cdot \tan \beta). \quad (12)$$

The values of coordinate x_k are:

$$x_k = r_k \quad (13)$$

$$x_k = -\frac{r_k \cdot t \cdot \tan \beta}{2 \cdot r_k + t \cdot \tan \beta}. \quad (14)$$

Because the target is the *minimum* value e_{k+1}^{\min} of the distance e_{k+1} , the correct solution is the one given by formula (14).

Introducing x_k in formula (9) the expression of e_{k+1}^{\min} is obtained:

$$e_{k+1}^{\min} = t - \sqrt{r_k^2 - \left(\frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_k}} \right)^2} - \sqrt{(r_k + t \cdot \tan \beta)^2 - \left(t \cdot \tan \beta - \frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_k}} \right)^2} \quad (15)$$

The type of spring characteristic depends on the order in which, when the spring is loaded, the spring coils come into contact. To clarify this situation, two stages must be covered. The first one is to calculate the values of the minimum distance between two adjacent coils, e_{k+1}^{\min} , for all the n active coils of the spring and to compare them with each other.

We use the following symbols for analysing the variation trend of the distance e_{k+1}^{\min} (eq. (15)) with the increase of wire's radius, for successive active coils:

– A_1 – absolute value of x_k :

$$A_1 = |x_k| = \frac{r_k \cdot t \cdot \tan \beta}{2 \cdot r_k + t \cdot \tan \beta} = \frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_k}} \quad (16)$$

– A_2 – formula:

$$A_2 = r_k^2 - \left(\frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_k}} \right)^2 \quad (17)$$

– A_3 – formula:

$$A_3 = (r_k + t \cdot \tan \beta)^2 \quad (18)$$

– A_4 – formula:

$$A_4 = \left(t \cdot \tan \beta - \frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_k}} \right)^2 \quad (19)$$

With these symbols, the formula (15) becomes:

$$e_{k+1}^{\min} = t - \sqrt{A_2} - \sqrt{A_3 - A_4}. \quad (20)$$

It can be easily noted that, if the wire cross section's radius r_k increases, then A_2 increases, A_3 increases and A_4 decreases, so the difference $A_3 - A_4$ increases. From formula (20) it results that, *if radius r_k increases, then the minimum distance e_{k+1}^{\min} between two adjacent coils decreases.*

The computation formula of the distance e_{k+1}^{\min} can be customized for all active coils, k undertaking successively the values $k = 0, 1, 2, \dots, k, \dots, n-1$. Thus, for some coils:

$$e_1^{\min} = t - \sqrt{r_0^2 - \left(\frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_0}} \right)^2} - \sqrt{(r_0 + t \cdot \tan \beta)^2 - \left(t \cdot \tan \beta - \frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_0}} \right)^2} \quad (21)$$

$$e_2^{\min} = t - \sqrt{r_1^2 - \left(\frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_1}} \right)^2} - \sqrt{(r_1 + t \cdot \tan \beta)^2 - \left(t \cdot \tan \beta - \frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_1}} \right)^2} \quad (22)$$

$$e_n^{\min} = t - \sqrt{r_{n-1}^2 - \left(\frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_{n-1}}} \right)^2} - \sqrt{(r_{n-1} + t \cdot \tan \beta)^2 - \left(t \cdot \tan \beta - \frac{1}{\frac{2}{t \cdot \tan \beta} + \frac{1}{r_{n-1}}} \right)^2} \quad (23)$$

Based on the above reasoning, it can be written:

$$e_1^{\min} > e_2^{\min} > e_3^{\min} > \dots > e_{k+1}^{\min} > \dots > e_n^{\min}. \quad (24)$$

3. CROSS SECTION'S ANGLE OF ROTATION φ

The first coil unfolded is considered (Fig. 3). The diameters of its extremities are d_0 and d_1 .

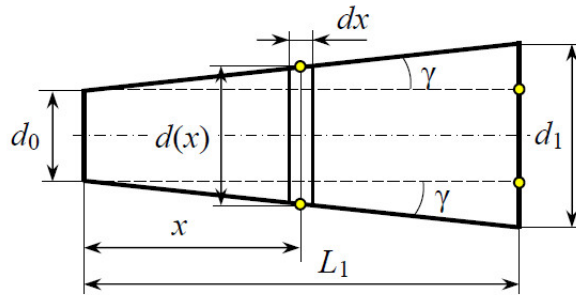


Fig. 3 – First unfolded coil.

