ABAQUS/EXPLICIT IMPLEMENTATION OF A CONSTITUTIVE MODEL FOR THIN SHEET METALS SUBJECTED TO FORMING PROCEDURES. PART I: THEORY

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Abstract. This paper deals with the Abaqus/Explicit implementation of a constitutive model for orthotropic thin sheet metals subjected to forming procedures dominated by stretching and bending effects. The metallic sheet is assumed to behave as an elastoplastic shell body, its mechanical response being described by Hooke's law combined with a plastic potential and the associated flow rule. The constitutive model is kept in a general form so that it can easily accommodate different expressions of the plastic potential. Such a particularization involving the BBC2005 effective stress combined with two analytically defined hardening laws has been implemented by the authors as a VUMAT subroutine of the Abaqus/Explicit finite element programme. The predictive capabilities of the constitutive model will be analysed in the second part of the paper.

Keywords: sheet metals, orthotropic plasticity, constitutive modelling, finite element analysis.

1. INTRODUCTION

The material model presented in this paper operates with components of the Cauchy stress and logarithmic strain tensors expressed in a corotational frame. The rate-type constitutive equations involving plain time derivatives of such quantities are always objective [1]. The corotational frame also characterizes the orthotropy of cold-rolled metallic sheets [2], being initially coincident with the frame defined by the rolling direction (axis 1), transverse direction (axis 2), and normal direction (axis 3).

Throughout the text, Latin and Greek lower-right indices are used to individualize tensor components expressed in orthonormal bases. Whenever not explicitly mentioned, the following sets of admissible values are assumed for these

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Rom. J. Techn. Sci. – Appl. Mechanics, Vol. 59, N° 3, P. 125–135, Bucharest, 2014

qualifiers: Latin indices $i, j, k, \ldots \in \{1,2,3\}$; Greek indices $\alpha, \beta, \gamma, \ldots \in \{1,2\}$. The summation rule of the repeated lower-right index is also adopted in tensor relationships.

The following symbols denote strain and stress quantities involved in the constitutive model:

$\varepsilon_{ij} = \varepsilon_{ji}$ corotational components of the logarithmic strain tensor additively separable into elastic $\varepsilon_{ij}^{(e)} = \varepsilon_{ji}^{(e)}$ and plastic $\varepsilon_{ij}^{(p)} = \varepsilon_{ji}^{(p)}$ parts, i.e.

$$\varepsilon_{ij} = \varepsilon_{ij}^{(e)} + \varepsilon_{ij}^{(p)} \tag{1}$$

$\sigma_{ij} = \sigma_{ji}$ corotational components of the Cauchy stress tensor

$\bar{\sigma} \geq 0$ effective stress defined as a first-degree homogeneous function

$$\bar{\sigma} = \bar{\sigma}\left(\sigma_{11} - \sigma_{22}, \sigma_{22} - \sigma_{33}, \sigma_{33} - \sigma_{11}, \sigma_{12} = \sigma_{21}, \sigma_{23} = \sigma_{32}, \sigma_{31} = \sigma_{13}\right) \tag{2}$$

$\bar{\varepsilon}^{(p)} \geq 0$ effective plastic strain emerging from the equivalence principle

$$\bar{\sigma}\dot{\bar{\varepsilon}}^{(p)} = \sigma_{ij}\dot{\varepsilon}_{ij}^{(p)} \tag{3}$$

$Y > 0$ yield stress defined by a non-decreasing function

$$Y = Y\left[\bar{\varepsilon}^{(p)}\right] \tag{4}$$

(the so-called hardening law).

The orthotropic plasticity of sheet metals is described by means of the effective stress $\bar{\sigma}$. Several researchers focused their efforts on elaborating anisotropic expressions of $\bar{\sigma}$. An exhaustive literature review surveying this topic can be found in [2]. In what follows, only the significant contributions will be mentioned.

One of the earliest $\bar{\sigma}$ functions of orthotropic type was proposed by Hill in 1948 [3]. Due to its quadratic form, straightforward identification and computational efficiency, the Hill1948 expression of the effective stress is still widely used in the numerical simulation of sheet metal forming processes, despite the fact that it poorly approximates the plasticity of aluminium alloys [2]. With the aim of overcoming this drawback, Hill developed a series of non-quadratic $\bar{\sigma}$ functions [4-6]. Although the newer expressions of the effective stress proposed by Hill perform better, their predictions are sometimes in disagreement with crystal plasticity models.

