CHARACTERIZATION OF ANISOTROPIC MATERIALS USING A NON-DESTRUCTIVE FREE VIBRATION METHOD

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Abstract. The outcome of a proposed analysis procedure aimed to determine the elastic properties of fiber reinforced polymer (FRP) profiles is presented and discussed in the current paper. An experimental modal analysis was used to obtain the resonance frequencies and mode shapes of a single glass FRP profile, and a similar finite element model simulation was created to estimate natural frequencies and mode shapes numerically. Corresponding frequencies were allocated using the modal assurance criterion. By comparing results obtained by different methods, it is proved that a non-destructive technique based on the free vibration response can be successfully used to evaluate the elastic properties of fiber reinforced polymers.

Key words: Composite, FRP, Vibrations, Dynamic analysis, Genetic algorithm, Modal identification.

1. INTRODUCTION

Generally, the elastic behavior of a structural element is evaluated experimentally by static test methods like flexural, tensile, compressive and shear tests. Nonetheless, these methods require close to ideal conditions, take up a significant amount of time due to the number of specimens needed to be prepared and tested, are costly because of the nature of composite materials, and rely on simplification hypotheses. The static tests may also include uncertainties such as anisotropic coupling effects, boundary conditions and material heterogeneities, among others [1].

In the past two decades a lot of effort has been made towards the evaluation of elastic properties of anisotropic materials using non-destructive techniques [2–7]. One of these techniques which mitigates part of the aforementioned drawbacks of the standard destructive methods is based on measuring the dynamic properties of specimens. The dynamic characteristics are determined by the geometry, boundary conditions, elastic constants and densities of the composing materials. Hence, by adopting an inverse approach, these properties can be used to estimate the elastic constants if the other parameters are assumed to be known [8–10]. Moreover, by using an iterative procedure, the engineering constants can be updated in a finite element model of the test specimens in such a way that the

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computed dynamic properties match the measured ones. Therefore, the purpose of this study was to compare the mechanical properties determined with the proposed non-destructive method with the values offered by the manufacturer and the ones obtained from static tests reported in [11] for a glass FRP profile, and to prove the feasibility of this method in characterizing real-scale composite members.

The employed characterization procedure consisted of an experimental modal analysis and a finite element modal analysis coupled with a parameter identification method based on a multiple objective genetic algorithm.

2. METHODOLOGY

Before discussing the procedure and results of the study, a brief description of the employed methods is necessary to be carried.

2.1. EXPERIMENTAL MODAL ANALYSIS

Experimental modal analysis is a method used to empirically estimate the dynamic properties of a linear, time-invariant structure, based on the relation between excitation and dynamic response. The procedure is also built on Maxwell's reciprocity theorem and on the assumption that the vibrational response of a linear, dynamic system can be expressed as a linear combination of simple, harmonic movements or normal modes [5]. Ideally, a vibrational normal mode of an oscillating structure is a pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a stable phase relation. The free motion described by normal modes takes place at fixed frequencies also known as natural or resonant frequencies. In addition, each normal mode has a modal damping value and a mode shape which defines the spatial deformation of the structure due to the resonance. Results and methodologies of various modal characterization tests performed on FRP beams, light structures and footbridges have been reported in [12–16]. Comparable analyses and methods have been published in [17], [18] regarding the characterization of composite material vibration-induced structural degradation.

In general, during an experimental modal analysis the structure is artificially exited using an instrument capable of registering the input signal while the response obtained is measured with a translational transducer. In the particular case of using a single impact hammer to induce vibrations and a single accelerometer to record the response, the position of the accelerometer may be fixed while the excitation is applied in various points across the discretized surface of the structure – method known as roving exciter test. Secondly, by using a Fast Fourier Transformation (FFT) analysis of the measurements, a response model of the physical structure may be recreated by calculating a spectrum in the form of

Frequency Response Functions (FRFs) from the time domain signal. Subsequently, the experimental modal parameters – natural frequencies, modal damping and modal shapes – can be estimated by curve-fitting a set of the registered FRFs. In this process, a mode indicator function is commonly adopted to help identify how many modes are contained in a frequency band of FRF data.

After the evaluation of the modal properties, a quality control check of the data is usually required. In this sense, the Modal Assurance Criterion (MAC) is useful to validate experimental modal models and to map a correlation matrix between analytical, experimental or numerical modal models. The criterion is defined as a scalar constant relating the degree of consistency (linearity) between one modal and another reference modal vector [19]. Normally, MAC values superior to 0.8 are found to be acceptable to establish a certain correspondence between two shapes [20]. It is noted that its reliability is highly dependent on the number of elements (i.e., measured degrees of freedom) in the modal vectors.

