

TUBULAR THIN-WALLED ENERGY ABSORBING STRUCTURES WITH TRIGGERS – RECENT ADVANCES

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Abstract. In the present paper the state of art review concerning recent advances in research into crashworthiness performance and energy absorption capacity of tubular thin-walled energy absorbing structures with triggers promoting the most desirable crushing process is presented. Particularly recent results of the crushing behavior analysis of thin-walled rectangular cross-section columns with non-periodically situated flaws (dents or slots) are presented. Some final remarks about further research perspectives in this field are derived.

Key words: energy absorption, impact, thin-walled structures.

1. INTRODUCTION

Increasing number of impacting events of many types like traffic accidents, collisions of ships or collisions of a ship either with an iceberg or ship grounding on a narrow rock, etc. induced the rapid development of the *impact crashworthiness* dealing with research into impact engineering problems, particularly in the field of dynamic response of structures in the plastic range and the design of *energy absorbers*. Since demands of general public of the safe design of components of vehicles, ships, etc. have increased substantially in the last few decades, a new challenge appeared to design special structural members which would dissipate the impact energy in order to limit the deceleration and finally to stop a moveable mass (e.g. vehicle) in a controlled manner. Such a structural member termed energy absorber converts totally or partially the kinetic energy into another form of energy. One of the possible design solutions is the conversion of the kinetic energy of impact into the energy of plastic deformation of a thin-walled metallic structural member.

In the early sixties of the 20th century automotive safety regulations stimulated the development of the new concept of a *crashworthy (safe) vehicle* that had to

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fulfil integrity and impact energy management requirements. A designer of any impact attenuation device must meet two main, sometimes contrary) requirements: The initial collapse load has to be not too high in order to avoid unacceptably high impact velocities of the vehicle. On the other extreme, the main requirement is a possibly highest energy dissipation capacity, which may not be achieved if the collapse load of the impact device is too low. The latter may result in dangerously high occupant “ridedown” decelerations.

Thus, maximizing energy absorption and minimizing peak to mean force ratio by seeking for the optimal design of these components are of great significance.

There are numerous types of energy absorbers of that kind that are cited in the literature [5]. Namely, there are steel drums, thin tubes or multi-corner columns subject to compression, compressed frusta (truncated circular cones), simple struts under compression, sandwich plates or beams (particularly honeycomb cells) and many others. Among all those design solutions, mentioned above, thin-walled metal tubes are widely used as energy absorption systems in automotive industry due to their high energy absorption capability, easy to fabricate, relatively low price and sustainability at collapse. Figure 1 shows a cylindrical tube subjected to axial compressive static or quasi-dynamic force.

After reaching the first peak load at point 1 the tube exhibits an unstable behavior associated with repeated pattern. Each pair of peaks in Fig. 1 (e.g. 2 and 3) is caused by the development of a fold (wrinkle) or buckle (shown in Fig. 3a). Usually, these folds (buckles) develop sequentially from one end of the tube, so that the phenomenon is termed as progressive buckling. In the case of impact load, it is a dynamic progressive buckling [1]. The load-deformation characteristics of the tube shown in Fig. 1 is a typical example of structural behavior of an energy absorber.

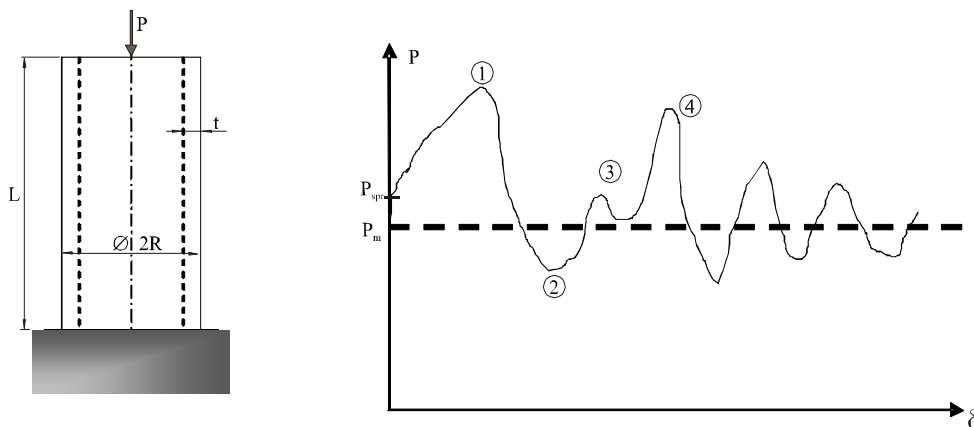


Fig. 1 – Structural behavior of axially compressed cylindrical tube [1].