Barlat and his co-workers elaborated another class of orthotropic $\bar{\sigma}$ functions having non-quadratic expressions. The formulations called Yld1989 [7] and Yld2000 [8] are the most extensively used in practice. It is worth mentioning that Barlat's models generally provide more realistic predictions than Hill's [2].

Using the non-quadratic formulation Yld1989 [7] as a basis, Banabic and his team developed a family of more flexible $\bar{\sigma}$ functions by adding extra material coefficients [2]. Among the $\bar{\sigma}$ expressions obtained in this manner, the formulation called BBC2005 [2] is the most used in the numerical simulation of sheet metal forming processes.

The main objective of this paper consists in describing the Abaqus/Explicit implementation of a constitutive model for thin sheet metals subjected to forming procedures dominated by stretching and bending effects. Section 2 briefly presents the relationships that reflect the mechanical behaviour of sheet metals:

- Hooke's law defining the elastic response
- Plastic potential and the associated flow rule defining the inelastic response.

Section 3 details the Abaqus/Explicit implementation of the material model. After rewriting the constitutive relationships in an incremental form, the authors gather a set of nonlinear equations that defines the current state of the metallic sheet. The numerical procedure used to solve the set of nonlinear equations is also explained. The incremental constitutive model is kept in a general form so that it can be easily particularized for different expressions of the $\bar{\sigma}$ and Y functions. Such a particularization involving the BBC2005 effective stress combined with two analytically defined hardening laws (Swift and modified Voce formulations) is described at the end of Section 3.

2. GENERAL DESCRIPTION OF THE CONSTITUTIVE MODEL

From the total strain accumulated by metallic sheets during typical forming processes dominated by stretching and bending effects, the recoverable part usually represents only a few percent. Under these circumstances, the elastic response of such materials can be described with sufficient accuracy by the inverted rate-version of Hooke's isotropic law

$$\dot{\varepsilon}_{ij}^{(e)} = \frac{1}{2\mu}\left(\dot{\sigma}_{ij} - \frac{\lambda}{3\lambda + 2\mu}\delta_{ij}\dot{\sigma}_{\ell\ell}\right), \tag{5}$$

in which δ_{ij} is Kronecker's symbol

$$\delta_{ij} = \begin{cases} 0, & \text{if } i \neq j, \\ 1, & \text{if } i = j, \end{cases} \tag{6}$$

while

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{E}{2(1+\nu)} \tag{7}$$

are Lamé's constants deduced from Young's modulus E and Poisson's ratio ν.

The plastic part of the constitutive model is based on the potential function

$$\phi = \bar{\sigma} - Y. \tag{8}$$

A stress state is admissible if and only if $\phi \leq 0.$ In general,

$$\begin{cases} \phi < 0 & \text{individualizes all elastic states, while} \\ \phi = 0 & \text{(the so-called yield criterion) individualizes all elastoplastic states.} \end{cases} \tag{9}$$

The second ingredient of the plastic constitutive model is the flow rule

$$\dot{\varepsilon}_{ij}^{(p)} = \dot{\bar{\varepsilon}}^{(p)} \frac{\partial \phi}{\partial \sigma_{ij}} = \dot{\bar{\varepsilon}}^{(p)} \frac{\partial \bar{\sigma}}{\partial \sigma_{ij}}, \tag{10}$$

in which

$$\begin{cases} \dot{\bar{\varepsilon}}^{(p)} = 0, & \text{if } \phi < 0, \\ \dot{\bar{\varepsilon}}^{(p)} \geq 0, & \text{if } \phi = 0. \end{cases} \tag{11}$$

Any $\bar{\sigma}$ function defined in accordance with Eq. (2) has a traceless gradient, i.e. $\partial \bar{\sigma} / \partial \sigma_{\ell\ell} = 0.$ By means of Eq. (10), this property is transferred to the plastic part of the strain-rate tensor: $\dot{\varepsilon}_{\ell\ell}^{(p)} = 0.$