2.2. FINITE ELEMENT MODAL ANALYSIS

The second stage of the proposed non-destructive procedure consists in performing a finite element (FE) modal analysis in order to determine the vibrational characteristics of the specimens. The assumptions and restrictions accounted are that the structure is time-invariant and linear, and that there is no external force applied to the mass.

Initial material input, in the form of elastic constants and densities, is needed to carry out the FE modal analysis. Thus, the glass FRP (GFRP) profile is regarded in a simplified manner as a homogenous orthotropic linearly-elastic material with transverse isotropy. A composite member having transverse isotropy, such as the one illustrated in Fig. 1, has five independent elastic constants: longitudinal and transverse elastic modulus; in-plane longitudinal shear modulus; and two Poisson's ratios, as exemplified in Table 1. The rest of the material constants can be determined from the independent constants.

Besides material information, the FE modal analysis demands geometry and boundary conditions data that reflect the physical structural model. To obtain satisfactory results, conditions that are easy to simulate should be considered.

The results of the finite element analysis are in the form of eigenfrequencies and corresponding eigenvectors specific to each specimen investigated. To study the relation between the input and output values of the FE model, a sensitivity study may be performed that can determine which material properties have the most or the least impact on a specific set of dynamic characteristics. In this way, minor input parameters can be disabled to generate a more accurate and less expensive simulation, while the highest impact parameters can later be used in conjunction with the results of the experimental modal analysis to set the objectives and constraints of a parameter identification method that can lead to the numerical estimation of material properties.

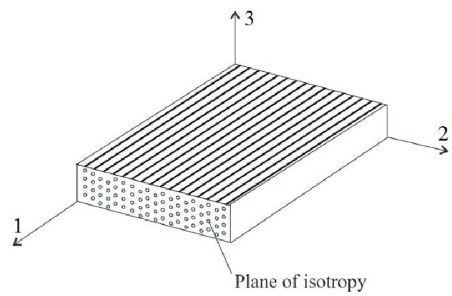

Fig. 1 – Fiber reinforced polymer with transverse isotropy.

Table 1

Elastic constants of orthotropic and transverse isotropic materials

Material	**Elastic constants**	
	Independent	Dependent
Orthotropic	E_1, E_2, E_3 G_{12}, G_{13}, G_{23} $\nu_{12}, \nu_{13}, \nu_{23}$	
Transverse isotropic	E_1, E_2 G_{12} ν_{12}, ν_{23}	$E_3 = E_2$ $G_{13} = G_{12}$ $G_{23} = E_2 / 2(1+\nu_{23})$ $\nu_{13} = \nu_{12}$

2.3. PARAMETER IDENTIFICATION METHOD

The parameter identification method that was found suitable for evaluating the material properties of the pultruded glass FRP profile is contained within the ANSYS software solution [21], [22]. The technique employs the Direct Optimization single-component system which utilizes real solvers instead of standard response surface evaluations. The optimization method preferred for this scenario was the Adaptive Multiple-Objective Genetic Algorithm (Adaptive MOGA), in which the “best” possible designs candidates are obtained from a sample set, given a list of specified objectives and constraints. It represents a hybrid optimization method that combines a Latin Hypercube Sampling (LHS) method, a Kriging error predictor to reduce the number of evaluations needed to locate the global optimum, and the MOGA algorithm where objectives can be weighted in terms of importance.

In detail, the influence of an input to an output parameter is determined from their correlation. The samples used for the parameter correlation study and optimization method were obtained using the Latin Hypercube Sampling, a statistical method for generating a set of plausible collections of parameter values from a multidimensional distribution. The LHS tries to locate the sampling points such that the space of random input parameters is explored in the most efficient way or acquire the necessary information with a number of minimum sampling points. The presence of points in efficient locations reduces the number of sample points required and increases the accuracy of the results.

The employed genetic algorithm uses a Kriging response surface that allows for a more rapid optimization process because it does not evaluate all design points, except when necessary, and because part of the sample population is simulated by evaluations. It is an accurate multidimensional interpolation combining a polynomial model which provides a global model of the design space and local deviations so that the model interpolates the design points.

To conclude, in the parameter identification method, the objectives are set so that the dynamic properties evaluated in the experimental modal analysis have to match the dynamic properties of the finite element model. In completion, constraints are added to define the variation boundaries for the material elastic constants so as to simplify the optimization process and improve its accuracy. The material input data is then generated and the resulting modal properties of the specimens are updated in an iterative procedure until the best solution is found for the problem.