2. THIN-WALLED TUBULAR ABSORBERS – STATE OF ART REVIEW

In the case of thin-walled members subjected to axial compression acting as energy absorbers, it is essential to come up with a design that would promote a progressive buckling mechanism to stimulate the highest energy absorption capacity. One of the possible design solutions is the application of a trigger (notch or dent) to release the desired crushing mechanism. There are numerous publications concerning the problem of energy absorption of thin-walled tubes [1, 5, 7, 18], however, only few of them are devoted to tubular structures with dents or other flaws.

The problem of impact loading of metal tubes, particularly those with a prismatic cross-section, has been investigated since the 1960s of the 20th century [3]. Many efforts have so far been made by several researchers to reduce peak loads at the initial stage of the crushing process and at the same time to increase the energy absorption capacity of members acting as energy absorbers [2, 18, 24].

At the turn of the centuries, a new class of tubular energy absorbers have been introduced, namely – axially loaded thin-walled foam-filled metal columns made of different foam materials like polyurethane, metal, polypropylene, etc. The energy absorbed during the crushing process by such members consists of the energy dissipated in the metal tube and the energy dissipated in the filler.

The problem of the crushing behavior of these members was initially investigated by Hanssen et al. [12, 13]. Then, Wang et al. [25] published the results concerning closed-hat section columns filled with aluminum foam. They derived a theoretical model of such columns and developed a constitutive material model of the aluminum foam on the basis of experimental tests.

In last two decades numerous research results concerning the problem of foam-filled metal columns acting as energy absorbers have been published [7]. Given the existence of many parameters affecting the energy absorption capability of such columns and their more and more complex structural design, optimization procedures had to be established. One of the first attempts in this area was published by Zarei and Kroger [27], who performed the optimization of a foam-filled aluminum tube.

In the first decade of the 21st century multi-cell columns, both hollow and foam-filled, have been introduced as energy absorbers. Chen and Wierzbicki [7] published theoretical results concerning the crushing behavior of such structural members. They investigated the axial crushing of multi-cell hollow columns via analytical and numerical methods and of double-cell and triple-cell foam-filled columns by the FE method. Very recently, Yin et al. [26] performed a multi-objective crashworthiness optimization of foam-filled multi-cell columns by different metamodel techniques. The optimization criteria were maximum energy absorption

and minimum peak crushing force. They carried out the multi-objective optimization using four metamodels and found that the Radial Basis Function (RBF) model was the most effective one.

In last few years, research was carried out into another solution, namely – the application of multi-cornered thin-walled tubes as energy absorbing members [7]. Abbasi et al. [2] investigated the crushing behavior and energy absorption of hexagonal, octagonal and 12-edge thin-walled columns. Reddy et al. [19] investigated a similar problem for 12-edge columns with varying geometry. Ali et al. [16] analyzed the axial crushing of pentagonal and cross-shape thin-walled tubes. A different solution was a bitubal energy absorber investigated experimentally under quasi-static load by Sharifi et al. [20]. A similar solution was investigated by Dass Goel M., who analysed crashworthiness performance of thin concentric cylindrical tubes by replacing the inner tube with half cylindrical shells acting as stiffeners along the circumference of the external tube [23]. Xiang Zou et al. [21] investigated the influences of multi-cell configuration and loading arrangement on the crashworthiness of square tubes, eight kinds of multi-cell square tubes were numerically analysed under axial and oblique loads.

Another solution investigated Tong Pang et al. [22], namely a functionally graded thin-walled structure with variable diameter (functionally graded tapered tube, FGTT).

Very recently, W.Liu et al. [17] performed a crashworthiness multi-objective optimization of sandwich star-shaped tubes. They investigated a multi-wall structure, with a star-shaped tube between two circulars. Similar configuration of the multi-cornered tube (cross-shaped) was investigated by Reddy et al. [19].

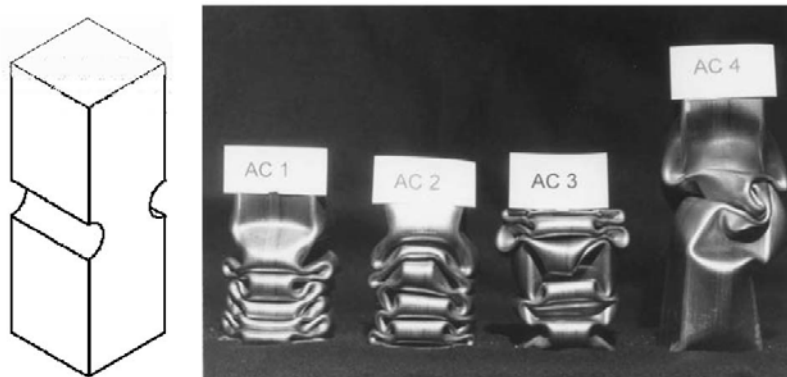


Fig. 2 – Quasi statically crushed square tubes with opposing parallel cylindrical indentations; depth of indentations increases from left to right [24].