3. IMPLEMENTATION OF THE CONSTITUTIVE MODEL IN THE ABAQUS/EXPLICT FINITE-ELEMENT PROGRAMME

The numerical simulation of forming processes follows a sequence of small time steps $t \rightarrow t + \Delta t.$ At each time step, the configuration of the metallic sheet corresponding to the moment t acts as a reference state, its parameters being known quantities. In the particular case of the constitutive model presented in the previous section of the paper, these parameters are the stress components ${}^{t}\sigma_{ij}$ and the effective plastic strain ${}^{t}\bar{\varepsilon}^{(p)}.$ When the metallic sheet is assumed to behave as a shell body, the generalized plane-stress condition ${}^{t}\sigma_{33} = 0$ should be met. Unfortunately, the numerical schemes used to update the corotational frame do not guarantee the rigorous fulfilment of this constraint. Such an aspect cannot be ignored when implementing constitutive models for shell bodies. If not taken into account and compensated, the inaccurate fulfilment of the generalized plane-stress condition leads to numerical errors that tend to grow in an uncontrollable manner during the simulation of the forming process.

The current state parameters ${}^{t+\Delta t}\sigma_{ij}$ and ${}^{t+\Delta t}\bar{\varepsilon}^{(p)}$ are expected as output from the constitutive model. The procedure used to evaluate these quantities is consistent with the mechanics of shell bodies only if the condition

$$^{t+\Delta t}\sigma_{33} = 0 \tag{12}$$

is fulfilled. The $t \to t + \Delta t$ evolution of the state parameters is controlled by strain increments

$$\Delta\varepsilon_{ij} = \int_t^{t+\Delta t} {}^{\tau}\dot{\varepsilon}_{ij}\,\mathrm{d}\tau. \tag{13}$$

Except for $\Delta\varepsilon_{33}$, all the other components $\Delta\varepsilon_{ij}$ are calculated in advance and passed as input to the constitutive model. In the case of shell bodies, the thickness strain increment $\Delta\varepsilon_{33}$ is fully defined by Eq. (12). As a consequence, the constitutive model must evaluate $\Delta\varepsilon_{33}$ together with the current state parameters $^{t+\Delta t}\sigma_{ij}$ and $^{t+\Delta t}\bar{\varepsilon}^{(p)}$.

Eq. (5) constrained by Eq. (12) allows deducing the formula

$$^{t+\Delta t}\sigma_{ij} = {}^{t}\sigma_{ij} + \lambda\,\delta_{ij}\,\Delta\varepsilon_{\ell\ell}^{(e)} + 2\mu\,\Delta\varepsilon_{ij}^{(e)}, \tag{14}$$

in which

$$\Delta\varepsilon_{ij}^{(e)} = \int_t^{t+\Delta t} {}^{\tau}\dot{\varepsilon}_{ij}^{(e)}\,\mathrm{d}\tau \tag{15}$$

are elastic parts of the incremental strains, with $\Delta\varepsilon_{33}^{(e)}$ acting as a dependent quantity defined as follows:

$$\Delta\varepsilon_{33}^{(e)} = -\frac{1}{\lambda + 2\mu}\left[{}^{t}\sigma_{33} + \lambda\,\Delta\varepsilon_{\gamma\gamma}^{(e)}\right]. \tag{16}$$

Eq. (14) and Eq. (16) are useful in the first stage of the computations, when the elastic or elastoplastic character of the strain increments $\Delta\varepsilon_{ij}$ must be identified.

Under the hypothesis of a purely elastic evolution during the time interval $[t, t + \Delta t]$, i.e. $\Delta\varepsilon_{ij} = \Delta\varepsilon_{ij}^{(e)}$, a set of trial strain increments $\Delta\varepsilon_{ij}^{(e,trial)}$ can be defined (see also Eq. (16)):

$$\begin{cases} \Delta\varepsilon_{ij}^{(e,trial)} = \Delta\varepsilon_{ij}, \quad \text{if } (i,j) \neq (3,3), \\ \Delta\varepsilon_{33}^{(e,trial)} = -\dfrac{1}{\lambda + 2\mu}\left({}^{t}\sigma_{33} + \lambda\,\Delta\varepsilon_{\gamma\gamma}\right). \end{cases} \tag{17}$$