For a dx element situated at distance x from the end with the minimum diameter of the wire d_0 , the cross section's elemental angle of twist is:

$$d\varphi_1 = \frac{T \cdot dx}{G \cdot I_{p1}(x)}, \quad (25)$$

where: T – the torque to which the arc wire is subjected over its entire length, in N·mm;
 G – modulus of rigidity, in MPa;
 $I_{p1}(x)$ – polar moment of inertia of the section situated at distance x , in mm^4 .

The torque T is:

$$T = F \cdot \frac{D_{mn}}{2}, \quad (26)$$

where: F – the axial force acting axially on the spring, in N;

D_{mn} – mean diameter of the active coil with the largest diameter, in mm.

The rotation angle of the cross section with the diameter d_0 versus the one with the diameter d_1 is:

$$\varphi_1 = \int_0^{L_1} \frac{T \cdot dx}{G \cdot I_{p1}(x)}. \quad (27)$$

From Fig. 3 it results:

$$d(x) = d_0 + 2 \cdot x \cdot \tan \gamma = d_0 + \frac{d_1 - d_0}{L_1} \cdot x \quad (28)$$

$$I_{p1}(x) = \frac{\pi \cdot d^4(x)}{32} = \frac{\pi}{32} \cdot \left(d_0 + \frac{d_1 - d_0}{L_1} \cdot x \right)^4. \quad (29)$$

Replacing (29) and (28) in (27) is achieved by integration:

$$\varphi_1 = 3.395 \cdot \frac{T \cdot L_1}{G \cdot (d_1 - d_0)} \cdot \left(\frac{1}{d_0^3} - \frac{1}{d_1^3} \right). \quad (30)$$

The same way, the angle of rotation for the other coils is written:

Coil 2:

$$\varphi_2 = 3.395 \cdot \frac{T \cdot L_2}{G \cdot (d_2 - d_1)} \cdot \left(\frac{1}{d_1^3} - \frac{1}{d_2^3} \right) \quad (31)$$

Coil k+1:

$$\varphi_{k+1} = 3.395 \cdot \frac{T \cdot L_{k+1}}{G \cdot (d_{k+1} - d_k)} \cdot \left(\frac{1}{d_k^3} - \frac{1}{d_{k+1}^3} \right) \quad (32)$$

Coil n:

$$\varphi_n = 3.395 \cdot \frac{T \cdot L_n}{G \cdot (d_n - d_{n-1})} \cdot \left(\frac{1}{d_{n-1}^3} - \frac{1}{d_n^3} \right) \quad (33)$$

The formulas for lengths' computation $L_1, L_2, \dots, L_{k+1}, \dots, L_n$ are set out below.

For the computation of the unfolded L_1 of the middle fiber of the first coil, it can be observed that the projection on a perpendicular plane to the spring axis of the *folded* coil, is an Archimedean spiral with the pitch:

$$a = \frac{d_1 - d_0}{4 \cdot \pi} \quad (34)$$

and the angle θ_0 :

$$\theta_0 = \frac{D_{m0}}{2 \cdot a} \quad (35)$$

where:

$$D_{m0} = D_i + d_0. \quad (36)$$

The length of the spiral arc corresponding to the first coil is:

$$L_{sp1} = \frac{a}{2} \cdot \sqrt{\theta_1 \cdot \sqrt{1 + \theta_1^2} - \theta_0 \cdot \sqrt{1 + \theta_0^2}} + \ln \frac{\theta_1 + \sqrt{1 + \theta_1^2}}{\theta_0 + \sqrt{1 + \theta_0^2}} \quad (37)$$

where:

$$\theta_1 = \theta_0 + 2 \cdot \pi. \quad (38)$$

Developed length L_1 is:

$$L_1 = \sqrt{L_{sp1}^2 + t^2}. \quad (39)$$

For any coil $k+1$ we have, analogously:

$$L_{sp(k+1)} = \frac{a}{2} \cdot \sqrt{\theta_{k+1} \cdot \sqrt{1 + \theta_{k+1}^2} - \theta_k \cdot \sqrt{1 + \theta_k^2}} + \ln \frac{\theta_{k+1} + \sqrt{1 + \theta_{k+1}^2}}{\theta_k + \sqrt{1 + \theta_k^2}} \quad (40)$$

where:

$$\theta_k = \theta_0 + 2 \cdot k \cdot \pi \quad (41)$$

$$\theta_{k+1} = \theta_0 + 2 \cdot (k+1) \cdot \pi. \quad (42)$$

Developed length L_{k+1} is:

$$L_{k+1} = \sqrt{L_{sp(k+1)}^2 + t^2}. \quad (43)$$

For coil n :

$$L_{spn} = \frac{a}{2} \cdot \sqrt{\theta_n \cdot \sqrt{1 + \theta_n^2} - \theta_{n-1} \cdot \sqrt{1 + \theta_{n-1}^2}} + \ln \frac{\theta_n + \sqrt{1 + \theta_n^2}}{\theta_{n-1} + \sqrt{1 + \theta_{n-1}^2}} \quad (44)$$

where:

$$\theta_{n-1} = \theta_0 + 2 \cdot (n-1) \cdot \pi \quad (45)$$

$$\theta_n = \theta_0 + 2 \cdot n \cdot \pi. \quad (46)$$

Developed length L_n is:

$$L_n = \sqrt{L_{spn}^2 + t^2}. \quad (47)$$

4. DEFLECTIONS' COMPUTATION

It was shown in chapter 2 that the first step for assessing the order in which the spring's coils are touching at a certain axial force is to determine the minimum distance between adjacent coils for the unloaded spring.

The second step consists in calculating the deflections for each active coil.

The wire has a variable diameter along its length. Consequently, coil stiffness differs from one coil to another. Therefore, the deflections for each active coil shall be calculated (Fig. 4).

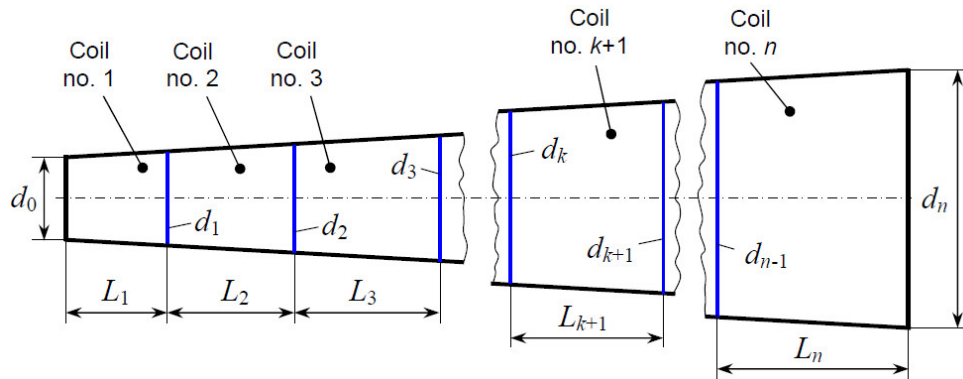


Fig. 4 – The spring with the n unfolded active coils.

Coil no. 1. In formula (30) of the rotation angle φ_1 we note:

$$E_1 = \frac{3.395 \cdot L_1}{G \cdot (d_1 - d_0)} \cdot \left(\frac{1}{d_0^3} - \frac{1}{d_1^3} \right) \quad (48)$$

Formula (30) becomes:

$$\varphi_1 = E_1 \cdot T. \quad (49)$$

Because E_1 (coil elasticity to torsion stress) is constant, the potential energy of coil's deflection is:

$$U_1 = \frac{1}{2} \cdot T \cdot \varphi_1 = \frac{1}{2} \cdot E_1 \cdot T^2. \quad (50)$$

Taking into account the formula (26), the energy U_1 has the form:

$$U_1 = \frac{1}{8} \cdot E_1 \cdot D_{mn}^2 \cdot F^2. \quad (51)$$

Applying the first theorem of Castigliano, the deflection corresponding to the first coil is:

$$f_1 = \frac{dU_1}{dF} = \frac{1}{4} \cdot E_1 \cdot D_{mn}^2 \cdot F = \frac{1}{2} \cdot E_1 \cdot D_{mn} \cdot T. \quad (52)$$

For the other active coils proceed analogously.