An alternative solution of thin-walled tubular energy absorber, which promotes a progressive buckling mechanism, stimulating the highest energy absorption consists in

the use of flaws or dents acting as triggers. A trigger may induce the desired crushing (collapse) mode, leading not only to higher energy absorption and a higher mean-to-peak crushing force ratio, but also to lower peak crushing force. Flaws or dents acting as triggers can be periodically or non-periodically situated.

Relatively few publications are devoted to tubular structures with dents, dimples or other flaws. An overview of tubular sections with different kinds of imperfections and fillers subjected to axial impact loads was published by Chung and Nurick [24], who analysed crashworthiness performance of tubular structures with different types of imperfections (flaws), including prebuckle, parallel and dished indentations, cutouts, stiffeners, fillers, and wrapping.

An interesting solution has been proposed by Z. Yang et al. [28], who investigated the crushing behavior of a thin-walled circular tube with internal gradient grooves. The authors fabricated a stainless steel thin-walled tube with preset internal circumferential rectangular groove defects using the SLM 3D printing method. They observed a double buckling-splitting crushing mode. Empty and foam-filled circumferentially grooved thick-walled circular tubes under axial low velocity impact were investigated theoretically and experimentally by Darvizeh et al. [9]. In [10] the effect of discontinuity size on the energy absorption performance of square profiles was reported. In [16] the authors studied the behavior of tubular columns with dents, but these flaws were treated as imperfections resulting from damage.

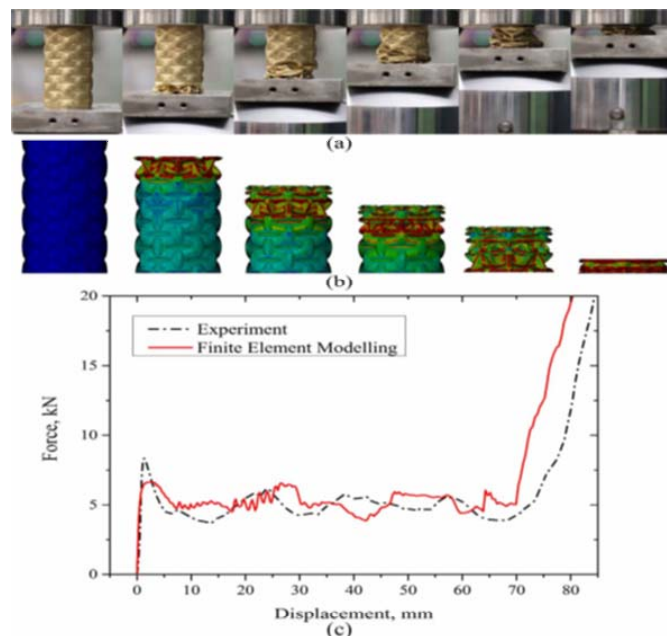


Fig. 3 – Yang et al., in: *Thin-Walled Structures*, vol. 112, 2017 [15].

An attempt to use columns with dents as energy absorbers, i.e., a static analysis of axially compressed square section tubes with dents in the corners, is presented in [11]. The subject of the investigation was a square-section column with periodically (along its height) situated dimples (dents produced by redrawing) in the corners. FE simulations of the crushing behavior were performed. Very recently, Yang et al. [15] investigated circular tubes with periodically situated (both along the circumference and the height) ellipsoidal dimples, subjected to axial crushing force. FE simulations of the crushing behavior were performed and experimentally validated by quasi-static tests on 3D printed brass tubes.

Different types of crushing mechanisms are observed in tubular structures made of composite materials, which recently are also used as energy absorbers, although their energy absorption effectiveness is generally lower in comparison with metallic structures. A. Rabiee and H. Ghasemnejad [29] published recently an overview of crushing process and energy absorption capacity of fibre-reinforced polymer (FRPs) composites tubular structures with different crush initiators, like notches, steeples, flanges, tulip triggers and others.

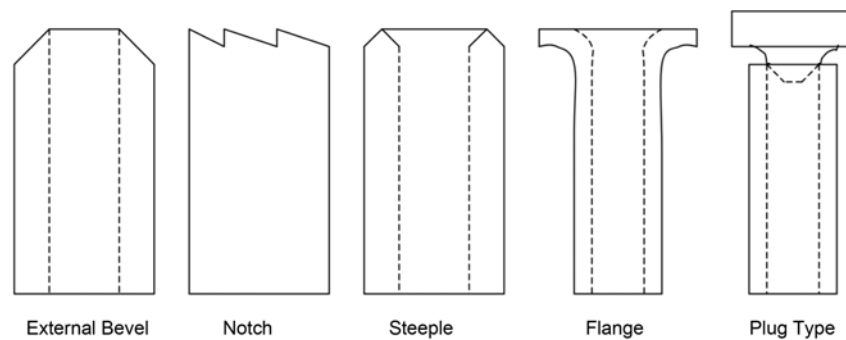


Fig. 4 – Various types of trigger mechanism in tubes made of fiber composite material [29].