The corresponding trial stress components $^{t+\Delta t}\sigma_{ij}^{(trial)}$ result from Eq. (14) after replacing $\Delta\varepsilon_{ij}^{(e)}$ by $\Delta\varepsilon_{ij}^{(e,trial)}$:

$$^{t+\Delta t}\sigma_{ij}^{(trial)} = {}^{t}\sigma_{ij} + \lambda\,\delta_{ij}\,\Delta\varepsilon_{\ell\ell}^{(e,trial)} + 2\mu\,\Delta\varepsilon_{ij}^{(e,trial)}. \tag{18}$$

As soon as the quantities ${}^{t+\Delta t}\sigma_{ij}^{(trial)}$ are available, Eq. (8), Eq. (2) and Eq. (4) allow the calculation of the associated potential

$$ {}^{t+\Delta t}\phi^{(trial)} = \bar{\sigma}\Big|_{\sigma_{ij}={}^{t+\Delta t}\sigma_{ij}^{(trial)}} - Y\left[{}^{t}\bar{\varepsilon}^{(p)}\right]. \tag{19} $$

If ${}^{t+\Delta t}\phi^{(trial)} \le 0,$ the hypothesis of a purely elastic evolution is correct. In such a case, the thickness strain increment and the parameters of the current configuration can be set as follows: $\Delta\varepsilon_{33} = \Delta\varepsilon_{33}^{(e,trial)}$, ${}^{t+\Delta t}\sigma_{ij} = {}^{t+\Delta t}\sigma_{ij}^{(trial)}$, and ${}^{t+\Delta t}\bar{\varepsilon}^{(p)} = {}^{t}\bar{\varepsilon}^{(p)}$. The generalized plane-stress condition ${}^{t+\Delta t}\sigma_{33}^{(trial)} = 0$ is automatically satisfied when Eq. (17) and Eq. (18) are used.

On the other hand, if ${}^{t+\Delta t}\phi^{(trial)} > 0,$ the sheet metal evolves through elastoplastic states during the time interval $\left[t, t+\Delta t\right],$ i.e. (see Eq. (1))

$$ \Delta\varepsilon_{ij} = \Delta\varepsilon_{ij}^{(e)} + \Delta\varepsilon_{ij}^{(p)}. \tag{20} $$

The unrecoverable parts of the strain increments, denoted as $\Delta\varepsilon_{ij}^{(p)}$ in Eq. (20), are now different from zero. These quantities can be evaluated using backward Euler approximations of the time integrals

$$ \Delta\varepsilon_{ij}^{(p)} = \int_{t}^{t+\Delta t} {}^{\tau}\dot{\varepsilon}_{ij}^{(p)}\,\mathrm{d}\tau, \qquad \Delta\bar{\varepsilon}^{(p)} = \int_{t}^{t+\Delta t} {}^{\tau}\dot{\bar{\varepsilon}}^{(p)}\,\mathrm{d}\tau, \tag{21} $$

i.e.

$$ \Delta\varepsilon_{ij}^{(p)} = {}^{t+\Delta t}\dot{\varepsilon}_{ij}^{(p)}\Delta t, \qquad \Delta\bar{\varepsilon}^{(p)} = {}^{t+\Delta t}\bar{\varepsilon}^{(p)} - {}^{t}\bar{\varepsilon}^{(p)} = {}^{t+\Delta t}\dot{\bar{\varepsilon}}^{(p)}\Delta t. \tag{22} $$

When combined with these approximations, Eq. (10) becomes

$$ \Delta\varepsilon_{ij}^{(p)} = \left[{}^{t+\Delta t}\bar{\varepsilon}^{(p)} - {}^{t}\bar{\varepsilon}^{(p)}\right]\frac{\partial\bar{\sigma}}{\partial\sigma_{ij}}\bigg|_{\sigma_{ij}={}^{t+\Delta t}\sigma_{ij}}. \tag{23} $$