3. EXPERIMENTAL AND NUMERICAL PROCEDURE

A two meters long glass fiber reinforced polymer profile was chosen for the non-destructive characterization tests. The experimental modal analysis of the composite profile was performed in both vertical and horizontal directions, on a number of three surfaces: top flange, bottom flange, and web. Due to the inherent low mass, the specimen was investigated under free boundary conditions.

Before the impact testing could commence, the surfaces of the profile which had to be studied were meshed. The element size of the mesh usually depends on the geometry of the specimen and the required spatial resolution of the modal vector. Thus, a fine mesh will provide better results but will increase the complexity of the experiment and resulting modal model. On the contrary, a less refined mesh may generate insufficient or poor data. Maximum longitudinal spacing between points was about 5% of total length. The GFRP computer model that was used to visualize the experimental modal results had a similar mesh.

Elastic vibrations were induced in the profile with the help of a small impact hammer with a metal tip, capable of recording signals in a frequency range up to 10 kHz. One of the key aspects in capturing as many vibrational modes as possible is fixing the accelerometer in a proper position and setting an appropriate frequency range for the analysis. Thus, for the GFRP profile, the transducer was placed on a point near one of the corners of each subsequent surface and the frequency range was established as 0–800 Hz.

For the experimental modal analysis, the time domain signals coming from the impact exciter and accelerometer were recorded using two data channels of a data acquisition system and converted to frequency spectrums (FRFs) within the accompanying software analyzer.

The last phase of the procedure consisted in determining the experimental dynamic properties of the specimens. To accomplish this objective, the data recorded during the tests was post-processed. Three-dimensional models of the tested specimens were recreated and meshed. The FRFs were then imported and assigned to each corresponding mesh point from the experiment. Modal parameters were estimated by curve-fitting the responses using the Complex Mode Indicator Function (CMIF) and a polynomial method. The CMIF was used to determine how many modes are contained in a frequency band of data by counting the resonance peaks above a threshold level. It is useful for finding closely coupled modes – two or more modes represented by a single resonance peak – and repeated roots – two or more modes at the same frequency but with different mode shapes. In addition, it estimates parameters more accurately from each reference measurement.

The dynamic parameters of the tested specimens estimated in the experimental modal analysis served as seek targets for the objectives of the numerical material parameter identification method.

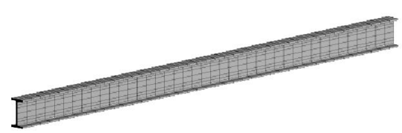

Fig. 2 – Meshed finite element model of the glass FRP profile.

Finite element models made of solid elements were built for the GFRP profile following the nominal fabrication dimensions. To mimic the modal tests, the GFRP model seen in Fig. 2 had simulated free boundary conditions. For the composite profile, the five independent elastic properties of the orthotropic material were assigned as input parameters with initial values, while the remaining four dependent elastic constants were expressed as parameters which derive from the former ones.

The output parameters defined for the profile's finite element model were the first three modal frequencies attributed to the longitudinal bending modes, transverse bending modes and torsional modes.

A sensitivity study made between the input and output parameters of the composite shape evaluated which elastic constants have the most or the least impact on eigenfrequencies, and in this way the minor contributing factors could be eliminated when building the optimization method, by treating them as deterministic parameters.