3. ENERGY ABSORBING EFFECTIVENESS

The crashworthiness performance of energy absorbers can be specified in many ways. Thus, there are several crashworthiness indicators used to evaluate the crashworthiness of the energy absorbing structure [1, 14, 33]. The typical crushing force-displacement curve for a thin-walled member subjected to axial impact is shown in Fig. 5. The factor, which directly indicates absorbing effectiveness is the energy absorbed (EA) given by relation (1), or specific energy (energy per unit mass) SEA, given by (2), where m is a mass of the absorber.

$$E_A(d_x) = \int_0^{d_x} F(x)dx, \quad (1)$$

where d_x is a crushing distance and $F(x)$ is the crushing force as a function of crush distance x (see Fig. 5).

$$SEA = \frac{E_A}{m} \quad (2)$$

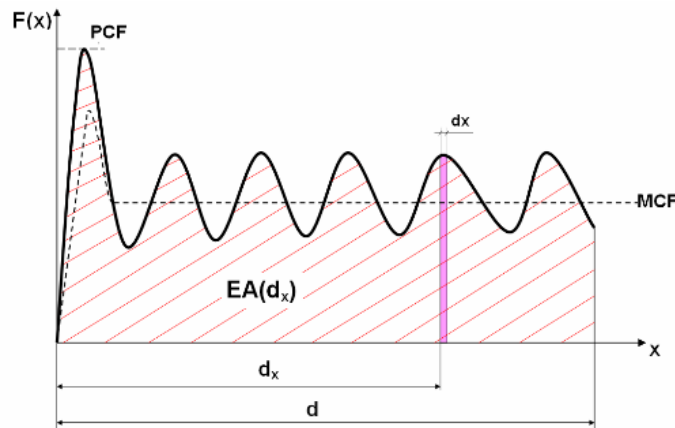


Fig. 5 – Exemplary load-shortening diagram of thin-walled column under axial impact.

Two factors, mentioned above, (EA and SEA) are not direct measures of crashworthiness. There are some several other factors, which help in assessing energy absorption capability and provide a deeper understanding of crashworthiness performance of energy absorbing structure.

Since the reduction of an initial peak crushing force (PCF) is mostly desirable from biomechanical reasons, this factor is an important indicator of the absorber effectiveness.

The mean crushing force (MCF – Fig. 5) for a given crushing deformation d_x is calculated as:

$$MCF = \frac{E_A(d_x)}{d_x}. \quad (3)$$

Crash load efficiency is defined as a *mean* crushing force (MCF) to peak crushing force (PCF – see Fig. 5) ratio:

$$CLE = \frac{MCF}{PCF} \cdot 100\%. \quad (4)$$

The equivalent efficiency parameter, termed by some authors [24] e_E is defined as a ratio of the energy absorbed to the theoretical maximum energy, which can be absorbed:

$$e_E = \frac{E_A}{PCF \cdot d_{\max}}. \quad (4a)$$

The CLE factor should be as close to 100% as possible in practice, thus, CLE of an ideal energy absorber is 100%.

Another crashworthiness indicator is the stroke efficiency, which represents the deformation capacity of an absorber. Stroke efficiency St_e is defined as follows:

$$St_e = \frac{L_o - U}{L_o}. \quad (5a)$$

where: L_o – initial length (characteristic dimension) of the member [mm]; U – maximum shortening (maximum characteristic deformation) of the member.

Some authors [18] define the stroke efficiency as:

$$SE = \frac{U}{L_o}. \quad (5b)$$

The most desirable is the lowest value of St_e (or the highest value of SE) factor, corresponding to the highest value of the crushing distance. It is one of the fundamental performance indicators. An ideal structure should involve the complete available length in crushing (plastic deformation) to enable the structure utilization of the entire length to absorb an impact energy.

A combination of the CLE and the stroke efficiency was defined by Hanssen et al. [13] as total efficiency TE to assess the whole performance of an energy absorber. It is a dimensionless number and is given by

$$TE = CLE \times St_e. \quad (6)$$

However, this total efficiency does not include the factor of the weight of the structures and therefore another index was introduced by Zhang et al. [32], who proposed to divide the total efficiency by the mass. This factor is defined as specific total efficiency and expressed as follows:

$$STE = \frac{TE}{m} = \frac{CLE \times St_e}{m}. \quad (7)$$

All indicators, mentioned above express an absorber characteristics within one specific structural geometry and one absorber's material. The first indicator,

which included the material factor was introduced by Pugsley and Jones [14], termed structural efficiency and defined as:

$$\eta = \frac{MCF}{A \cdot \sigma_0} \quad (8)$$

where: σ_0 – yield stress, A – area of the absorber cross-section.