Eq. (20), Eq. (5), Eq. (23), and Eq. (12) allow deducing the relationships

$$ \begin{aligned} \Delta\varepsilon_{\alpha k} = \left[{}^{t+\Delta t}\bar{\varepsilon}^{(p)} - {}^{t}\bar{\varepsilon}^{(p)}\right]\frac{\partial\bar{\sigma}}{\partial\sigma_{\alpha k}}\bigg|_{\substack{\sigma_{ij}={}^{t+\Delta t}\sigma_{ij} \\ {}^{t+\Delta t}\sigma_{33}=0}} + \\ \frac{1}{2\mu}\left({}^{t+\Delta t}\sigma_{\alpha k} - \frac{\lambda}{3\lambda+2\mu}\delta_{\alpha k}\,{}^{t+\Delta t}\sigma_{\gamma\gamma}\right) - {}^{t}\varepsilon_{\alpha k}^{(e)}, \qquad \text{for } \alpha \le k, \end{aligned} \tag{24} $$

and

$$\Delta\varepsilon_{33} = \left[{}^{t+\Delta t}\overline{\varepsilon}^{(p)} - {}^{t}\overline{\varepsilon}^{(p)} \right] \frac{\partial \overline{\sigma}}{\partial \sigma_{33}} \Bigg|_{\substack{\sigma_{ij} = {}^{t+\Delta t}\sigma_{ij} \\ {}^{t+\Delta t}\sigma_{33} = 0}} - \frac{\lambda}{2\mu\left(3\lambda + 2\mu\right)} {}^{t+\Delta t}\sigma_{\gamma\gamma} - {}^{t}\varepsilon_{33}^{(e)}, \tag{25}$$

in which

$${}^{t}\varepsilon_{ij}^{(e)} = \frac{1}{2\mu}\left({}^{t}\sigma_{ij} - \frac{\lambda}{3\lambda + 2\mu}\delta_{ij}\, {}^{t}\sigma_{\ell\ell} \right) \tag{26}$$

are components of the elastic strain tensor associated to the reference configuration. It is not difficult to notice that Eq. (25) expresses $\Delta\varepsilon_{33}$ as a function of ${}^{t+\Delta t}\sigma_{\alpha k} = {}^{t+\Delta t}\sigma_{k\alpha}\left(\alpha \le k\right)$ and ${}^{t+\Delta t}\overline{\varepsilon}^{(p)}$. The strain increment $\Delta\varepsilon_{33}$ and the normal stress ${}^{t+\Delta t}\sigma_{33} = 0$ can be thus removed from the set of primary unknowns which reduces to ${}^{t+\Delta t}\sigma_{\alpha k}\left(\alpha \le k\right)$ and ${}^{t+\Delta t}\overline{\varepsilon}^{(p)}$.

The quantities ${}^{t+\Delta t}\sigma_{\alpha k}\left(\alpha \le k\right)$ and ${}^{t+\Delta t}\overline{\varepsilon}^{(p)}$ must satisfy Eq. (24) together with the yield criterion (see Eq. (9) and Eq. (8))

$$\overline{\sigma}\Big|_{\substack{\sigma_{ij} = {}^{t+\Delta t}\sigma_{ij} \\ {}^{t+\Delta t}\sigma_{33} = 0}} - Y\left[{}^{t+\Delta t}\overline{\varepsilon}^{(p)} \right] = 0. \tag{27}$$

The nonlinear system consisting of Eq. (24) and Eq. (27) is solved numerically using a forward-difference Newton scheme combined with a simplified line-search strategy [9]. The reference state parameters ${}^{t}\sigma_{\alpha k}\left(\alpha \le k\right)$ and ${}^{t}\overline{\varepsilon}^{(p)}$ define the start point of the solution procedure. This initial guess ensures a rapid convergence of the Newton iterations for sufficiently small strain increments $\Delta\varepsilon_{\alpha k}\left(\alpha \le k\right)$ passed as input to the constitutive model. The performances of the solution procedure tend to degrade when the carrying capability of the metallic sheet suffers a drop due to necking and the continuous decrease of the material strength leads to an accelerated strain accumulation. The solution procedure is able to overcome almost all the convergence difficulties that may occur in such cases by activating an adaptive subincrementation mechanism. The principle of this mechanism consists in splitting the input values $\Delta\varepsilon_{\alpha k}\left(\alpha \le k\right)$ into smaller fractions. The quantities $\Delta\varepsilon_{\alpha k}\left(\alpha \le k\right)$ are successively halved until the convergence of the Newton scheme is achieved or the total number of subincrementation attempts becomes too large (for example, greater than seven). In the latter case, the numerical simulation is stopped after issuing an error message. If the Newton scheme has converged, the strain increment fraction is doubled and the previously obtained state parameters are used to define the initial guess for a new iterative solution of Eq. (24) and Eq. (27). This procedure ends when the process of adding small fractions restores the full input values $\Delta\varepsilon_{\alpha k}\left(\alpha \le k\right)$. Having ${}^{t+\Delta t}\sigma_{\alpha k} = {}^{t+\Delta t}\sigma_{k\alpha}\left(\alpha \le k\right)$ and ${}^{t+\Delta t}\overline{\varepsilon}^{(p)}$ determined, Eq. (25) allows the calculation of the strain increment $\Delta\varepsilon_{33}$.