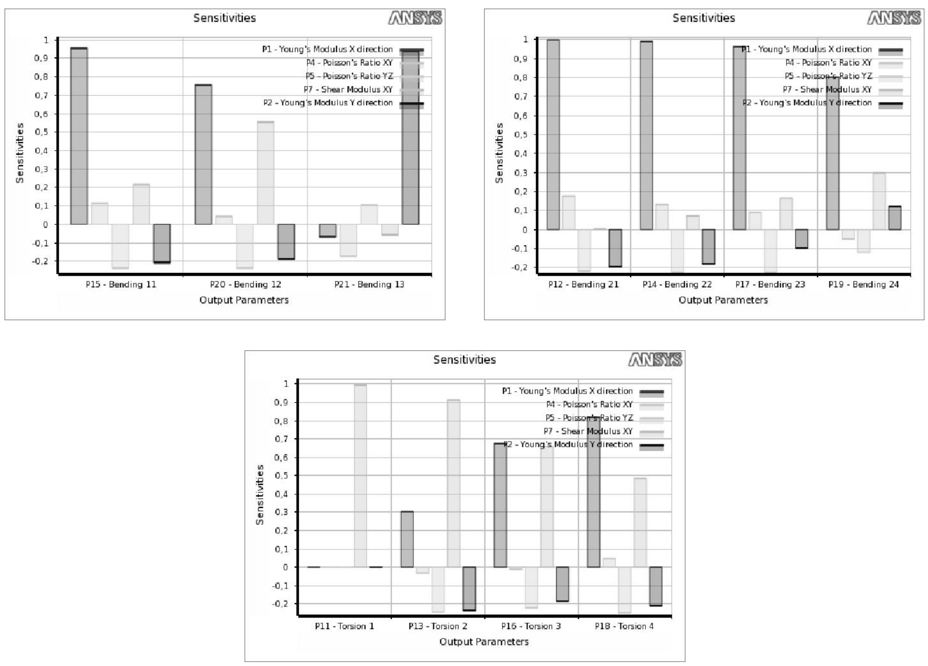

Fig. 3 – Sensitivity analysis for elastic constants in normal bending modes, transverse bending modes and torsional bending modes.

The numbers shown in the charts reflect the sensitivity of frequencies to material properties, where a positive sensitivity occurs when increasing the input leads to an increased output and where a negative sensitivity is computed when increasing the input decreases the output. The statistical sensitivities are based on Spearman's rank order correlation coefficients that simultaneously consider the amount by which an output parameter varies across the variation range of an input parameter and the variation range of an input parameter – the wider the range, the larger the impact.

The results reflect the fact that the first longitudinal and transverse bending modes (b_{11} and b_{21}) are dominated by the influence of the longitudinal modulus of elasticity of the profile, and that the first torsional mode (t_1) is heavily influenced by the in-plane shear modulus. The remaining correlation analyses indicate that for higher order vibrational modes ($b_{12}, b_{13}, b_{22}, b_{23}, t_2, t_3$), the modal frequencies start to be sensitive to multiple elastic properties.

Once the input parameters were established, the characterization procedure continued with the hybrid optimization method, based on the Adaptive Multiple-Objective Genetic Algorithm. For the pultruded composite profile, the objectives set were that the eigenfrequencies of the first three bending longitudinal and transversal modes as well as torsional modes, determined with the Finite Element Analysis (FEA), seek the corresponding natural frequencies of the empirical modes estimated in the experimental modal analysis, as illustrated in Fig. 4.

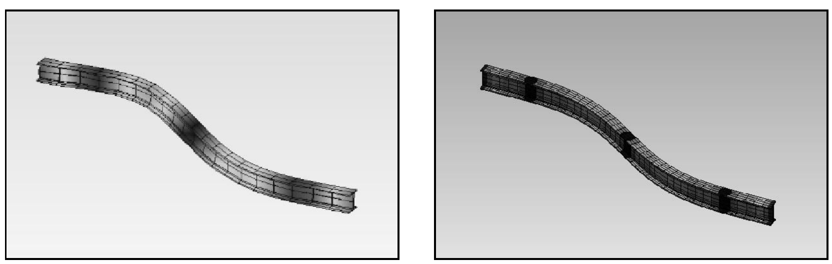

Fig. 4 – Example of matched experimental and numerical bending modes.

In accordance with the sensitivity study, the matching objectives covering the first mode frequencies had a higher importance set in the optimization process. Lower and upper bounds were set for the variation of the elastic properties of the glass fiber-reinforced plastic material by gathering possible interval values from literature and design guide manuals [23–25].

As mentioned before, the modal vectors to be matched during the parameter identification method were checked using the Modal Assurance Criterion (MAC). With the formulation in cause, a correlation matrix between the experimental and numerical vectors was built for the composite profile, as seen in Fig. 5. The bottom values of the experimental mode and eigenvector/numerical mode axis indicate the frequency order number of the mode being compared, and the vertical axis points to the MAC value obtained.

The displayed chart shows a major diagonal distribution demonstrating that the compared modal shapes were similar and correctly identified. The chart bars indicate high coherence levels for the GFRP profile.

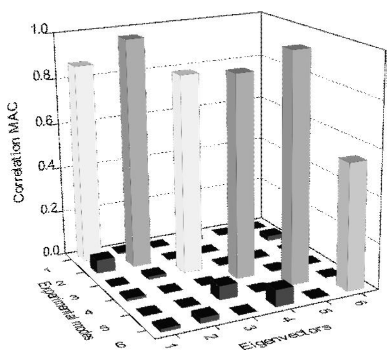

Fig. 5 – Modal Assurance Criterion matrix for the investigated composite member.

4. RESULTS AND DISCUSSION

After running the optimization procedure, the best material data candidates were found for the glass fiber reinforced polymer profile. The modal frequencies obtained from the experimental modal analysis and numerical optimization problem are summarized for the composite element in Table 2.