Any comparisons between the characteristics of energy absorbers, made from different materials are difficult using indicators, mentioned above, to some extent except structural efficiency given by (8). Thus, a new alternative approach to the assessment of energy absorbing effectiveness has been proposed by Jones [14]. He proposed the factor defined as:

$$\Psi_0 = \frac{\text{total elastic and plastic energy absorbed by a absorber}}{\text{energy absorbed in the same volume of material specimen up to failure in tension}} \quad (8)$$

In the case of axial crushing, Jones derived an approximate relation determining this factor as follows [14, 33]:

$$\Psi_0 = \frac{3mv^2}{8\sigma_0 A \delta_f \varepsilon_R} \quad (9)$$

where: m – mass of impactor, v – initial impact velocity, σ_0 – yield stress, A – area of the absorber cross-section, δ_f – final axial displacement (shortening), ε_R – rupture strain.

In the case of foam filled structure, the denominator in (9) incorporates energy of the same volume of foam material tensile test specimen, as follows:

$$\Psi_0 = \frac{3mv^2}{8\delta_f \varepsilon_R (\sigma_0 A_s + \sigma_f A_f)} \quad (10)$$

where: σ_f – yield stress of the foam material, A_s – area of the shell (face sheet), A_f – area of the foam.

The energy absorbing effectiveness factor Ψ_0 may be used in a preliminary analysis, leading to a selection of an absorber's type. After this initial selection, within one type of absorbing structure, other crashworthiness indicators, particularly important for specific absorber's purposes, should be taken into account. An exemplary comparison of Ψ_0 factor for two types of tubular absorbers, namely hollow and foam-filled prismatic frustum and tubular absorber with four dents in the corners made of aluminum and steel (type I – Fig. 7a) is shown in Fig. 6 [33].

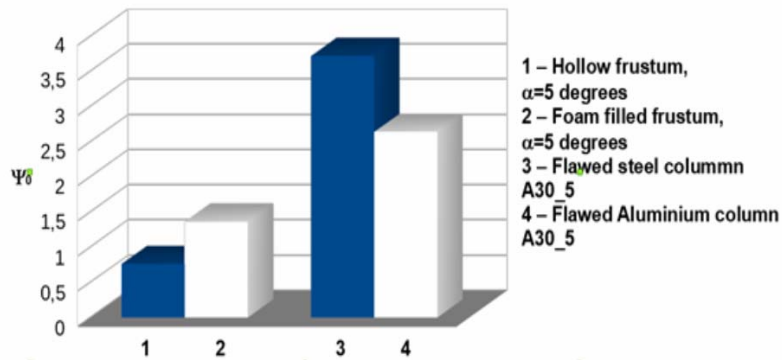


Fig. 6 – Energy absorbing effectiveness factor Ψ_0 for two types of absorbers – comparison [33].

In summarizing the above paragraph, it should be said, that no single crashworthiness factor can decide energy absorption capability of energy absorbing structure. An appropriate general indicator (or indicators) of crashworthiness is still an open question and demands further comparative analysis.

4. TUBULAR MEMBERS WITH NON-PERIODICALLY SITUATED TRIGGERS – SELECTED RESULTS

The state of art review presented above shows a gap in research undertaken so far into crushing behavior and crashworthiness of thin-walled columns of prismatic cross-section with non-periodically situated flaws. This problem is a subject of theoretical investigation, results of which are presented by Ferdynus et al. in [30]. Experimental validation of those results were published by the same authors in [31]. Some new results of that research program are presented below.

The subject of investigation was a thin-walled square section aluminum tube with four or two flaws (triggers), of different shapes shown in Fig. 7.

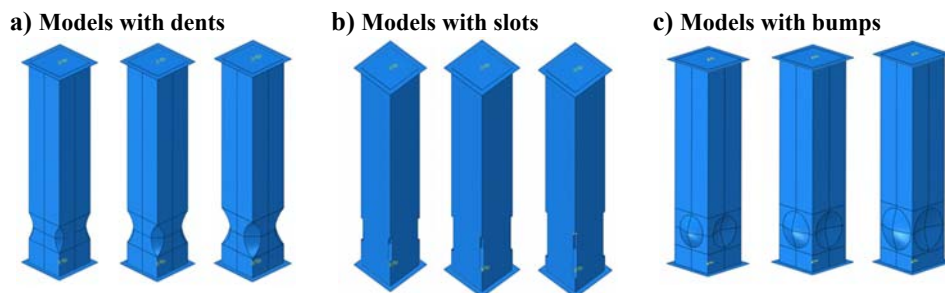


Fig. 7 – Examples of the typology of prismatic parallellepiped tubes under investigation.