The constitutive model presented above can easily accommodate full plane-stress yield criteria. Their specificity consists in the fact that only the in-surface components of the stress tensor are used to define $\bar{\sigma}$, i.e. [2]

$$\bar{\sigma} = \bar{\sigma}\left(\sigma_{11}, \sigma_{22}, \sigma_{12} = \sigma_{21}\right). \tag{28}$$

Eq. (28) is able to replace Eq. (2) as soon as the following hypotheses are added to the constitutive model:

- Through-thickness shear strains are purely elastic, i.e.

$$\varepsilon_{\alpha 3}^{(p)} = \varepsilon_{3\alpha}^{(p)} = 0. \tag{29}$$

- The plastic part of the strain tensor is traceless, i.e.

$$\varepsilon_{33}^{(p)} = -\varepsilon_{\gamma\gamma}^{(p)}. \tag{30}$$

Both assumptions are in good agreement with the mechanics of sheet metal forming processes dominated by stretching and bending effects.

In the particular situation defined by Eq. (28), Eq. (29) and Eq. (30), one should reorganize Eq. (24), Eq. (25) and Eq. (27) (see also Eq. (26)):

$$\begin{aligned} \Delta\varepsilon_{\alpha\beta} = & \left[{}^{t+\Delta t}\bar{\varepsilon}^{(p)} - {}^{t}\bar{\varepsilon}^{(p)} \right] \left. \frac{\partial \bar{\sigma}}{\partial \sigma_{\alpha\beta}} \right|_{\sigma_{\alpha\beta} = {}^{t+\Delta t}\sigma_{\alpha\beta}} + \\ & \frac{1}{2\mu}\left({}^{t+\Delta t}\sigma_{\alpha\beta} - \frac{\lambda}{3\lambda + 2\mu}\delta_{\alpha\beta}\, {}^{t+\Delta t}\sigma_{\gamma\gamma} \right) - {}^{t}\varepsilon_{\alpha\beta}^{(e)}, \qquad \text{for } \alpha \le \beta, \end{aligned} \tag{31}$$

$${}^{t+\Delta t}\sigma_{\alpha 3} = {}^{t}\sigma_{\alpha 3} + 2\mu\, \Delta\varepsilon_{\alpha 3}, \tag{32}$$

$$\Delta\varepsilon_{33} = -\left[{}^{t+\Delta t}\bar{\varepsilon}^{(p)} - {}^{t}\bar{\varepsilon}^{(p)} \right] \left. \frac{\partial \bar{\sigma}}{\partial \sigma_{\gamma\gamma}} \right|_{\sigma_{\alpha\beta} = {}^{t+\Delta t}\sigma_{\alpha\beta}} - \frac{\lambda}{2\mu\left(3\lambda + 2\mu\right)}\, {}^{t+\Delta t}\sigma_{\gamma\gamma} - {}^{t}\varepsilon_{33}^{(e)}, \tag{33}$$

and

$$\left. \bar{\sigma} \right|_{\sigma_{\alpha\beta} = {}^{t+\Delta t}\sigma_{\alpha\beta}} - Y\left[{}^{t+\Delta t}\bar{\varepsilon}^{(p)} \right] = 0. \tag{34}$$