Table 2

Experimental and numerical finite element analysis modal frequency results

Mode ID*	f_{EXP}	f_{FEA}	diff.
	(Hz)	(Hz)	(%)
b_{11}	176	179	+1.6
b_{12}	430	429	-0.3
b_{13}	720	717	-0.4
b_{21}	53	53	-0.5
b_{22}	140	143	+2.0
b_{23}	265	270	+2.1
t_1	48	41	-13.9
t_2	110	104	-5.8
t_3	212	207	-2.4

The percentile differences computed for the results of the profile show that for the longitudinal and transverse bending modes, the natural frequencies were very close, within a 2% limit. On the other hand, the error between the numerical and experimental values for torsional vibrational modes was negative, with the FEA model exhibiting less torsional stiffness.

A comparative chart of the experimentally and numerically estimated frequencies, depicted in Fig. 6, reveals that the optimization FE method does not prefer stiffer of more flexible designs, as the data markers are dispersed evenly along the spectrum's diagonal. More so, the optimization procedure is able to generate a material data candidate that can also satisfy less important objectives such as seeking to match higher order mode characteristics.

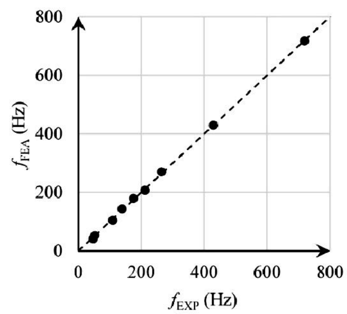

Fig. 6 – Comparative charts of the experimentally and numerically estimated natural frequencies.

Finally, as the aim of the proposed method was to characterize in a non-destructive manner the elastic properties of the glass fiber reinforced polymer, a comparison is made between the properties estimated following the numerical parameter identification method and the analogous values offered by the manufacturer and laboratory static tests [11]. Table 3 includes in the last couple of columns the percentile differences between the results of the numerical analyses and the other two sources. The computed differences are mostly positive.

Table 3

Comparison between the estimated elastic properties using the proposed non-destructive method and the properties offered by the manufacturer or obtained from the destructive static tests

Elastic property	Data obtained from			diff.manuf.	diff.static
	Manufacturer	Static tests	Non-destructive tests	(%)	(%)
E_1 (GPa)	41.40	39.11	42.45	+2.5	+8.5
E_2 (GPa)	-	10.77	10.80	-	+0.3
G_{12} (GPa)	-	3.98	4.47	-	+12.3
ν_{12}	-	0.27	0.28	-	+2.5
ν_{23}	-	-	0.33	-	-
G_{23} (GPa)	-	-	4.07	-	-
ν_{21}	-	0.07	0.07	-	-0.1

The proposed method is thus able to estimate the complete set of elastic constants of the anisotropic material within satisfactory error and time limits. As observed, the elastic properties of the GFRP profile stipulated by the manufacturer are clearly insufficient for analytic or numeric calculations, whereas the data gathered from the static tests, though sufficient, requires a great deal of preparation tasks and experimental trials.

5. CONCLUSIONS

In the present paper, a non-destructive characterization method was developed to obtain in a fast and reliable manner the mechanical elastic properties of anisotropic composite materials. The subsequent remarks are reported regarding the procedure's methodology and results:

- The proposed non-destructive method is based on the analysis of the free vibration response of fiber reinforced polymer specimens, and combines the results of an experimental and numerical modal analysis within an adaptive parameter identification method.
- The adaptive method consisted of an iterative procedure during which the elastic constants of the materials were sampled between set intervals and the dynamic properties of the specimens were updated so that the established multiple objectives and constraints could be satisfied with the use of a genetic algorithm. After a prior sensitivity study, the objectives that sought to equal the first dominant mode shape frequencies were ranked as more important within the algorithm.
- For the glass FRP profile, the first bending and torsional mode frequencies were fit with very good precision. Computed frequency errors for the element were in the range of 2% for the bending mode shapes and slightly higher for the torsional modes.
- The method proved to be a viable alternative to characterizing the elastic constants of FRP materials by means of static tests. The obtained mechanical properties resembled the previously determined laboratory values, with a maximum difference of 12% for the in-plane shear modulus. It is also noted that the proposed method has a minor tendency of overestimating results.
- Work is still needed on this topic but the initial findings and observations have demonstrated that the global elastic behavior of anisotropic materials can be accurately described in a short amount of time using a non-destructive technique.

Finally, it must be stated that the projected method requires precise dynamic measurements for a correct evaluation. At the same time, a large number of input parameters will demand high computational resources.

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