The first type (type I) were tubes with four cylindrical indentations (dents) in the corners, formed by means of cylindrical redrawing (Fig. 7a), the second (type II) – with slots (Fig. 7b) and the last (type III) – with “bumps” formed by means of spherical concave indentations or spherical convex redrawing (Fig. 7c).

Within the first type (Fig. 7a), the parametric study concerned the varying parameters: the dent’s depth and distance of the dent to the base. The dents were made either at the bottom of the column or at the top of it. The models with dents at the bottom of the column were denoted by the symbols from ANN_X to ANN_X , where the first number (NN) stands for the relative depth of the dent (in percentage) while X stands for the distance of the dent from the bottom. Also a position of a dent, either at the bottom, or at the top base (the load application point) was examined. Similarly, the models with dents situated at the top of the column (where the impact load was applied) were denoted by the column symbols from CNN_X . The column with smooth walls (without dents) was denoted as SM.

It was found, that the main factor influencing a crushing mode and, subsequently, energy absorption capability, is a dent depth. The dent distance from the base is of less importance. Also a position of a dent, either at the bottom, or at the top base (the load application point) does not influence the crushing behavior significantly.

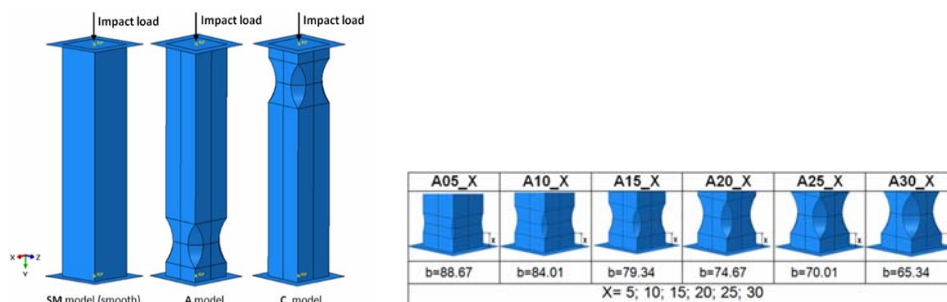


Fig. 8 – Typology of tubes type I (Fig. 7a) [30].

Three main modes of failure were observed [30]. For the smallest values of the dent depth, regardless of the dent position (X), the crushing process was initiated by a local plastic mechanism (first local fold) situated at the center of the column (Fig. 9a). For medium values of the dent depth ($X \leq 20$ mm), the crushing process was triggered by a local plastic mechanism situated above the dent (Fig. 9b). For the largest dent depths ($X > 20$ mm), regardless of the position of the dent, the crushing was initiated by a local plastic mechanism situated exactly at the dent. These three types of failure are denoted below as mode I, mode II and mode III, respectively.

The best energy absorption capability was obtained for crushing mode III.

This case produced the largest values of relative dent depth (A20-A30), regardless of the position of the dent. For this crushing mode the highest values of crash load efficiency (CLE) and the smallest values of peak crushing force (PCF) were obtained. For the deepest dents, the relative increase in the CLE amounted to about 25% with respect to the non-flawed column (Fig. 11). The corresponding relative decrease in the PCF amounted to about 24%. For the deepest dents, where crushing mode III could be observed, the CLE indicator decreased with elevating the dent's position. At a smaller depth of the dent, for crushing modes I and II, the dent position has little effect on the energy absorption capability of the column.

Exemplary load-shortening diagrams for tube, exhibiting three crushing modes, mentioned above, are shown in Fig. 10. The important advantage of dents application is a significant decrease of peak crushing force (PCF). As mentioned above, this type of tubes with dents was also tested experimentally. Results of the impact tests confirmed numerical results [31].