One may notice by examining Eq. (31), Eq. (32), Eq. (33), and (34) that $\Delta\varepsilon_{33}$, ${}^{t+\Delta t}\sigma_{33} = 0$ and ${}^{t+\Delta t}\sigma_{\alpha 3} = {}^{t+\Delta t}\sigma_{3\alpha}$ can be removed from the set of primary unknowns which now reduces to ${}^{t+\Delta t}\sigma_{\alpha\beta}\left(\alpha \le \beta\right)$ and ${}^{t+\Delta t}\bar{\varepsilon}^{(p)}$. The nonlinear system consisting of Eq. (31) and Eq. (34) must be solved to determine the quantities ${}^{t+\Delta t}\sigma_{\alpha\beta}\left(\alpha \le \beta\right)$ and ${}^{t+\Delta t}\bar{\varepsilon}^{(p)}$. The numerical procedure described previously is still usable for this purpose. Having ${}^{t+\Delta t}\sigma_{\alpha\beta} = {}^{t+\Delta t}\sigma_{\beta\alpha}\left(\alpha \le \beta\right)$ and ${}^{t+\Delta t}\overline{\varepsilon}^{(p)}$ determined, Eq. (33) allows the calculation of the strain increment $\Delta\varepsilon_{33}$, while Eq. (32) evaluates the shear stresses ${}^{t+\Delta t}\sigma_{\alpha 3} = {}^{t+\Delta t}\,\sigma_{3\alpha}$. In general, Eq. (32) overestimates the shear stiffness of shell elements. A formula that performs better is

$$ {}^{t+\Delta t}\sigma_{\alpha 3} = {}^{t}\sigma_{\alpha 3} + 2\kappa\mu\,\Delta\varepsilon_{\alpha 3}, \tag{35} $$

with $\kappa = 5/6$ acting as a correction factor.

A particular form of the constitutive theory presented above has been implemented as a VUMAT subroutine of the Abaqus/Explicit finite-element programme [10]. The particularization is individualized by the following characteristics of the plasticity model:

- BBC2005 formulation of the effective stress [2]
- Analytical description of the hardening law.

The BBC2005 effective stress is expressible in the form [2]

$$ \overline{\sigma} = \left[a\left(\Lambda + \Gamma\right)^{2k} + a\left(\Lambda - \Gamma\right)^{2k} + b\left(\Lambda + \Psi\right)^{2k} + b\left(\Lambda - \Psi\right)^{2k} \right]^{\frac{1}{2k}}, \tag{36} $$

with

$$ \begin{aligned} \Gamma &= L\sigma_{11} + M\sigma_{22}, \\ \Lambda &= \sqrt{\left(N\sigma_{11} - P\sigma_{22}\right)^2 + \sigma_{12}\sigma_{21}}, \\ \Psi &= \sqrt{\left(Q\sigma_{11} - R\sigma_{22}\right)^2 + \sigma_{12}\sigma_{21}}. \end{aligned} \tag{37} $$

The symbols $k \in \mathbb{N}_{>0}$, $a \in \mathbb{R}_{>0}$, $b \in \mathbb{R}_{>0}$, $L \in \mathbb{R}$, $M \in \mathbb{R}$, $N \in \mathbb{R}$, $P \in \mathbb{R}$, $Q \in \mathbb{R}$, and $R \in \mathbb{R}$ denote constant parameters. The $k-$exponent is always set in accordance with the material structure, the following values being suitable for most practical applications: $k = 3$ in the case of BCC sheet metals, and $k = 4$ in the case of FCC sheet metals. As for a, b, L, M, N, P, Q, and R, these parameters result by identification with experimental data [2]. The partial derivatives $\partial\overline{\sigma}/\partial\sigma_{\alpha\beta}$ involved in Eq. (31) and Eq. (33) can be evaluated using the chain rule (see Eq. (36) and Eq. (37))

$$ \frac{\partial\overline{\sigma}}{\partial\sigma_{\alpha\beta}} = \frac{\partial\overline{\sigma}}{\partial\Gamma}\frac{\partial\Gamma}{\partial\sigma_{\alpha\beta}} + \frac{\partial\overline{\sigma}}{\partial\Lambda}\frac{\partial\Lambda}{\partial\sigma_{\alpha\beta}} + \frac{\partial\overline{\sigma}}{\partial\Psi}\frac{\partial\Psi}{\partial\sigma_{\alpha\beta}}, \tag{38} $$