Coil no. $k+1$

$$f_{k+1} = \frac{1}{2} \cdot E_{k+1} \cdot D_{mn} \cdot T \quad (53)$$

where

$$E_{k+1} = \frac{3.395 \cdot L_{k+1}}{G \cdot (d_{k+1} - d_k)} \cdot \left(\frac{1}{d_k^3} - \frac{1}{d_{k+1}^3} \right). \quad (54)$$

Coil no. n

$$f_n = \frac{1}{2} \cdot E_n \cdot D_{mn} \cdot T \quad (55)$$

where

$$E_n = \frac{3.395 \cdot L_n}{G \cdot (d_n - d_{n-1})} \cdot \left(\frac{1}{d_{n-1}^3} - \frac{1}{d_n^3} \right). \quad (56)$$

5. THE ACTUAL DISTANCE BETWEEN COILS ON LOADED SPRING

The type of spring characteristic depends on the way the adjacent coils touch each other, at the axial load of the spring.

The minimum distances e_{k+1}^{\min} between adjacent coils of the free spring are given by the formulas of type (21–23), with the comment from formula (24).

On another hand, the deflections corresponding to active coils, for the spring loaded with a certain axial force, are those from the formulas of type (53–56).

When the spring is loaded, *the actual distance* e_{ef} between the adjacent coils is:

$$e_{ef1} = e_1^{\min} - f_1 \quad (57)$$

$$e_{ef2} = e_2^{\min} - f_2 \quad (58)$$

$$e_{ef(k+1)} = e_{k+1}^{\min} - f_{k+1} \quad (59)$$

$$e_{efn} = e_n^{\min} - f_n \quad (60)$$

The coils will successively touch each other, starting with the smallest actual distance $e_{ef(k+1)}$ and continuing, in increasing order of the actual distance between coils, with the other coils. For this reason, the spring stiffness is not constant, but increasing.

In Table 1 an application is presented for a helical spring with variable wire diameter.

Table 1

Numerical application for a spring with variable wire diameter

<u>Initial data:</u> $d_0 = 5 \text{ mm}; d_6 = 10 \text{ mm}; t = 30 \text{ mm}; n = 6;$ $D_i = 55 \text{ mm}; F = 50 \text{ N}; \beta = 0,796; \gamma = 0.12^\circ$						
Coil no.	Sizes	e_{k+1}^{\min} [mm]	L_{k+1} [mm]	φ_{k+1} [°]	f_{k+1} [mm]	$e_{ef(k+1)}$ [mm]
1		24.599	192.161	2.75	1.56	23.039
2		23.764	194.747	1.565	0.888	22.876
3		22.929	197.334	0.958	0.543	22.386
4		22.094	199.922	0.621	0.352	21.742
5		21.26	202.511	0.421	0.239	21.021
6		20.426	205.1	0.296	0.168	20.258

From the numerical application presented in Table 1 it can be noted that the coils shall touch each other starting with coil no. 6 and finishing with coil no. 1.

6. CONCLUSIONS

The helical spring with variable wire diameter has certain particularities arising from the theoretical arguments, as well as from the numerical application (Table 1).

From a geometric point of view, the minimum distance e_{k+1}^{\min} between the adjacent coils for the unloaded spring is not constant, as in helical cylindrical springs with constant wire diameter's case. It varies downward with increasing wire diameter.

As far as the spring loading with a certain axial load, it is found that the coils' stiffness increases with increasing wire diameter. Consequently, coil no. 1 has the largest deflection, the other coils' deflections decreasing with increasing wire diameter.

The actual distance between adjacent coils depends on the minimum one e_{k+1}^{\min} , as well as on the deflection f_{k+1} as results from formulas (57–60). Because the actual distances between the coils are not equal the coils shall successively touch. Consequently, the spring's stiffness shall be increasing.

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