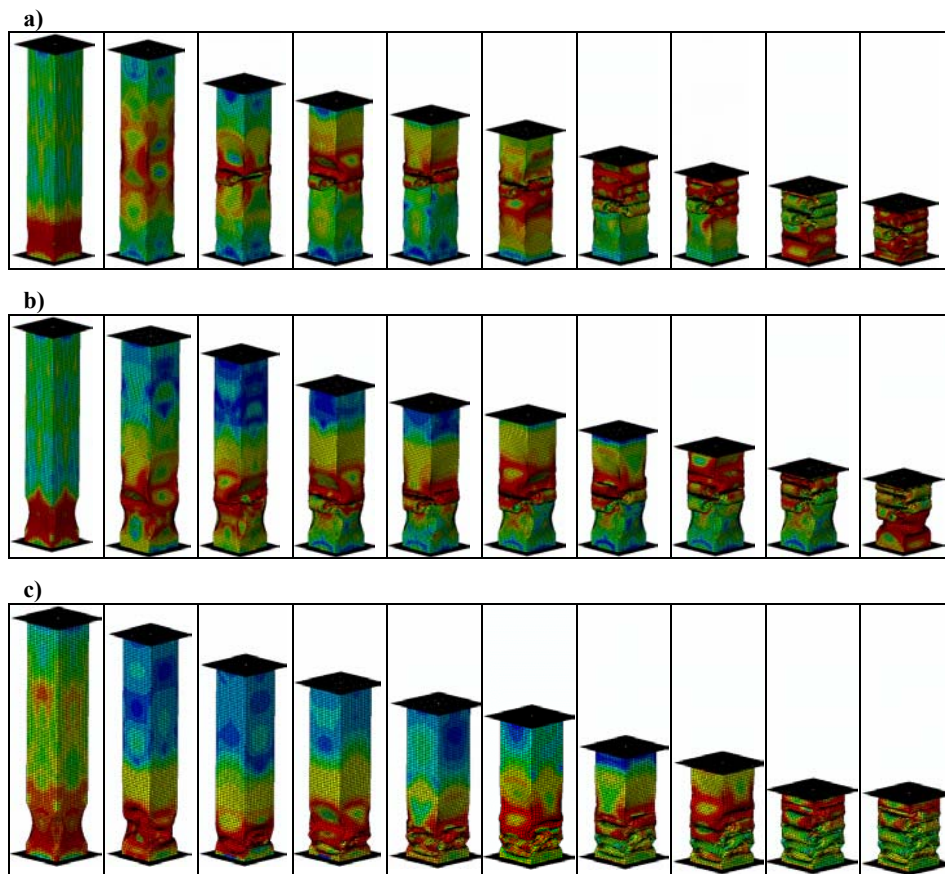


Fig. 9 – Crushing modes for tubes with dents (type I): a) mode I, b) mode II, c) mode III [30].

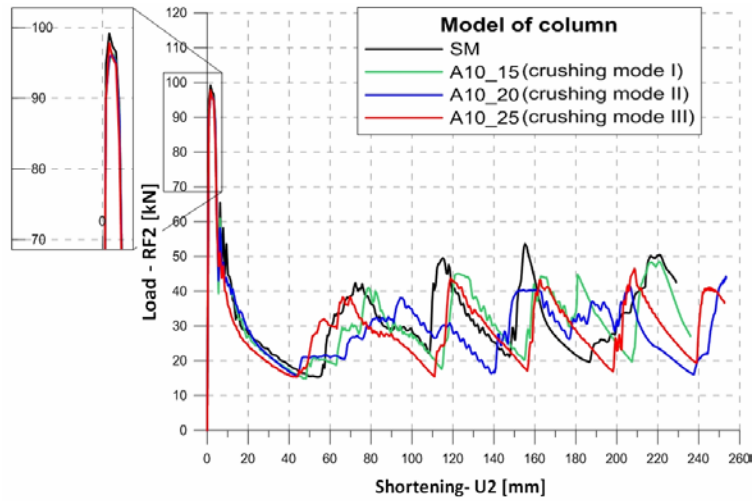


Fig. 10 – Load-shortening diagrams for columns *A* exhibiting three different crushing modes (constant value of relative dent depth = 10%) [30].

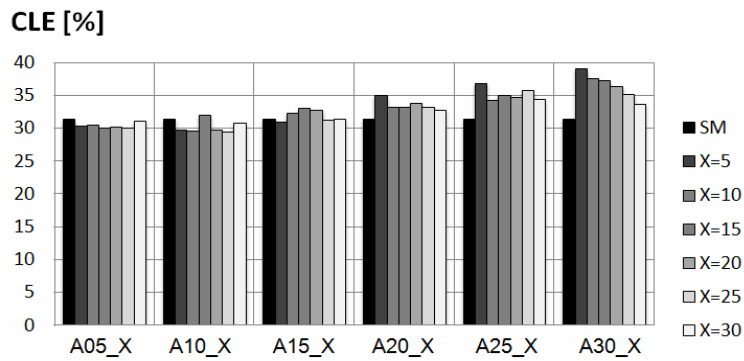


Fig. 11 – Crash load efficiency versus relative dent depth and distance from the bottom plate [30].

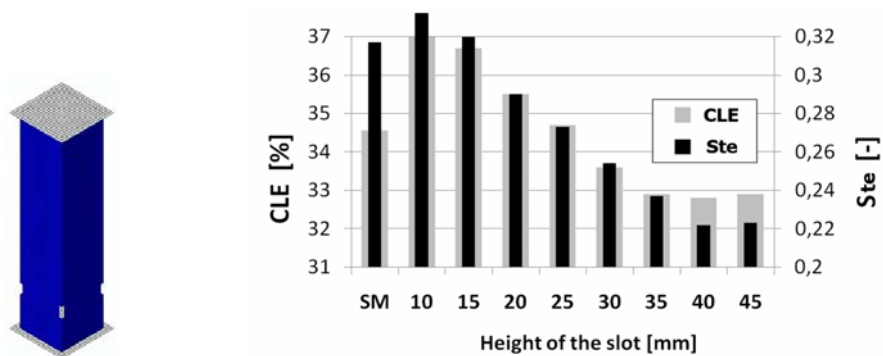


Fig. 12 – Crash load efficiency and stroke efficiency for tubes with slots (type II).