with

$$
\begin{aligned}
&\frac{\partial \bar{\sigma}}{\partial \Gamma} = \frac{1}{\bar{\sigma}^{2k-1}}\left[a(\Lambda+\Gamma)^{2k-1} - a(\Lambda-\Gamma)^{2k-1}\right],\\
&\frac{\partial \bar{\sigma}}{\partial \Lambda} = \frac{1}{\bar{\sigma}^{2k-1}}\left[a(\Lambda+\Gamma)^{2k-1} + a(\Lambda-\Gamma)^{2k-1} + b(\Lambda+\Psi)^{2k-1} + b(\Lambda-\Psi)^{2k-1}\right],\\
&\frac{\partial \bar{\sigma}}{\partial \Psi} = \frac{1}{\bar{\sigma}^{2k-1}}\left[b(\Lambda+\Psi)^{2k-1} - b(\Lambda-\Psi)^{2k-1}\right],
\end{aligned}
\tag{39}
$$

and

$$
\begin{aligned}
&\frac{\partial \Gamma}{\partial \sigma_{11}} = L, \quad \frac{\partial \Gamma}{\partial \sigma_{22}} = M, \quad \frac{\partial \Gamma}{\partial \sigma_{12}} = 0, \quad \frac{\partial \Gamma}{\partial \sigma_{21}} = 0,\\
&\frac{\partial \Lambda}{\partial \sigma_{11}} = \frac{N(N\sigma_{11} - P\sigma_{22})}{\Lambda}, \quad \frac{\partial \Lambda}{\partial \sigma_{22}} = -\frac{P(N\sigma_{11} - P\sigma_{22})}{\Lambda},\\
&\frac{\partial \Lambda}{\partial \sigma_{12}} = \frac{\sigma_{21}}{2\Lambda}, \quad \frac{\partial \Lambda}{\partial \sigma_{21}} = \frac{\sigma_{12}}{2\Lambda},\\
&\frac{\partial \Psi}{\partial \sigma_{11}} = \frac{Q(Q\sigma_{11} - R\sigma_{22})}{\Psi}, \quad \frac{\partial \Psi}{\partial \sigma_{22}} = -\frac{R(Q\sigma_{11} - R\sigma_{22})}{\Psi},\\
&\frac{\partial \Psi}{\partial \sigma_{12}} = \frac{\sigma_{21}}{2\Psi}, \quad \frac{\partial \Psi}{\partial \sigma_{21}} = \frac{\sigma_{12}}{2\Psi}.
\end{aligned}
\tag{40}
$$

Two analytical descriptions of the hardening law are currently implemented in the VUMAT subroutine:

$$
Y\left[\bar{\varepsilon}^{(p)}\right] = \begin{cases} K\left[\varepsilon_0 + \bar{\varepsilon}^{(p)}\right]^n & \text{swift,} \\ A - B\exp\left[-C\,\bar{\varepsilon}^{(p)}\right] + D\,\bar{\varepsilon}^{(p)} & \text{voce (modified).} \end{cases}
\tag{41}
$$

The symbols K, ε_0, n, A, B, C, and D in Eq. (41) denote constant parameters determined by fitting experimental data.

4. CONCLUSIONS

This paper describes the Abaqus/Explicit implementation of a constitutive model for thin sheet metals subjected to forming procedures dominated by stretching and bending effects. Thin metallic sheets are assumed to behave as orthotropic elastoplastic shells. The mechanical response of such bodies is described by Hooke's law combined with a plastic potential and the associated flow rule. A set of nonlinear equations is obtained by rewriting the constitutive relationships in an incremental form. The convergence and stability of the numerical procedure used to solve the set of nonlinear equations is ensured by means of a subincrementation technique. The constitutive model is particularized using the BBC2005 effective stress combined with two analytically defined hardening laws (Swift and modified Voce formulations). The performances of the constitutive model will be analysed in the second part of this paper by comparing the theoretical predictions with experimental data obtained from deep-drawing tests.

Received on August 29, 2019

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