Preliminary numerical parametric study was also performed for tubes with slots. Crash load efficiency (CLE) and stroke efficiency in terms of slots height is shown in Fig. 12. The highest values of CLE were obtained for cutouts of 10 mm height. However, the smallest value of St_e (which corresponds to the best stroke efficiency) was obtained for slots of 40 mm height. Preliminary results of numerical parametric study for the third type of tubes (with “bumps”) are shown in Fig. 13. They concern tube with constant redrawing diameter (36 mm) and varying redrawing relative depth (multiple of tube’s wall thickness). Two cases were examined: A – with convex redrawing, B – with concave indentation. As for prismatic tubes of the first type (with four dents), energy absorption capability increases with the increase of relative redrawing depth.

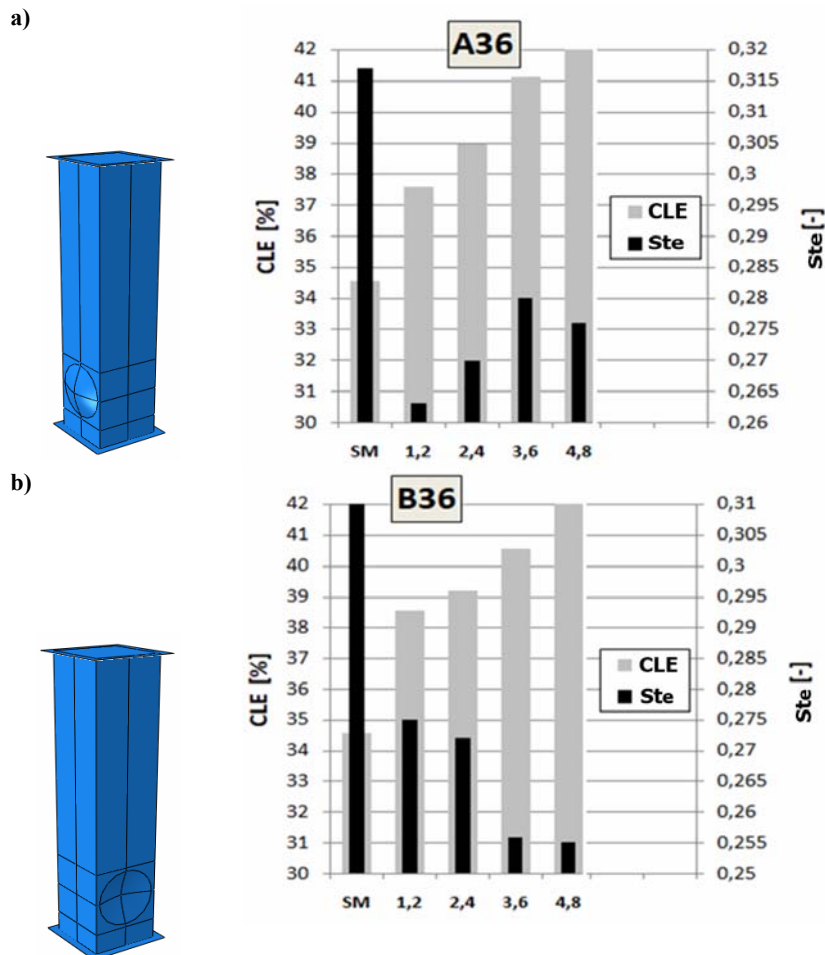


Fig. 13 – Crash load efficiency and stroke efficiency for tubes with bumps (type III): a) with convex redrawing, b) with concave indentations [35].

5. FINAL REMARKS

The results of several analyses of crashworthiness performance of tubular, thin-walled columns with different types of triggers, presented above, indicate that this design solution of an energy absorbing thin-walled structure is efficient. Triggers of different shapes (dents, slots, flanges, grooves) promote a progressive buckling mechanism and, subsequently, stimulate high energy absorption with simultaneous relatively low peak crushing force, high crash load efficiency and high stroke efficiency.

Thus, further research should be conducted into design of optimal shapes of triggers, not only from the crushing performance point of view, but also taking into account possibly low cost of manufacturing process.

In the particular case of thin-walled columns of prismatic, rectangular cross-section with non-periodically situated flaws (dents or slots) further investigation should be continued into multi-objective crashworthiness optimization, taking into account a wide range of geometric and material parameters. This theoretical investigation should be validated by means of extensive experimental impact tests.

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